Integral Calculus JEE Main PYQ - 2

Total Time: 25 Minute **Total Marks:** 40

Instructions

Instructions

1. Test will auto submit when the Time is up.
2. The Test comprises of multiple choice questions (MCQ) with one or more correct answers.
3. The clock in the top right corner will display the remaining time available for you to complete the examination.

Navigating & Answering a Question

1. The answer will be saved automatically upon clicking on an option amongst the given choices of answer.
2. To deselect your chosen answer, click on the clear response button.
3. The marking scheme will be displayed for each question on the top right corner of the test window.

Integral Calculus

1. $\lim_{x\to\infty} \frac{(\sqrt{3x+1}+\sqrt{3x-1})^6+(\sqrt{3x+1}-\sqrt{3x-1})^6}{(x+\sqrt{x^2-1})^6+(x-\sqrt{x^2-1})^6}x^3$ [27 Aug 2021 Shift 1] **(+4, -1)**

a. is equal to $\frac{27}{2}$

b. is equal to 9

c. does not exist

d. is equal to 27

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2. If 5f(x) + 4f $\left(\frac{1}{x}\right)$ = $\frac{1}{x}$+ 3, then $18\int_1^2$ f(x)dx is: [Sep. 05, 2020 (I)] **(+4, -1)**

a. 10 ln 3 - 6

b. 5 ln2 - 6

c. 10 ln 2 - 6

d. 5 ln 2 - 3

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3. $I(x)=\int\frac{x^2(xsec^2x+tanx)}{(xtanx+1)^2}dx$. If $I(0)=1$ then $I(\frac{\pi}{4})$ is equal to **(+4, -1)** [26-Jun-2022-Shift-1]

a. $-\frac{\pi^2}{4\pi+16}+2ln(\frac{\pi+4}{4\sqrt{2}})+1$

b. $\frac{\pi^2}{4\pi+16}-2ln(\frac{\pi+4}{4\sqrt{2}})+1$

c. $-\frac{\pi^2}{\pi+4}+2ln(\frac{\pi+1}{\sqrt{2}})+1$

d. $\frac{\pi^2}{\pi+16}+2ln(\frac{\pi+1}{4\sqrt{2}})+1$

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4. The area enclosed by $y=|x-1|+|x-2|$ and $y=3$ **(+4, -1)** [25-Jun-2022-Shift-1]

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5. Find area bounded by the curves y = $max\{sinx, cosx\}$ and $x-axis$ between $x=-\pi$ and $x=\ \pi$ **(+4, -1)** [24-Jun-2022-Shift-2]

a. $2+\sqrt{2}$

b. $\sqrt{2}$

c. $1+\sqrt{2}$

d. $2\sqrt{2}$

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6. If $\int \frac{dx}{x^3(1+x^6)^{\frac{2}{3}}} = f(x)(1+x^6)^{\frac{1}{3}} + C$ where C is a constant of integration, then $f(x)$ is equal to : **(+4, -1)**
[31-Jan-2023 Shift 1]

a. $-\frac{1}{6x^3}$

b. $-\frac{3}{x^2}$

c. $-\frac{1}{2x^2}$

d. $-\frac{1}{2x^3}$

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7. The value of the integral, $\int_\limits{1}^{3}\left[x ^{2}-2 x -2\right] dx ,$ where $[x]$ denotes the greatest integer less than or equal to x, is : **(+4, -1)**
[29-Jan-2023 Shift 2]

a. $-\sqrt{2}-\sqrt{3}+1$

b. $-\sqrt{2}-\sqrt{3}-1$

c. -5

d. -4

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8. The integral $\int \frac{\sin^2 \quad \cos^2}{(\sin^5 \quad + \cos^3 \quad \sin^2 \quad + \quad \cos^2 \quad + \cos^5 \quad)^2}$ is equal to: **(+4, -1)**

[29-Jan-2023 Shift 2]

a. (A) $\frac{1}{1+\cot^3 x}+C$

b. (B) $\frac{-1}{1+\cot^3 x}+C$

c. (C) $\frac{1}{3\left(1+\cot^3 x\right)}+C$

d. (D) $\frac{-1}{3\left(1+\cot^3 x\right)}+C$

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9. Let $f(x)=\begin{vmatrix}1+\sin^2 x & \cos^2 x & \sin 2x\\ \sin^2 x & 1+\cos^2 x & \sin 2x\\ \sin^2 x & \cos^2 x & 1+\sin 2x\end{vmatrix}, x\in\left[\frac{\pi}{6},\frac{\pi}{3}\right]$ If α and β respectively are the maximum and the minimum values of f, then **(+4, -1)**

[27-Jan-2024 Shift 2]

a. $\beta^2+2\sqrt{\alpha}=\frac{19}{4}$

b. $\alpha^2+\beta^2=\frac{9}{2}$

c. $\alpha^2-\beta^2=4\sqrt{3}$

d. $\beta^2-2\sqrt{\alpha}=\frac{19}{4}$

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10. Number of integral solutions to the equation $x+y+z=21$, where $x\geq 1, y\geq 3, z\geq 4$, is equal to ___ **(+4, -1)**

[27-Jan-2024 Shift 2]

Answers

1. Answer: d

Explanation:

$$\lim_{x\to\infty}\frac{(\sqrt{3x+1}+\sqrt{3x-1})^6+(\sqrt{3x+1}-\sqrt{3x-1})^6}{\left(x+\sqrt{x^2-1}\right)^6+\left(x-\sqrt{x^2-1}\right)^6}x^3$$

$$\lim_{x\to\infty} x^3\times\left\{\frac{x^3\left\{\left(\sqrt{3+\frac{1}{x}}+\sqrt{3-\frac{1}{x}}\right)^6+\left(\sqrt{3+\frac{1}{x}}-\sqrt{3-\frac{1}{x}}\right)^6\right\}}{x^6\left\{\left(1+\sqrt{1-\frac{1}{x^2}}\right)^6+\left(1-\sqrt{1-\frac{1}{x^2}}\right)^6\right\}}\right\}$$

$$=\frac{(2\sqrt{3})^6+0}{2^6+0}=3^3=(27)$$

So, the correct answer is (D) : is equal to 27

Concepts:

1. Applications of Integrals:

There are distinct applications of integrals, out of which some are as follows:

In Maths

Integrals are used to find:

- The center of mass (centroid) of an area having curved sides
- The area between two curves and the area under a curve
- The curve's average value

In Physics

Integrals are used to find:

- Centre of gravity
- Mass and momentum of inertia of vehicles, satellites, and a tower
- The center of mass
- The velocity and the trajectory of a satellite at the time of placing it in orbit
- Thrust

2. Answer: c

Explanation:

$5f(x) + 4f(\frac{1}{x}) = \frac{1}{x} + 3......(i)$
Replace $x \to \frac{1}{x}$
$5f(\frac{1}{x}) + 4f(x) = x + 3.....(ii)$
By (i) and (ii)
$9f(x) = \frac{5}{x} - 4x + 3$
$18\int_1^2 f(x)dx = \int_1^2 (\frac{10}{x} - 8x + 6)dx$
$18\int_1^2 f(x)dx =$ 10 ln 2 - 6

So, the correct option is (C): 10 ln 2 - 6

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3. Answer: a

Explanation:

Using integration by parts

$I(x) = x^2.\frac{(-1)}{x\ tanx+1} - \int 2x.\frac{(-1)}{x\ tanx+1}dx$

$= -\frac{x^2}{x\ tanx+1} + 2\int \frac{x\cos x}{x\sin x+\cos x}dx$

$\Rightarrow I(x) = -\frac{x^2}{x\tan x+1} + 2ln|x\ sin\,x + cos\,x| + c$

put $x = 0$

$c = 1$

$\therefore I(\frac{\pi}{4}) = \frac{\frac{-\pi^2}{16}}{\frac{\pi}{4}+1} + 2ln(\frac{\frac{\pi}{4}+1}{\sqrt{2}}) + 1$

$I(\frac{\pi}{4}) = -\frac{\pi^2}{4\pi+16} + 2ln(\frac{\pi+4}{4\sqrt{2}}) + 1$

So, the correct answer is (A): $-\frac{\pi^2}{4\pi+16} + 2ln(\frac{\pi+4}{4\sqrt{2}}) + 1$

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4. Answer: 4 - 4

Explanation:

The correct answer is 4.

Concepts:

1. **Area under Simple Curves:**

- The area of the region bounded by the curve y = f (x), x-axis and the lines x = a and x = b (b > a) - given by the formula:

$$\text{Area} = \int_a^b y dx = \int_a^b f(x)dx$$

- The area of the region bounded by the curve x = φ (y), y-axis and the lines y = c, y = d - given by the formula:

$$\text{Area} = \int_c^d x dy = \int_c^d \phi(y)dy$$

Read More: Area under the curve formula

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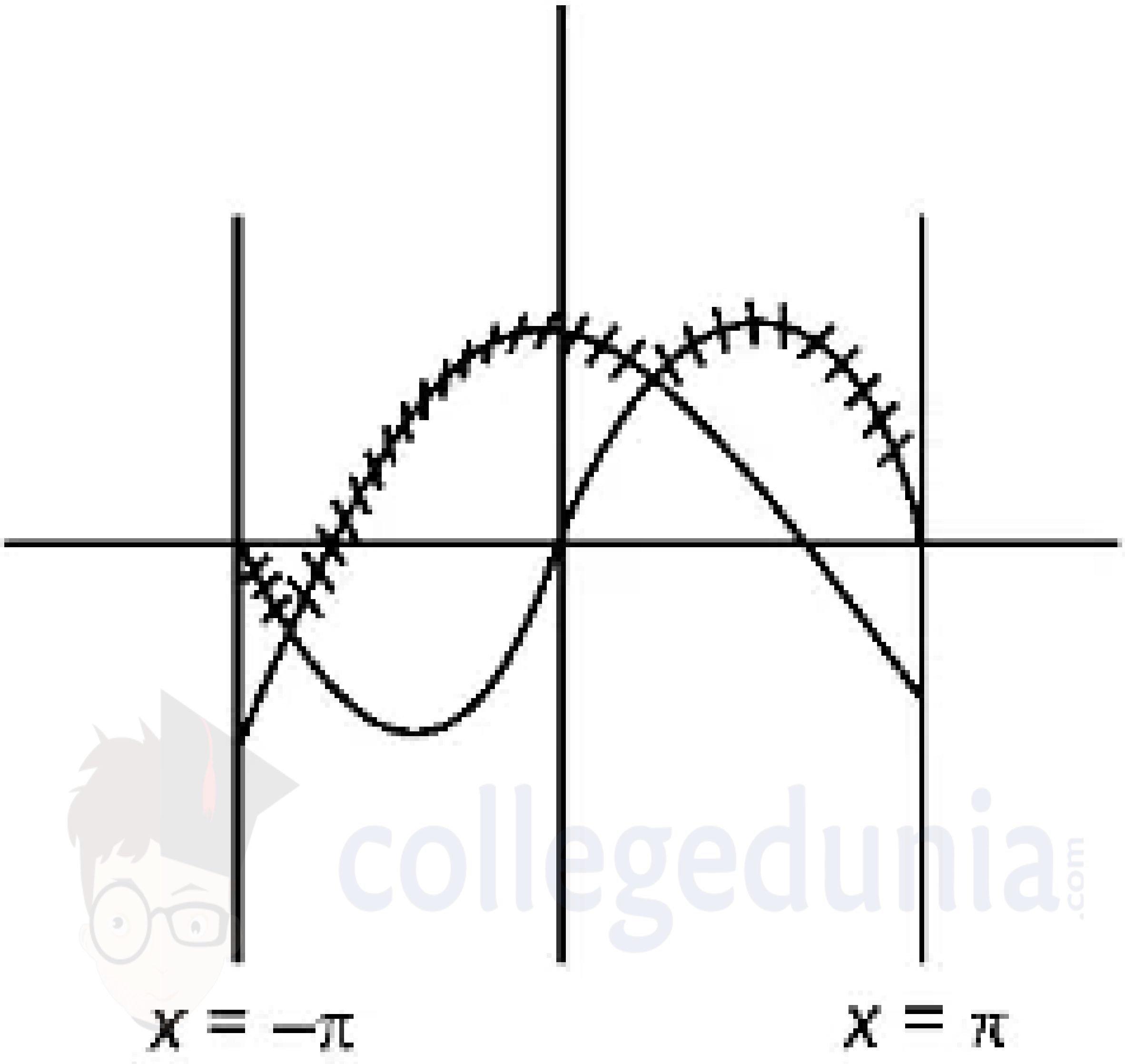
5. **Answer: d**

Explanation:



$\int_{-\pi}^{-3\frac{\pi}{4}} sinx\, dx + \int_{-3\frac{\pi}{4}}^{\frac{\pi}{4}} cosx\, dx + \int_{\frac{\pi}{4}}^{\pi} sinx\, dx$

$-cos\, x|_{-\pi}^{-3\frac{\pi}{4}} + sin\, x|_{-3\frac{\pi}{4}}^{\frac{\pi}{4}} + -cos\, x|_{\frac{\pi}{4}}^{\pi}$

$= (\frac{1}{\sqrt{2}} - 1) + (\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}) + (1 + \frac{1}{\sqrt{2}})$

$= 2\sqrt{2}$

The correct option is (D): $2\sqrt{2}$

Concepts:

1. Applications of Integrals:

There are distinct applications of integrals, out of which some are as follows:

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6. **Answer: d**

Explanation:

$\int \frac{dx}{x^3(1+x^6)^{\frac{2}{3}}}$

$= \int \frac{dx}{x^7(1+x^6)^{\frac{2}{3}}}$

Let $1 + \frac{1}{x^6} = t \quad \Rightarrow \quad -6x^7\, dx = dt$

$\therefore\ I = -\frac{1}{6} \int \frac{dt}{t^{\frac{2}{3}}}$

$= -\frac{3}{6} t^{\frac{1}{3}} + C$

$= -\frac{1}{2}(1 + \frac{1}{x^6})^{\frac{1}{3}} + C$

$= -\frac{1}{2x^2}(1 + x^6)^{\frac{1}{3}} + C$

$\therefore\ f(x) = -\frac{1}{2x^3}$

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7. **Answer: b**

Explanation:

$\int\limits_{1}^{3}\left(\left[(x-1)^{2}\right]-3\right) d x$
$=\int\limits_{1}^{2}\left[x^{2}\right]-3 \int\limits_{1}^{3} d x$
$=\int\limits_{1}^{3} 0 d x+\int\limits_{1}^{\sqrt{2}} 1 .d x+\int\limits_{\sqrt{2}}^{\sqrt{3}} 2.d x+\int\limits_{\sqrt{3}}^{2} 3. d x-6$ $= \sqrt{2} - 1 + 2(\sqrt{3} - \sqrt{2}) + 3(2 - \sqrt{3}) - 6 = -\sqrt{2} - \sqrt{3} - 1$

Concepts:

1. Integral:

The representation of the area of a region under a curve is called to be as integra l. The actual value of an integral can be acquired (approximately) by drawing rectangles.

- The definite integral of a function can be shown as the area of the region bounded by its graph of the given function between two points in the line.
- The area of a region is found by splitting it into thin vertical rectangles and applying the lower and the upper limits, the area of the region is summarized.
- An integral of a function over an interval on which the integral is described.

Also, F(x) is known to be a Newton-Leibnitz integral or antiderivative or primitive of a function f(x) on an interval I.

F'(x) = f(x)

For every value of x = I.

Types of Integrals:

Integral calculus helps to resolve two major types of problems:

1. The problem of getting a function if its derivative is given.
2. The problem of getting the area bounded by the graph of a function under given situations.

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8. Answer: d

Explanation:

Explanation:

$\int \frac{\sin^2 \quad \cos^2}{(\sin^5 \quad + \cos^3 \quad \sin^2 \quad + \sin^3 \quad \cos^2 \quad + \cos^5 \quad)^2} \int \frac{\sin^2 \quad \cos^2}{\{(\sin^2 \quad (\sin^3 \quad + \cos^3 \quad) + \cos^2 \quad (\sin^3 \quad + \cos^3 \quad)\}^2}$

$\int \frac{\sin^2 \quad \cos^2}{\{(\sin^2 \quad + \cos^2 \quad)(\sin^3 \quad + \cos^3 \quad)\}^2} \int \frac{\sin^2 \quad \cos^2}{(\sin^3 \quad + \cos^3 \quad)^2}$ Divide by $\cos^3$ in numerator and denominator

we get = $\int \frac{\sec^2 \quad \tan^2}{(\tan^3 \quad + 1)^2}$ Let $1 + \tan^3$ = $3\tan^2 \quad \sec^2$ = $= \frac{1}{3} \int \frac{}{^2} = -\frac{1}{3}\frac{1}{} +$

$= -\frac{1}{3(1 + \tan^3 \quad)} +$ Hence, the correct option is (D).

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9. Answer: d

Explanation:

$C_1 \to C_1 + C_2 + C_3$

$$f(x) = \begin{vmatrix} 2+\sin 2x & \cos^2 x & \sin 2x \\ 2+\sin 2x & 1+\cos^2 x & \sin 2x \\ 2+\sin 2x & \cos^2 x & 1+\sin 2x \end{vmatrix}$$

$$f(x) = (2+\sin 2x)\begin{vmatrix} 1 & \cos^2 x & \sin 2x \\ 1 & 1+\cos^2 x & \sin 2x \\ 1 & \cos^2 x & 1+\sin 2x \end{vmatrix}$$

$R_2 \to R_2 - R_1$

$R_3 \to R_3 - R_1$

$$f(x) = 2+\sin 2x)\begin{vmatrix} 1 & \cos^2 x & \sin 2x \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$= (2+\sin 2x)(1) = 2+\sin 2x$

$= \sin 2x \in \left[\frac{\sqrt{3}}{2}, 1\right]$

Hence $2+\sin 2x \in \left[2+\frac{\sqrt{3}}{2}, 3\right]$

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10. Answer: 105 - 105

Explanation:

The correct answer is 105.

$^{15}C_2 = \frac{15\times14}{2} = 105$

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