JEE Main 2023 30 Jan Shift 1 Mathematics Question Paper

Time Allowed :180 minutes	**Maximum Marks :**300	**Total questions :**90

General Instructions

Read the following instructions very carefully and strictly follow them:

1. The test is of 3 hours duration.
2. The question paper consists of 90 questions, out of which 75 are to attempted. The maximum marks are 300.
3. There are three parts in the question paper consisting of Physics, Chemistry and Mathematics having 30 questions in each part of equal weightage.
4. Each part (subject) has two sections.
   (i) Section-A: This section contains 20 multiple choice questions which have only one correct answer. Each question carries 4 marks for correct answer and –1 mark for wrong answer.
   (ii) Section-B: This section contains 10 questions. In Section-B, attempt any five questions out of 10. The answer to each of the questions is a numerical value. Each question carries 4 marks for correct answer and –1 mark for wrong answer. For Section-B, the answer should be rounded off to the nearest integer

Mathematics

Section A

1. Let

$$A = \begin{bmatrix} m & n \\ p & q \end{bmatrix},\ d = |A| \neq 0, \textbf{ and } |A - d(\textbf{Adj}A)| = 0.$$

Then:

(1) $(1+d)^2 = (m+q)^2$

(2) $1 + d^2 = (m+q)^2$

(3) $(1+d)^2 = m^2 + q^2$

(4) $1 + d^2 = m^2 + q^2$

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2. The line ℓ_1 passes through the point $(2, 6, 2)$ and is perpendicular to the plane $2x + y - 2z = 10$. Then the shortest distance between the line ℓ_1 and the line

$$\frac{x+1}{2} = \frac{y+4}{-3} = \frac{z}{2}$$

is:

(1) 7

(2) $\frac{19}{3}$

(3) $\frac{19}{2}$

(4) 9

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3. If an unbiased die, marked with $-2, -1, 0, 1, 2, 3$ on its faces, is thrown five times, then the probability that the product of the outcomes is positive, is:

(1) $\frac{881}{2592}$

(2) $\frac{521}{2592}$

(3) $\frac{440}{2592}$

(4) $\frac{27}{288}$

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4. Let the system of linear equations

x + y + kz = 2

2x + 3y - z = 1

3x + 4y + 2z = k

have infinitely many solutions. Then the system

(k+1)x + (2k-1)y = 7

(2k+1)x + (k+5)y = 10

has:

(1) infinitely many solutions

(2) unique solution satisfying $x - y = 1$

(3) no solution

(4) unique solution satisfying $x + y = 1$

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5. If

$$\tan 15^\circ + \frac{1}{\tan 75^\circ} + \tan 105^\circ + \tan 195^\circ = 2a,$$

then the value of $a + \frac{1}{a}$ **is:**

(1) 4

(2) $4 - 2\sqrt{3}$

(3) 2

(4) $5 - 3\sqrt{3}$

6.

Suppose $f : \mathbb{R} \to (0, \infty)$ **be a differentiable function such that**

$5f(x + y) = f(x) \cdot f(y), \forall x, y \in \mathbb{R}$**. If** $f(3) = 320$**, then** $\sum_{n=0}^{5} f(n)$ **is equal to:**

(1) 6875

(2) 6575

(3) 6825

(4) 6528

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7. If

$$a_n = \frac{-2}{4n^2 - 16n + 15}, \quad \text{then} \quad a_1 + a_2 + \cdots + a_5 \text{ is equal to:}$$

(1) $\frac{51}{144}$

(2) $\frac{49}{138}$

(3) $\frac{50}{141}$

(4) $\frac{52}{147}$

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8. If the coefficient of x^{15} in the expansion of

$$\left(ax^3 + \frac{1}{bx^3}\right)^{15}$$

is equal to the coefficient of x^{-15} in the expansion of

$$\left(\frac{a}{x^3} - \frac{1}{bx^3}\right)^{15},$$

where a and b are positive real numbers, then for each such ordered pair (a, b):

(1) $a = b$

(2) $ab = 1$

(3) $a = 3b$

(4) $ab = 3$

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9. If $\mathrm{a}, \mathrm{b}, \mathrm{c}$ are three non-zero vectors and $\hat{n}$ is a unit vector perpendicular to c such that

$$\mathbf{a} = \alpha\mathbf{b} - \hat{n}, \quad (\alpha \neq 0)$$

and

$$\vec{b} \cdot \vec{c} = 12, \quad \text{then} \quad \left|\vec{c} \times (\vec{a} \times \vec{b})\right|$$

is equal to:

(1) 15

(2) 9

(3) 12

(4) 6

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10. The number of points on the curve

$$y = 54x^5 - 135x^4 - 70x^3 + 180x^2 + 210x$$

at which the normal lines are parallel to

$$x + 90y + 2 = 0$$

is:

(1) 2

(2) 3

(3) 4

(4) 0

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11. Let

$$y = x + 2, \quad 4y = 3x + 6, \quad \text{and} \quad 3y = 4x + 1$$

be three tangent lines to the circle

$$(x - h)^2 + (y - k)^2 = r^2.$$

Then $h + k$ **is equal to:**

(1) 5

(2) $5(1 + \sqrt{2})$

(3) 6

(4) $5\sqrt{2}$

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12. Let the solution curve $y = y(x)$ **of the differential equation**

$$\frac{dy}{dx} - \frac{3x^5 \tan^{-1}(x^3)}{(1 + x^6)^{3/2}} y = 2x$$

$$exp \frac{x^3 - \tan^{-1} x^3}{\sqrt{(1 + x)^6}}$$

pass through the origin. Then $y(1)$ is equal to:

(1) $\exp\left(\frac{4-\pi}{4\sqrt{2}}\right)$

(2) $\exp\left(\frac{\pi-4}{4\sqrt{2}}\right)$

(3) $\exp\left(\frac{1-\pi}{4\sqrt{2}}\right)$

(4) $\exp\left(\frac{4+\pi}{4\sqrt{2}}\right)$

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13. Let a unit vector $\overrightarrow{OP}$ make angles α, β, γ with the positive directions of the coordinate axes OX, OY, OZ respectively, where $\beta \in \left(0, \frac{\pi}{2}\right)$, and $\overrightarrow{OP}$ is perpendicular to the plane through points $(1,2,3)$, $(2,3,4)$, and $(1,5,7)$. Then which one of the following is true?

(1) $\alpha \in \left(\frac{\pi}{2}, \pi\right)$ and $\gamma \in \left(\frac{\pi}{2}, \pi\right)$

(2) $\alpha \in \left(0, \frac{\pi}{2}\right)$ and $\gamma \in \left(0, \frac{\pi}{2}\right)$

(3) $\alpha \in \left(\frac{\pi}{2}, \pi\right)$ and $\gamma \in \left(0, \frac{\pi}{2}\right)$

(4) $\alpha \in \left(0, \frac{\pi}{2}\right)$ and $\gamma \in \left(\frac{\pi}{2}, \pi\right)$

14.

If $[t]$ denotes the greatest integer ≤ 1, then the value of

$$\frac{3(e-1)^2}{e} \int_1^2 x^2 e^{[x]+[x^3]} dx$$

is:

(1) $e^9 - e$

(2) $e^8 - e$

(3) $e^7 - 1$

(4) $e^8 - 1$

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15. If $P(h, k)$ be a point on the parabola $x = 4y^2$, which is nearest to the point $Q(0, 33)$, then the distance of P from the directrix of the parabola $y^2 = 4(x + y)$ is equal to:

(1) 2

(2) 4

(3) 8

(4) 6

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16. A straight line cuts off the intercepts $OA = a$ and $OB = b$ on the positive directions of the x-axis and y-axis, respectively. If the perpendicular from the origin O to this line makes an angle of $\frac{\pi}{6}$ with the positive direction of the y-axis and the area of $\triangle OAB$ is $\frac{98}{3}\sqrt{3}$, then $a^2 - b^2$ is equal to:

(1) $\frac{392}{3}$

(2) 196

(3) $\frac{196}{3}$

(4) 98

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17. The coefficient of x^{301} in

$$(1 + x)^{500} + x(1 + x)^{499} + x^2(1 + x)^{498} + \cdots + x^{500}$$

is:

(1) ${}^{501}C_{302}$

(2) ${}^{500}C_{301}$

(3) ${}^{500}C_{300}$

(4) ${}^{501}C_{200}$

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18. Among the statements:

(S1) $((p \vee q) \Rightarrow r) \Leftrightarrow (p \Rightarrow r)$

(S2) $((p \vee q) \Rightarrow r) \Leftrightarrow ((p \Rightarrow r) \vee (q \Rightarrow r))$

Which of the following is true?

(1) Only (S1) is a tautology

(2) Neither (S1) nor (S2) is a tautology

(3) Only (S2) is a tautology

(4) Both (S1) and (S2) are tautologies

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19. The minimum number of elements that must be added to the relation

$$R = \{(a, b), (b, c)\}$$

on the set

$$\{a, b, c\}$$

so that it becomes symmetric and transitive is:

(1) 4

(2) 7

(3) 5

(4) 3

20.

If the solution of the equation

$$\log_{\cos x} \cot x + 4 \log_{\sin x} \tan x = 1,\ x \in \left(0, \frac{\pi}{2}\right),$$

is

$$\sin^{-1}\left(\frac{\alpha + \sqrt{\beta}}{2}\right),$$

where α, β are integers, then $\alpha + \beta$ is equal to:

(1) 3

(2) 5

(3) 6

(4) 4

Section B

21. Let $S = \{1, 2, 3, 4, 5, 6\}$. Then the number of one-one functions $f : S \to P(S)$, where $P(S)$ denotes the power set of S, such that $f(n) \subset f(m)$ where $n < m$, is ____.

22. Let α be the area of the larger region bounded by the curve

$$y^2 = 8x$$

and the lines

$$y = x \quad \text{and} \quad x = 2,$$

which lies in the first quadrant. Then the value of 3α is equal to:

23. $\lambda_1 < \lambda_2$ are two values of λ such that the angle between the planes

$$P_1 : \vec{r} \cdot (3\hat{i} - 5\hat{j} + \hat{k}) = 7$$

and

$$P_2 : \vec{r} \cdot (\lambda\hat{i} + \hat{j} - 3\hat{k}) = 9$$

is $\sin^{-1}\left(\frac{2\sqrt{6}}{5}\right)$, then the square of the length of the perpendicular from the point $(38\lambda, 10\lambda, 2)$ to the plane P_1 is _____.

24.

Let $z = 1 + i$ and $z_1 = \frac{1+i\bar{z}}{\bar{z}(1-z)+\frac{1}{z}}$. Then $\frac{12}{\pi}$ arg(z_1) is equal to ____.

25.

$$\lim_{x \to 0} \frac{48}{x^4} \int_0^x \frac{t^3}{t^6 + 1}\, dt \text{ is equal to \_\_\_\_\_.}$$

26. The mean and variance of 7 observations are 8 and 16, respectively. If one observation 14 is omitted and a and b are respectively the mean and variance of the remaining 6 observations, then $a + 3b - 5$ is equal to:

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27. If the equation of the plane passing through the point $(1, 1, 2)$ and perpendicular to the line

$$x - 3y + 2z - 1 = 0, \quad 4x - y + z = 0 \quad \text{is} \quad Ax + By + Cz = 1,$$

then $140(C - B + A)$ is equal to:

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28. Let

$$\sum_{n=0}^{\infty} \frac{n^3\big((2n)!\big) + (2n-1)(n!)}{(n!)(2n)!} = ae + \frac{b}{e} + c,$$

where $a, b, c \in \mathbb{Z}$ and $e = \sum_{n=0}^{\infty} \frac{1}{n!}$. Then $a^2 - b + c$ is equal to ______.

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29. Number of 4-digit numbers (the repetition of digits is allowed) which are made using the digits 1, 2, 3, and 5 and are divisible by 15 is equal to:

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30. Let

$$f^1(x) = \frac{3x+2}{2x+3}, \quad x \in \mathbb{R}, \quad R - \left(-\frac{3}{2}\right).$$

For $n \geq 2$, **define** $f^n(x) = f^1 0 f^{n-1}(x)$ and if

$$f^5(x) = \frac{ax+b}{bx+a}, \quad \gcd(a, b) = 1, \quad \text{then} \quad a + b \text{ is equal to:}$$
