Integral Calculus JEE Main PYQ - 1

Total Time: 25 Minute **Total Marks:** 40

Instructions

Instructions

1. Test will auto submit when the Time is up.
2. The Test comprises of multiple choice questions (MCQ) with one or more correct answers.
3. The clock in the top right corner will display the remaining time available for you to complete the examination.

Navigating & Answering a Question

1. The answer will be saved automatically upon clicking on an option amongst the given choices of answer.
2. To deselect your chosen answer, click on the clear response button.
3. The marking scheme will be displayed for each question on the top right corner of the test window.

Integral Calculus

1. The area (in sq units) of the region described by $A = \{(x,y) : x^2 + y^2 \le 1$ and $y^2 \le 1 - x\}$ is: [27-Jan-2024 Shift 1] **(+4, -1)**

   a. $\frac{\pi}{2} + \frac{4}{3}$

   b. $\frac{\pi}{2} - \frac{4}{3}$

   c. $\frac{\pi}{2} - \frac{2}{3}$

   d. $\frac{\pi}{2} + \frac{2}{3}$

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2. The area of the region $A = [(x,y) : 0 \le y \le x|x| + 1$ and $-1 \le x \le 1]$ in s units is: [27-Jan-2024 Shift 2] **(+4, -1)**

   a. $\frac{2}{3}$

   b. $\frac{1}{3}$

   c. 2

   d. $\frac{4}{3}$

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3. The area (in s units) of the quadrilateral formed by the tangents at the end points of the latera recta to the ellipse $\frac{x^2}{9} + \frac{y^2}{5} = 1$, is: [Online May 19, 2012] **(+4, -1)**

   a. $\frac{27}{4}$

   b. 18

   c. $\frac{27}{2}$

   d. 27

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4. The area (in s units) of the region $\{(x,y) : x \ge 0, x + y \le 3, x^2 \le 4y$ and $y \le 1 + \sqrt{x}\}$ is [Online May 26, 2012] **(+4, -1)**

   a. $\frac{3}{2}$

   b. $\frac{7}{3}$

c. $\frac{5}{2}$

d. $\frac{59}{12}$

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5. For $a > 0$, let the curves $C_1 : y^2 = ax$ and $C_2 : x^2 = ay$ intersect at origin O and a point P. Let the line $x = b(0 < b < a)$ intersect the chord OP and the x-axis at points Q and R, respectively. If the line x = b bisects the area bounded by the curves, C_1 and C_2, and the area of $\Delta OQR = \frac{1}{2}$, then 'a' satisfies the equation : [24 Feb 2021 Shift 2] **(+4, -1)**

a. $x^6 - 12x^3 + 4 = 0$

b. $x^6 - 12x^3 - 4 = 0$

c. $x^6 + 6x^3 - 4 = 0$

d. $x^6 - 6x^3 + 4 = 0$

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6. If P is a point on the parabola $y = x^2 + 4$ which is closest to the straight line $y = 4x - 1$, then the co-ordinates of P are [18 Mar 2021 Shift 1] **(+4, -1)**

a. (3,13)

b. (1,5)

c. (-2,8)

d. (2,8)

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7. If the curve $y = ax^2 + bx + c, x \in R$, passes through the point (1,2) and the tangent line to this curve at origin is $y = x$, then the possible values of a, b, c are : [17 Mar 2021 Shift 2] **(+4, -1)**

a. $a = \frac{1}{2}, b = \frac{1}{2}, c = 1$

b. $a = 1, b = 0, c = 1$

c. $a = 1, b = 1, c = 0$

d. $a = -1, b = 1, c = 1$

8. Let α be the area of the larger region bounded by the curve $y^2 = 8x$ and the lines $y = x$ and $x = 2$, which lies in the first quadrant Then the value of 3α is equal to _____ [20 Jul 2021 Shift 2] **(+4, -1)**

9. Let $\sum_{n=0}^{\infty} \frac{n^3((2n)!+(2n-1)(n!))}{(n!)((2n)!)} = ae + \frac{b}{e} + c$, where $a, b, c \in Z$ and $e = \sum_{n=0}^{\infty} \frac{1}{n!}$ Then $a^2 - b + c$ is equal to _______ [27 Aug 2021 Shift 1] **(+4, -1)**

10. The area of the region given by $\left\{(x, y) : xy \le 8, 1 \le y \le x^2\right\}$ is : **(+4, -1)**

[Sep. 03, 2020 (I)]

a. $16 \log_e 2 - \frac{14}{3}$

b. $8 \log_e 2 - \frac{13}{3}$

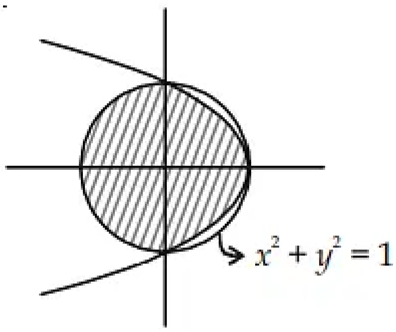
c. $8 \log_e 2 + \frac{7}{6}$

d. $16 \log_e 2 + \frac{7}{3}$

Answers

1. **Answer: a**

Explanation:



Shaded area

$$= \frac{\pi(1)^2}{2} + 2\int_0^1 \sqrt{(1-x)}dx$$
$$= \frac{\pi}{2} + \frac{2(1-x)^{3/2}}{3/2}(-1)\Big|_0^1$$
$$= \frac{\pi}{2} + \frac{4}{3}(0-(-1))$$
$$= \frac{\pi}{2} + \frac{4}{3}$$

Concepts:

1. Applications of Integrals:

There are distinct applications of integrals, out of which some are as follows:

In Maths

Integrals are used to find:

- The center of mass (centroid) of an area having curved sides
- The area between two curves and the area under a curve
- The curve's average value

In Physics

Integrals are used to find:

- Centre of gravity
- Mass and momentum of inertia of vehicles, satellites, and a tower
- The center of mass
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- Thrust

---

2. Answer: c

Explanation:

The graph is a follows
$\int_{-1}^{0}\left(-x^2+1\right)dx+\int_{0}^{1}\left(x^2+1\right)dx=2$

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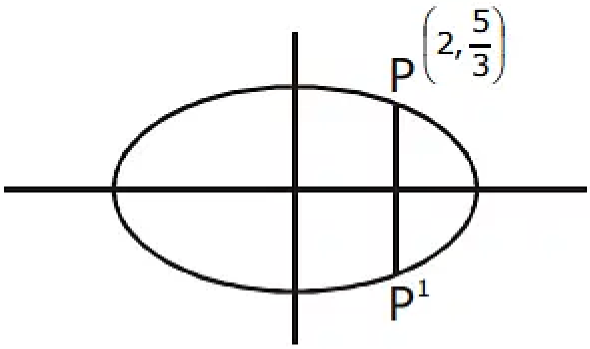
3. Answer: d

Explanation:



$\frac{x^2}{9}+\frac{y^2}{5}=1$
$a=3$
$b=\sqrt{5}$
$e^2=1-\frac{b^2}{a^2}$
$=1-\frac{5}{9}=\frac{4}{9}$
$e=\frac{2}{3}$
now the quadrilateral formed will be a rhombu with area $=\frac{2a^2}{e}$
$=\frac{2.9}{2}\times 3$
$=27$

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4. **Answer: c**

Explanation:

Area of shaded region

$$= \int_0^1 \left(\sqrt{x} + 1 - \frac{x^2}{4}\right) dx + = \int_1^2 \left((3 - x) - \frac{x^2}{4}\right) dx$$

$= \frac{5}{2}$ s unit

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5. **Answer: a**

Explanation:

$$\int_0^b \left(\sqrt{ax} - \frac{x^2}{a}\right) dx = \frac{1}{2} \times \frac{16\left(\frac{a}{4}\right)\left(\frac{a}{4}\right)}{3}$$

$$\Rightarrow \left[\frac{2\sqrt{a}}{3}x^{3/2} - \frac{x^3}{3a}\right]_0^b = \frac{a^2}{6}$$

$$\Rightarrow \frac{2\sqrt{a}}{3}b^{3/2} - \frac{b^3}{3a} = \frac{a^2}{6} \ldots (i)$$

Also, $\frac{1}{2} \times b^2 = \frac{1}{2} \Rightarrow b = 1$

so, $\frac{2\sqrt{a}}{3} - \frac{1}{3a} = \frac{a^2}{6} \Rightarrow a^3 - 4a^{3/2} + 2 = 0$

$$\Rightarrow a^6 + 4a^3 + 4 = 16a^3 \Rightarrow a^6 - 12a^3 + 4 = 0$$

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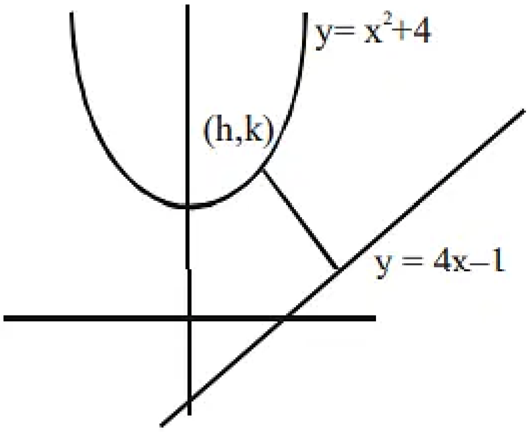
6. Answer: d

Explanation:



$P: y = x^2 + 4$ $k = h^2 + 4$ $L: y = 4x - 1$ $y - 4x + 1 = 0$ $d = AB = \left|\frac{k-4h+1}{\sqrt{5}}\right| = \left|\frac{h^2-4-4h+1}{\sqrt{5}}\right|$ $\frac{d(d)}{dh} = \frac{2h-4}{\sqrt{5}} = 0$ $h = 2$ $\frac{d^2(d)}{dh^2} = \frac{2}{\sqrt{5}} > 0$ $\therefore k = 4 + 4 = 8$ $\therefore$ Point (2,8)

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7. Answer: c

Explanation:

$a+b+c=2 ...(1)$ and $\left.\frac{dy}{dx}\right|_{(0,0)}=1$ $2ax+b|_{(0,0)}=1$ $b=1$ Curve passes through origin So, $c=0$ and $a=1$

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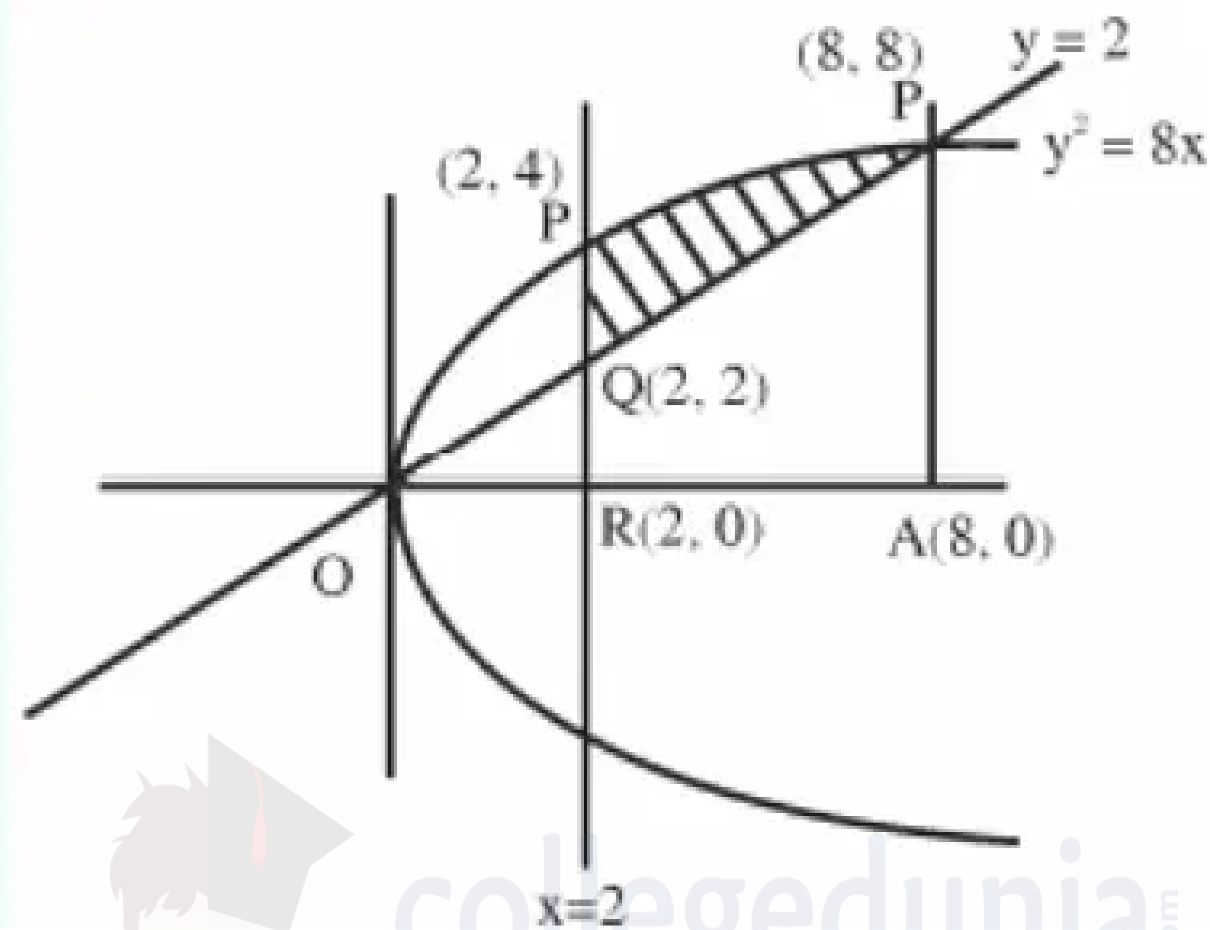
8. Answer: 22 - 22

Explanation:

The correct answer is 22.



$y = x$

& $y^2 = 8x$

Solving it

$x^2 = 8x$

$\therefore x = 0, 8$

$\therefore y = 0, 8$

$x = 2$ will intersect occur at

$y^2 = 16 \Rightarrow y = \pm 4$

$\therefore$ Area of shaded

$= \int_2^8 (\sqrt{8x} - x)dx = \int_2^8 (2\sqrt{2}\sqrt{x} - x)dx$

$= \left[2\sqrt{2} \cdot \frac{x^{3/2}}{3/2} - \frac{x^2}{2}\right]_0^8$

$= \left(\frac{4\sqrt{2}}{3} \cdot 2^{0/2} - 32\right) - \left(\frac{4\sqrt{2}}{3} \cdot 2^{0/2} - 2\right)$

$= \frac{128}{3} - 32 - \frac{16}{3} + 2 = \frac{112-90}{3} = \frac{22}{3} = A$

$\therefore 3A = 22$

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9. Answer: 26 - 26

Explanation:

The correct answer is 26.

$$\sum_{n=0}^{\infty} \frac{n^3((2n)!) + (2n-1)(n!)}{(n!)((2n)!)}$$
$$= \sum_{n=0}^{\infty} \frac{1}{(n-3)!} + \sum_{n=0}^{\infty} \frac{3}{(n-2)!} + \sum_{n=0}^{\infty} \frac{1}{(n-1)!} + \sum_{n=0}^{\infty} \frac{1}{(2n-1)!} - \sum_{n=0}^{\infty} \frac{1}{(2n)!}$$
$$= e + 3e + e + \tfrac{1}{2}\left(e - \tfrac{1}{e}\right) - \tfrac{1}{2}\left(e + \tfrac{1}{e}\right)$$
$$= 5e - \tfrac{1}{e}$$
$$\therefore a^2 - b + c = 26$$

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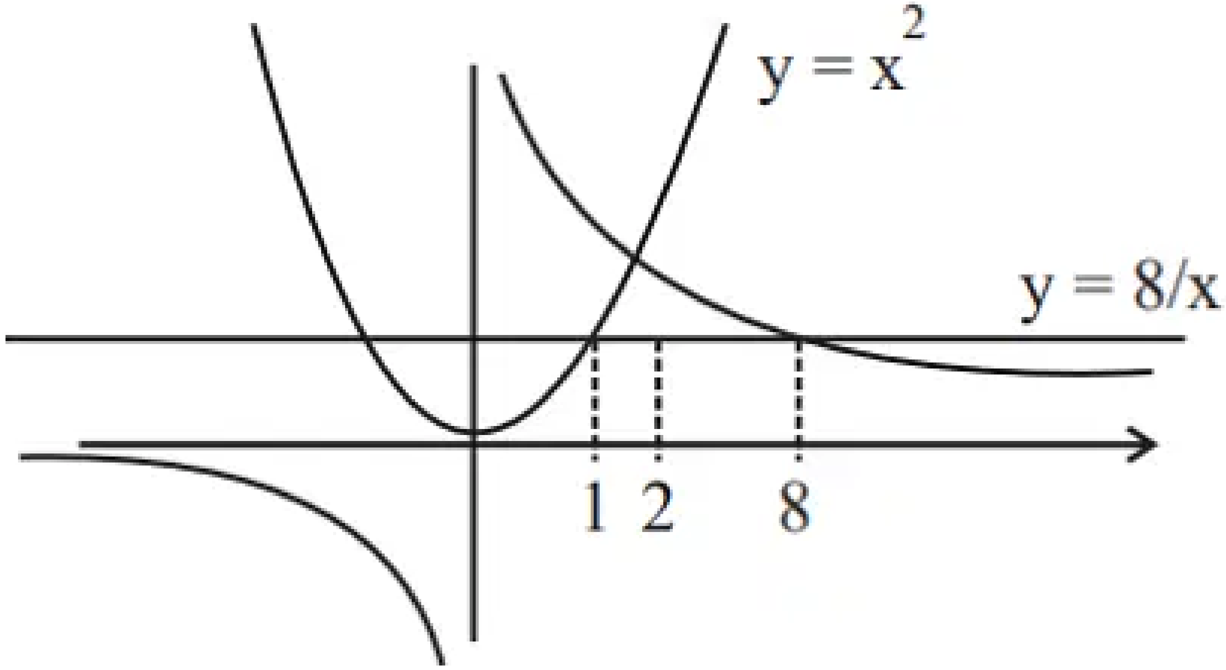
---

10. **Answer: a**

Explanation:

$xy \leq 8 \qquad 1 \leq y \leq x^2$



Area =1∫2(x2-1)dx+2∫8(x8-1)dx
=(3x3)12+8(lnx)28-(x)18
=37+8(2ln2)-7
=16ln2-314

So, the correct option is (A): $16\log_e 2-314$

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