Integral Calculus JEE Main PYQ - 3

Total Time: 25 Minute **Total Marks:** 40

Instructions

Instructions

1. Test will auto submit when the Time is up.
2. The Test comprises of multiple choice questions (MCQ) with one or more correct answers.
3. The clock in the top right corner will display the remaining time available for you to complete the examination.

Navigating & Answering a Question

1. The answer will be saved automatically upon clicking on an option amongst the given choices of answer.
2. To deselect your chosen answer, click on the clear response button.
3. The marking scheme will be displayed for each question on the top right corner of the test window.

Integral Calculus

1. The value of $\frac{8}{\pi}\int_{0}^{\frac{\pi}{2}}\frac{(\cos x)^{2023}}{(\sin x)^{2023}+(\cos x)^{2023}}dx$ is **(+4, -1)** [27-Jan-2024 Shift 2]

---

2. Let $f(x)=\begin{vmatrix}1+\sin^2 x & \cos^2 x & \sin 2x\\ \sin^2 x & 1+\cos^2 x & \sin 2x\\ \sin^2 x & \cos^2 x & 1+\sin 2x\end{vmatrix}, x\in\left[\frac{\pi}{6},\frac{\pi}{3}\right]$ If α and β respectively are the maximum and the minimum values of f, then **(+4, -1)** [17 Mar 2021 Shift 2]

a. $\beta^2+2\sqrt{\alpha}=\frac{19}{4}$

b. $\alpha^2+\beta^2=\frac{9}{2}$

c. $\alpha^2-\beta^2=4\sqrt{3}$

d. $\beta^2-2\sqrt{\alpha}=\frac{19}{4}$

---

3. The integral $\int_0^{\infty}\frac{6}{(e^{3x}+6e^{2x}+11ex+6)}dx$ **(+4, -1)** [25 Jul 2021 Shift 1]

a. In 32

b. In 27

c. In $\frac{32}{27}$

d. In $\frac{27}{32}$

---

4. Let $l_n=\int\tan^n x\,dx,(n>1).l_4+l_6=a\ \tan^5\ x+bx^5+C$, where C is a constant of integration, then the ordered pair (a,b) is equal to : **(+4, -1)** [20 Jul 2021 Shift 1]

a. $\left(\frac{1}{5},0\right)$

b. $\left(\frac{1}{5},-1\right)$

c. $\left(-\frac{1}{5},0\right)$

d. $\left(-\frac{1}{5},1\right)$

---

5. The integral $\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{dx}{1+\cos x}$ is equal to : [2021] **(+4, -1)**

a. 2

b. 4

c. -1

d. -2

---

6. If $f(x) = \frac{2-x\cos x}{2+x\cos x}$ and $g(x) = \log_e x., (x > 0)$ then the value of integral $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} g(f(x))\,dx$ is : [27 Aug 2021 Shift 1] **(+4, -1)**

a. $\log_e 3$

b. $\log_e 2$

c. $\log_e e$

d. $\log_e 1$

---

7. If f(x) = ∫x0 t(sin x-sin t)dt then [31-Jan-2024 Shift 2] **(+4, -1)**

a. $f'''(x) + f''(x) = \sin x$

b. $f'''(x) + f''(x) - f(x) = \cos x$

c. $f'''(x) + f'(x) = \cos x - 2x\sin x$

d. $f'''(x) - f''(x) = \cos x - 2x\sin x$

---

8. If $\int \frac{\sqrt{1-x^2}}{x^4}dx = A(x)\left(\sqrt{1-x^2}\right)^m + C$, for a suitable chosen integer m and a function $A(x)$, where C is a constant of integration then $(A(x))^m$ equals : [1-Feb-2024 Shift 2] **(+4, -1)**

a. $\frac{-1}{3x^3}$

b. $\frac{-1}{27x^9}$

c. $\frac{1}{9x^4}$

d. $\frac{1}{27x^6}$

---

9. If $\int \frac{dx}{x^3(1+x^6)^{\frac{2}{3}}} = f(x)\left(1+x^6\right)^{\frac{1}{3}} + C$, where C is a constant of integration, then the function $f(x)$ is equal to- **(+4, -1)**
[1-Feb-2024 Shift 1]

a. $-\frac{1}{6x^3}$

b. $\frac{3}{x^2}$

c. $-\frac{1}{2x^2}$

d. $-\frac{1}{2x^3}$

---

10. If the area of the region bounded by the curves $y^2 - 2y = -x, x + y = 0$ is A, then $8A$ is equal to __________ **(+4, -1)**
[29-Jul-2022-Shift-2]

Answers

1. **Answer: 2 - 2**

Explanation:

The correct answer is 2.

$I=\frac{8}{\pi}\int\limits_{0}^{\frac{\pi}{2}}\frac{(\cos x)^{2023}}{(\sin x)^{2023}+(\cos x)^{2023}}dx$......(1)

Using $\int\limits_{0}^{a}f(x)dx=\int\limits_{0}^{a}f(a-x)dx$

$I=\frac{8}{\pi}\int\limits_{0}^{\frac{\pi}{2}}\frac{(\sin x)^{2023}}{(\sin x)^{2023}+(\cos x)^{2023}}dx$........(2)

Adding (1) & (2)

$2I=\frac{8}{\pi}\int\limits_{0}^{\frac{\pi}{2}}1dx$

$I=2$

Concepts:

1. Integral:

The representation of the area of a region under a curve is called to be as integral. The actual value of an integral can be acquired (approximately) by drawing rectangles.

- The definite integral of a function can be shown as the area of the region bounded by its graph of the given function between two points in the line.
- The area of a region is found by splitting it into thin vertical rectangles and applying the lower and the upper limits, the area of the region is summarized.
- An integral of a function over an interval on which the integral is described.

Also, F(x) is known to be a Newton-Leibnitz integral or antiderivative or primitive of a function f(x) on an interval I.

F'(x) = f(x)

For every value of x = I.

Types of Integrals:

Integral calculus helps to resolve two major types of problems:

1. The problem of getting a function if its derivative is given.
2. The problem of getting the area bounded by the graph of a function under given situations.

---

2. Answer: d

Explanation:

$$C_1 \to C_1 + C_2 + C_3$$
$$f(x) = \begin{vmatrix} 2+\sin 2x & \cos^2 x & \sin 2x \\ 2+\sin 2x & 1+\cos^2 x & \sin 2x \\ 2+\sin 2x & \cos^2 x & 1+\sin 2x \end{vmatrix}$$
$$f(x) = (2+\sin 2x) \begin{vmatrix} 1 & \cos^2 x & \sin 2x \\ 1 & 1+\cos^2 x & \sin 2x \\ 1 & \cos^2 x & 1+\sin 2x \end{vmatrix}$$
$$R_2 \to R_2 - R_1$$
$$R_3 \to R_3 - R_1$$
$$f(x) = 2+\sin 2x) \begin{vmatrix} 1 & \cos^2 x & \sin 2x \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$
$$= (2+\sin 2x)(1) = 2+\sin 2x$$
$$= \sin 2x \in \left[\frac{\sqrt{3}}{2}, 1\right]$$
Hence $2+\sin 2x \in \left[2+\frac{\sqrt{3}}{2}, 3\right]$

Concepts:

1. Integral:

The representation of the area of a region under a curve is called to be as integra l. The actual value of an integral can be acquired (approximately) by drawing rectangles.

- The definite integral of a function can be shown as the area of the region bounded by its graph of the given function between two points in the line.
- The area of a region is found by splitting it into thin vertical rectangles and applying the lower and the upper limits, the area of the region is summarized.
- An integral of a function over an interval on which the integral is described.

Also, F(x) is known to be a Newton-Leibnitz integral or antiderivative or primitive of a function f(x) on an interval I.

F'(x) = f(x)

For every value of x = I.

Types of Integrals:

Integral calculus helps to resolve two major types of problems:

1. The problem of getting a function if its derivative is given.
2. The problem of getting the area bounded by the graph of a function under given situations.

---

3. Answer: c

Explanation:

The correct option is (C): ln $\frac{32}{27}$

$$I = \int_0^\infty \frac{6}{(e^x+1)(e^x+2)(e^x+3)}dx$$

$$= 6\int_0^\infty \left(\frac{\frac{1}{2}}{e^x+1} + \frac{-1}{e^x+2} + \frac{\frac{1}{2}}{e^x+3} \right)dx$$

$$= 3\int_0^\infty \frac{e^{-x}}{1+e^{-x}}dx - 6\int_0^\infty \frac{e^{-x}dx}{1+2e^{-x}} + 3\int_0^\infty \frac{e^{-x}}{1+3e^{-x}}dx$$

$$= 3\left[-\ln\left(1+e^{-x}\right)\right]_0^\infty + 6\frac{1}{2}\left[\ln\left(1+2e^{-x}\right)\right]_0^\infty$$

$$-\frac{3}{3}\left[\ln\left(1+3e^{-x}\right)\right]_0^\infty$$

$$= 3\ln 2 - 3\ln 3 + \ln 4$$

$$= 3\ln\frac{2}{3} + \ln 4$$

$$= \ln \frac{32}{27}$$

Concepts:

1. Integral:

The representation of the area of a region under a curve is called to be as integra l. The actual value of an integral can be acquired (approximately) by drawing rectangles.

- The definite integral of a function can be shown as the area of the region bounded by its graph of the given function between two points in the line.
- The area of a region is found by splitting it into thin vertical rectangles and applying the lower and the upper limits, the area of the region is summarized.
- An integral of a function over an interval on which the integral is described.

Also, F(x) is known to be a Newton-Leibnitz integral or antiderivative or primitive of a function f(x) on an interval I.

F'(x) = f(x)

For every value of x = I.

Types of Integrals:

Integral calculus helps to resolve two major types of problems:

1. The problem of getting a function if its derivative is given.
2. The problem of getting the area bounded by the graph of a function under given situations.

---

4. Answer: a

Explanation:

$$l_n = \int \tan^n x dx, n > 1$$
$$l_4 + l_6 = \int \left(\tan^4 x + \tan^6 x\right) dx$$

$= \int \tan^4 x \sec^2 x dx$

Let $\tan x = t$

$\sec^2 x dx = dt$

$= \int t^4 dt$

$= \frac{t^5}{5} + C$

$= \frac{1}{5} \tan^5 x + C$

$a = \frac{1}{5}, b = 0$

Concepts:

1. Integrals of Some Particular Functions:

There are many important integration formulas which are applied to integrate many other standard integrals. In this article, we will take a look at the integrals of these particular functions and see how they are used in several other standard integrals.

Integrals of Some Particular Functions:

- $\int 1/(x^2 – a^2)\, dx = (1/2a) \log|(x – a)/(x + a)| + C$
- $\int 1/(a^2 – x^2)\, dx = (1/2a) \log|(a + x)/(a – x)| + C$
- $\int 1/(x^2 + a^2)\, dx = (1/a) \tan^{-1}(x/a) + C$
- $\int 1/\sqrt{(x^2 – a^2)}\, dx = \log|x + \sqrt{(x^2 – a^2)}| + C$
- $\int 1/\sqrt{(a^2 – x^2)}\, dx = \sin^{-1}(x/a) + C$
- $\int 1/\sqrt{(x^2 + a^2)}\, dx = \log|x + \sqrt{(x^2 + a^2)}| + C$

These are tabulated below along with the meaning of each part.

S.No	Integral function	Integral value
1	$\int \frac{dx}{x^2-a^2}$	$\frac{1}{2a}\log\left\|\frac{x-a}{x+a}\right\| + C$
2	$\int \frac{dx}{a^2-x^2}$	$\frac{1}{2a}\log\left\|\frac{a+x}{a-x}\right\| + C$
3	$\int \frac{dx}{x^2+a^2}$	$\frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$
4	$\int \frac{dx}{\sqrt{x^2-a^2}}$	$\log\left\|x + \sqrt{x^2 - a^2}\right\| + C$
5	$\int \frac{dx}{\sqrt{a^2-x^2}}$	$\sin^{-1}\left(\frac{x}{a}\right) + C$
6	$\int \frac{dx}{\sqrt{x^2+a^2}}$	$\log\left\|x + \sqrt{x^2 + a^2}\right\| + C$

5. **Answer: a**

Explanation:

$$\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{dx}{2\cos^2\frac{x}{2}} dx = \frac{1}{2}\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \sec^2\frac{x}{2} dx$$

$$= \frac{1}{2}\left[\frac{\tan\frac{x}{2}}{\frac{1}{2}}\right]_{\frac{\pi}{4}}^{\frac{3\pi}{4}}$$

$$= \tan\frac{3\pi}{8} - \tan\frac{\pi}{8}$$

$$\left[\tan\frac{\pi}{8} = \sqrt{\frac{1-\cos\frac{\pi}{4}}{1+\cos\frac{\pi}{4}}} = \sqrt{\frac{\sqrt{2}-1}{\sqrt{2}+1}} = \frac{\sqrt{2}-1}{1} \tan\frac{3\pi}{8} = \sqrt{\frac{1-\cos\frac{3\pi}{4}}{1+\cos\frac{3\pi}{4}}} = \sqrt{\frac{\sqrt{2}+1}{\sqrt{2}-1}} = \sqrt{2}+1\right]$$

$$= (\sqrt{2}+1) - (\sqrt{2}-1)$$

$$= 2$$

Concepts:

1. **Integrals of Some Particular Functions:**

There are many important integration formulas which are applied to integrate many other standard integrals. In this article, we will take a look at the integrals of these particular functions and see how they are used in several other standard integrals.

Integrals of Some Particular Functions:

- ∫1/(x^2 – a^2) dx = (1/2a) log|(x – a)/(x + a)| + C
- ∫1/(a^2 – x^2) dx = (1/2a) log|(a + x)/(a – x)| + C
- ∫1/(x^2 + a^2) dx = (1/a) tan-1(x/a) + C
- ∫1/√(x^2 – a^2) dx = log|x + √(x^2 – a^2)| + C
- ∫1/√(a^2 – x^2) dx = sin-1(x/a) + C
- ∫1/√(x^2 + a^2) dx = log|x + √(x^2 + a^2)| + C

These are tabulated below along with the meaning of each part.

S.No	Integral function	Integral value
1	$\int \frac{dx}{x^2-a^2}$	$\frac{1}{2a}\log\left\lvert\frac{x-a}{x+a}\right\rvert + C$
2	$\int \frac{dx}{a^2-x^2}$	$\frac{1}{2a}\log\left\lvert\frac{a+x}{a-x}\right\rvert + C$
3	$\int \frac{dx}{x^2+a^2}$	$\frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$
4	$\int \frac{dx}{\sqrt{x^2-a^2}}$	$\log\left\lvert x + \sqrt{x^2-a^2}\right\rvert + C$
5	$\int \frac{dx}{\sqrt{a^2-x^2}}$	$\sin^{-1}\left(\frac{x}{a}\right) + C$
6	$\int \frac{dx}{\sqrt{x^2+a^2}}$	$\log\left\lvert x + \sqrt{x^2+a^2}\right\rvert + C$

---

6. **Answer: d**

Explanation:

$$g(f(x)) = \ell n(f(x)) = \ell n\left(\frac{2-x.\cos x}{2+x.\cos x}\right)$$
$$\therefore I = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \ell n\left(\frac{2-x.\cos x}{2+x\cos x}\right) dx$$
$$= \int_0^{\frac{\pi}{4}} \left(\ell n\left(\frac{2-x\cos x}{2+x.\cos x}\right) + \ell n\left(\frac{2+x.\cos x}{2-x.\cos x}\right)\right) dx$$
$$= \int_0^{\frac{\pi}{2}} (0)\, dx = 0 = \log_e(1)$$

Concepts:

1. **Integrals of Some Particular Functions:**

There are many important integration formulas which are applied to integrate many other standard integrals. In this article, we will take a look at the integrals of these particular functions and see how they are used in several other standard integrals.

Integrals of Some Particular Functions:

- $\int 1/(x^2 - a^2)\, dx = (1/2a) \log|(x - a)/(x + a)| + C$
- $\int 1/(a^2 - x^2)\, dx = (1/2a) \log|(a + x)/(a - x)| + C$
- $\int 1/(x^2 + a^2)\, dx = (1/a) \tan-1(x/a) + C$
- $\int 1/\sqrt{(x^2 - a^2)}\, dx = \log|x + \sqrt{(x^2 - a^2)}| + C$
- $\int 1/\sqrt{(a^2 - x^2)}\, dx = \sin-1(x/a) + C$
- $\int 1/\sqrt{(x^2 + a^2)}\, dx = \log|x + \sqrt{(x^2 + a^2)}| + C$

These are tabulated below along with the meaning of each part.

S.No	Integral function	Integral value
1	$\int \frac{dx}{x^2-a^2}$	$\frac{1}{2a}\log\left\|\frac{x-a}{x+a}\right\|+C$
2	$\int \frac{dx}{a^2-x^2}$	$\frac{1}{2a}\log\left\|\frac{a+x}{a-x}\right\|+C$
3	$\int \frac{dx}{x^2+a^2}$	$\frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+C$
4	$\int \frac{dx}{\sqrt{x^2-a^2}}$	$\log\left\|x+\sqrt{x^2-a^2}\right\|+C$
5	$\int \frac{dx}{\sqrt{a^2-x^2}}$	$\sin^{-1}\left(\frac{x}{a}\right)+C$
6	$\int \frac{dx}{\sqrt{x^2+a^2}}$	$\log\left\|x+\sqrt{x^2+a^2}\right\|+C$

---

7. Answer: c

Explanation:

$$f(x) = \int_0^x t(\sin x - \sin t)\,dt$$
$$f(x) = \sin x \int_0^x t\,dt - \int_0^x t\sin t\,dt$$
$$f'(x) = (\sin x)\,x + \cos x \int_0^x t\,dt - x\sin x$$
$$f'(x) = \cos x \int_0^x t\,dt = x\cos x$$
$$f''(x) = (\cos x)\,x - (\sin x)\int_0^x t\,dt$$
$$f'''(x) = x(-\sin x) + \cos x - (\sin x)\,x - (\cos x)\int_0^x t\,dt$$
$$f'''(x) + f'(x) = \cos x - 2x\sin x$$

Concepts:

1. Integrals of Some Particular Functions:

There are many important integration formulas which are applied to integrate many other standard integrals. In this article, we will take a look at the integrals of these particular functions and see how they are used in several other standard integrals.

Integrals of Some Particular Functions:

- $\int 1/(x^2 - a^2)\,dx = (1/2a)\log|(x - a)/(x + a)| + C$
- $\int 1/(a^2 - x^2)\,dx = (1/2a)\log|(a + x)/(a - x)| + C$
- $\int 1/(x^2 + a^2)\,dx = (1/a)\tan^{-1}(x/a) + C$
- $\int 1/\sqrt{(x^2 - a^2)}\,dx = \log|x + \sqrt{(x^2 - a^2)}| + C$
- $\int 1/\sqrt{(a^2 - x^2)}\,dx = \sin^{-1}(x/a) + C$

- ∫1/√(x^2 + a^2) dx = log|x + √(x^2 + a^2)| + C

These are tabulated below along with the meaning of each part.

S.No	Integral function	Integral value
1	$\int \frac{dx}{x^2-a^2}$	$\frac{1}{2a}\log\left\|\frac{x-a}{x+a}\right\|+C$
2	$\int \frac{dx}{a^2-x^2}$	$\frac{1}{2a}\log\left\|\frac{a+x}{a-x}\right\|+C$
3	$\int \frac{dx}{x^2+a^2}$	$\frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+C$
4	$\int \frac{dx}{\sqrt{x^2-a^2}}$	$\log\left\|x+\sqrt{x^2-a^2}\right\|+C$
5	$\int \frac{dx}{\sqrt{a^2-x^2}}$	$\sin^{-1}\left(\frac{x}{a}\right)+C$
6	$\int \frac{dx}{\sqrt{x^2+a^2}}$	$\log\left\|x+\sqrt{x^2+a^2}\right\|+C$

---

8. Answer: b

Explanation:

$\int \frac{\sqrt{1-x^2}}{x^4}dx = A(x)\left(\sqrt{1-x^2}\right)^m + C$

$\int \frac{|x|\sqrt{\frac{1}{x^2}-1}}{x^4}dx$

Put $\frac{1}{x^2}-1=t \Rightarrow \frac{dt}{dx}=\frac{-2}{x^3}$

Case-1 x $\geq$ 0

$-\frac{1}{2}\int\sqrt{t}dt \Rightarrow -\frac{t^{3/2}}{3}+C$

$\Rightarrow -\frac{1}{3}\left(\frac{1}{x^2}-1\right)^{3/2}$

$\Rightarrow \frac{\left(\sqrt{1-x^2}\right)^3}{-3x^2}+C$

$A(x)=-\frac{1}{3x^3}$

$(A(x))^m=\left(-\frac{1}{3x^3}\right)^3=-\frac{1}{27x^9}$

Case-II x $\leq$ 0

We get $\frac{\left(\sqrt{1-x^2}\right)^3}{-3x^3}+C$

$A(x)=\frac{1}{-3x^3}, m=3$

$(A(x))^m=\frac{-1}{27x^9}$

Concepts:

1. Integrals of Some Particular Functions:

There are many important integration formulas which are applied to integrate many other standard integrals. In this article, we will take a look at the integrals of these particular functions and see how they are used in several other standard integrals.

Integrals of Some Particular Functions:

- ∫1/(x^2 – a^2) dx = (1/2a) log|(x – a)/(x + a)| + C
- ∫1/(a^2 – x^2) dx = (1/2a) log|(a + x)/(a – x)| + C
- ∫1/(x^2 + a^2) dx = (1/a) tan-1(x/a) + C
- ∫1/√(x^2 – a^2) dx = log|x + √(x^2 – a^2)| + C
- ∫1/√(a^2 – x^2) dx = sin-1(x/a) + C
- ∫1/√(x^2 + a^2) dx = log|x + √(x^2 + a^2)| + C

These are tabulated below along with the meaning of each part.

S.No	Integral function	Integral value
1	$\int \frac{dx}{x^2-a^2}$	$\frac{1}{2a}\log\left\|\frac{x-a}{x+a}\right\|+C$
2	$\int \frac{dx}{a^2-x^2}$	$\frac{1}{2a}\log\left\|\frac{a+x}{a-x}\right\|+C$
3	$\int \frac{dx}{x^2+a^2}$	$\frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+C$
4	$\int \frac{dx}{\sqrt{x^2-a^2}}$	$\log\left\|x+\sqrt{x^2-a^2}\right\|+C$
5	$\int \frac{dx}{\sqrt{a^2-x^2}}$	$\sin^{-1}\left(\frac{x}{a}\right)+C$
6	$\int \frac{dx}{\sqrt{x^2+a^2}}$	$\log\left\|x+\sqrt{x^2+a^2}\right\|+C$

---

9. Answer: d

Explanation:

$\int \frac{dx}{x^3(1+x^6)^{\frac{2}{3}}} = xf(x)\left(1+x^6\right)^{\frac{1}{3}}+c$

$\int \frac{dx}{x^7\left(\frac{1}{x^6}+1\right)^{\frac{2}{3}}} = xf(x)\left(1+x^6\right)^{\frac{1}{3}}+c$

Let $t=\frac{1}{x^6}+1$

$dt=\frac{-6}{x^7}dx$

$=-\frac{1}{6}\int\frac{dt}{t^{\frac{2}{3}}}=-\frac{1}{2}t^{\frac{1}{3}}$

$=-\frac{1}{2}\left(\frac{1}{x^6}+1\right)^{\frac{1}{3}}=-\frac{1}{2}\frac{\left(1+x^6\right)^{\frac{1}{3}}}{x^2}$

$\therefore f(x)=-\frac{1}{2x^3}$

Concepts:

1. Integrals of Some Particular Functions:

There are many important integration formulas which are applied to integrate many other standard integrals. In this article, we will take a look at the integrals of these particular functions and see how they are used in several other standard integrals.

Integrals of Some Particular Functions:

- $\int 1/(x^2 - a^2)\, dx = (1/2a) \log|(x - a)/(x + a)| + C$
- $\int 1/(a^2 - x^2)\, dx = (1/2a) \log|(a + x)/(a - x)| + C$
- $\int 1/(x^2 + a^2)\, dx = (1/a) \tan{-1}(x/a) + C$
- $\int 1/\sqrt{(x^2 - a^2)}\, dx = \log|x + \sqrt{(x^2 - a^2)}| + C$
- $\int 1/\sqrt{(a^2 - x^2)}\, dx = \sin{-1}(x/a) + C$
- $\int 1/\sqrt{(x^2 + a^2)}\, dx = \log|x + \sqrt{(x^2 + a^2)}| + C$

These are tabulated below along with the meaning of each part.

S.No	Integral function	Integral value
1	$\int \frac{dx}{x^2-a^2}$	$\frac{1}{2a}\log\left\|\frac{x-a}{x+a}\right\|+C$
2	$\int \frac{dx}{a^2-x^2}$	$\frac{1}{2a}\log\left\|\frac{a+x}{a-x}\right\|+C$
3	$\int \frac{dx}{x^2+a^2}$	$\frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+C$
4	$\int \frac{dx}{\sqrt{x^2-a^2}}$	$\log\left\|x+\sqrt{x^2-a^2}\right\|+C$
5	$\int \frac{dx}{\sqrt{a^2-x^2}}$	$\sin^{-1}\left(\frac{x}{a}\right)+C$
6	$\int \frac{dx}{\sqrt{x^2+a^2}}$	$\log\left\|x+\sqrt{x^2+a^2}\right\|+C$

---

10. Answer: 36 - 36

Explanation:

The correct answer is 36

$y^2 - 2y = -x$

$\Rightarrow y^2 - 2y + 1 = -x + 1$

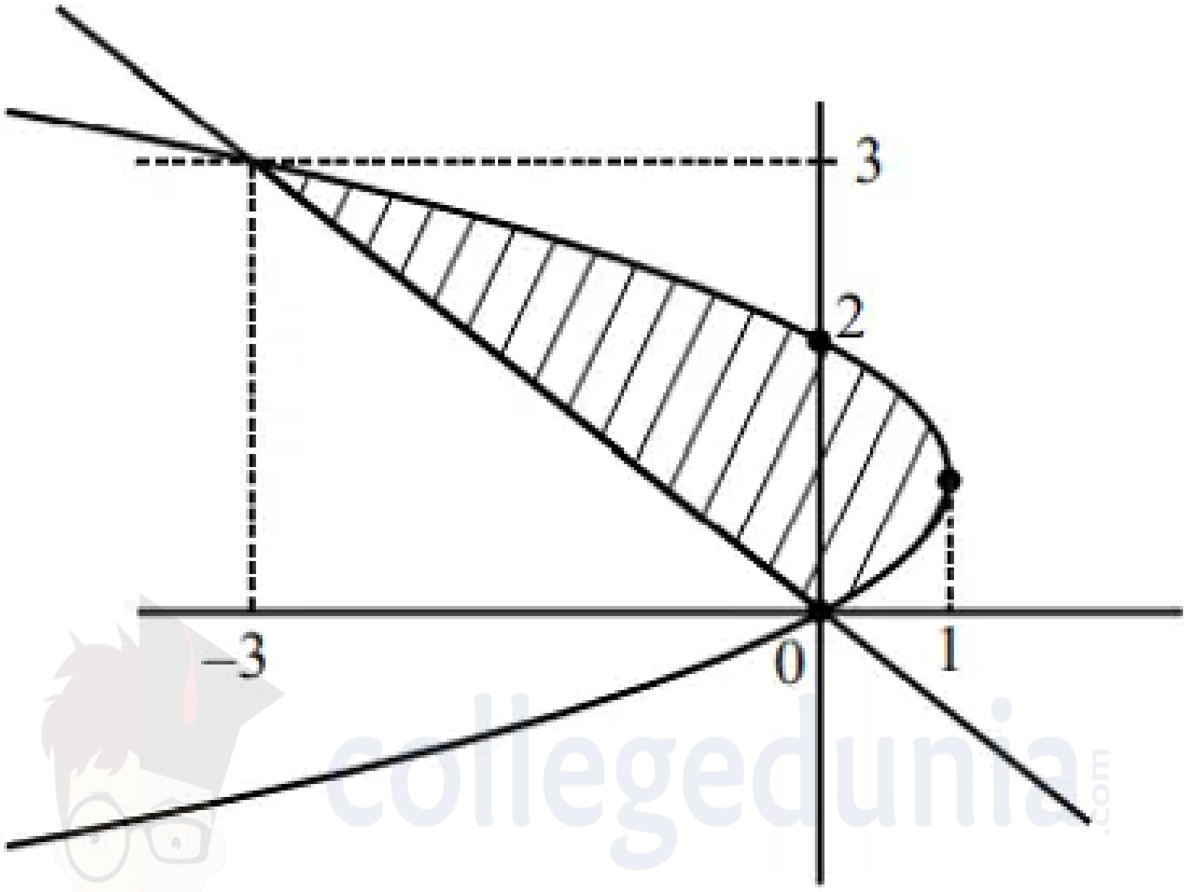
$(y-1)^2 = -(x-1)$

$y = -x$

Points of intersection

$x^2 + 2x = -x$

$x^2 + 3x = 0$
$x = 0, -3$



$A = \int_0^3 (-y^2 + 2y + y)\, dy$
$= \frac{3y^2}{2} - \frac{y^3}{3}\Big|_0^3 = \frac{9}{2}$
$8A = 36$

Concepts:

1. Applications of Integrals:

There are distinct applications of integrals, out of which some are as follows:

In Maths

Integrals are used to find:

- The center of mass (centroid) of an area having curved sides
- The area between two curves and the area under a curve
- The curve's average value

In Physics

Integrals are used to find:

- Centre of gravity
- Mass and momentum of inertia of vehicles, satellites, and a tower
- The center of mass
- The velocity and the trajectory of a satellite at the time of placing it in orbit
- Thrust