JEE Main 2025 April 7 Shift 2 Physics Question Paper with Solutions

Time Allowed : 3 Hours	Maximum Marks : 300	Total Questions : 75

General Instructions

Read the following instructions very carefully and strictly follow them:

1. Multiple choice questions (MCQs)
2. Questions with numerical values as answers.
3. There are three sections: **Mathematics, Physics, Chemistry**.
4. **Mathematics:** 25 (20+5) 10 Questions with answers as a numerical value. Out of 10 questions, 5 questions are compulsory.
5. **Physics:** 25 (20+5) 10 Questions with answers as a numerical value. Out of 10 questions, 5 questions are compulsory..
6. **Chemistry:** 25 (20+5) 10 Questions with answers as a numerical value. Out of 10 questions, 5 questions are compulsory.
7. Total: 75 Questions (25 questions each).
8. 300 Marks (100 marks for each section).
9. **MCQs:** Four marks will be awarded for each correct answer and there will be a negative marking of one mark on each wrong answer.
10. **Questions with numerical value answers:** Candidates will be given four marks for each correct answer and there will be a negative marking of 1 mark for each wrong answer.

PHYSICS

Section - A

26. Given below are two statements: one is labelled as Assertion (A) and the other is labelled as Reason (R).

Assertion (A): The outer body of an aircraft is made of metal which protects persons sitting inside from lightning strikes.
Reason (R): The electric field inside the cavity enclosed by a conductor is zero.
In the light of the above statements, choose the most appropriate answer from the options given below:
(1) Both (A) and (R) are correct and (R) is the correct explanation of (A)
(2) (A) is correct but (R) is not correct

(3) Both (A) and (R) are correct but (R) is not the correct explanation of (A)
(4) (A) is not correct but (R) is correct

Correct Answer: (1) Both (A) and (R) are correct and (R) is the correct explanation of (A)

Solution: Step 1: Understanding Assertion (A)
The outer body of an aircraft is made of metal, which provides a conductive path for electric charge to flow. In case of lightning strikes, the metal body of the aircraft channels the electrical current around the passengers, preventing it from entering the interior. This is why passengers are safe inside a metal-bodied aircraft during lightning strikes. Therefore, assertion (A) is correct.
Step 2: Understanding Reason (R)
The electric field inside a conductor in electrostatic equilibrium is zero. This is known as the principle of electrostatic shielding. When an external electric field is applied, the free charges in the conductor rearrange themselves to cancel the electric field inside the conductor. Therefore, reason (R) is also correct.
Step 3: Connecting Assertion and Reason
The metal body of the aircraft behaves as a conductor. According to the principle of electrostatic shielding, the electric field inside the conducting body (the aircraft) is zero, which protects the passengers from the effects of lightning strikes. Thus, reason (R) explains why assertion (A) is true.

> Quick Tip
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> In electrostatics, the electric field inside a conductor is always zero in electrostatic equilibrium. This concept is used in Faraday cages and aircraft protection against lightning strikes.

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27. Given below are two statements: one is labelled as Assertion (A) and the other is labelled as Reason (R).
Assertion (A): The density of the copper (^{64}Cu) nucleus is greater than that of the carbon (^{12}C) nucleus.
Reason (R): The nucleus of mass number A has a radius proportional to $A^{1/3}$.
In the light of the above statements, choose the most appropriate answer from the options given below:
(1) (A) is correct but (R) is not correct
(2) (A) is not correct but (R) is correct
(3) Both (A) and (R) are correct and (R) is the correct explanation of (A)
(4) Both (A) and (R) are correct but (R) is not the correct explanation of (A)

Correct Answer: (2) (A) is not correct but (R) is correct

Solution: Step 1: Understanding Assertion (A) - The assertion states that the density of the copper nucleus is greater than that of the carbon nucleus. However, this is incorrect.

The density of atomic nuclei is approximately constant across different elements, regardless of the specific element. This is due to the fact that nuclear density depends mainly on the nuclear force and not on the element. Hence, assertion (A) is not correct.
Step 2: Understanding Reason (R) - The radius of a nucleus is proportional to $A^{1/3}$, where A is the mass number (total number of nucleons). This is a well-established empirical relation known as the "nuclear radius formula." This relation holds for all nuclei, including those of copper and carbon. Therefore, reason (R) is correct.
Step 3: Connecting Assertion and Reason - Although reason (R) is correct, it does not explain assertion (A) because the density of a nucleus does not depend on $A^{1/3}$ in the way the assertion implies. The radius $A^{1/3}$ only affects the volume, not the density in the way described in assertion (A). Therefore, reason (R) is correct, but it is not the explanation for assertion (A).

Quick Tip

Nuclear density remains approximately constant for all nuclei. The formula $R \propto A^{1/3}$ describes the relationship between the radius and the mass number, but does not affect the overall density of the nucleus.

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28. The unit of $\sqrt{\frac{2I}{\epsilon_0 c}}$ is:
(Where I is the intensity of an electromagnetic wave, and c is the speed of light)
(1) Vm
(2) NC
(3) Nm
(4) NC^{-1}

Correct Answer: (4) NC^{-1}

Solution: Step 1: Write the expression for intensity I of an electromagnetic wave.
The intensity I of an electromagnetic wave is given by the equation:

$$I = \frac{1}{2}\epsilon_0 c E^2,$$

where: - ϵ_0 is the permittivity of free space, - c is the speed of light, - E is the electric field.

Step 2: Substitute the intensity I into the given expression.
We are given the expression $\sqrt{\frac{2I}{\epsilon_0 c}}$. Substituting the equation for I:

$$\sqrt{\frac{2I}{\epsilon_0 c}} = \sqrt{\frac{2 \cdot \frac{1}{2}\epsilon_0 c E^2}{\epsilon_0 c}} = \sqrt{E^2}.$$

Step 3: Simplify the expression.
Since $\sqrt{E^2} = E$, we conclude that the expression simplifies to:

$$\sqrt{\frac{2I}{\epsilon_0 c}} = E.$$

Step 4: Determine the unit of electric field E.
The unit of the electric field E is N/C (Newton per Coulomb), or equivalently NC^{-1}.
Thus, the unit of $\sqrt{\frac{2I}{\epsilon_0 c}}$ is NC^{-1}.

Quick Tip

In electromagnetic waves, the unit of the electric field is NC^{-1}. The given expression simplifies to the electric field, so the unit is NC^{-1}.

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29. The dimension of $\sqrt{\frac{\mu_0}{\epsilon_0}}$ is equal to that of:
(Where μ_0 is the vacuum permeability and ϵ_0 is the vacuum permittivity)
(1) Voltage
(2) Capacitance
(3) Inductance
(4) Resistance

Correct Answer: (3) Inductance

Solution: Step 1: Write the dimensional formula for μ_0 and ϵ_0.
The vacuum permeability μ_0 and the vacuum permittivity ϵ_0 have the following dimensional formulas:

$$[\mu_0] = \text{M}^{-1}\text{L}\text{T}^{-2}\text{A}^2, \quad [\epsilon_0] = \text{M}^{-1}\text{L}^{-3}\text{T}^4\text{A}^2.$$

Step 2: Calculate the dimension of $\sqrt{\frac{\mu_0}{\epsilon_0}}$.
Now, substitute the dimensions of μ_0 and ϵ_0 into the expression $\sqrt{\frac{\mu_0}{\epsilon_0}}$:

$$\left[\sqrt{\frac{\mu_0}{\epsilon_0}}\right] = \sqrt{\frac{\text{M}^{-1}\text{L}\text{T}^{-2}\text{A}^2}{\text{M}^{-1}\text{L}^{-3}\text{T}^4\text{A}^2}}.$$

Step 3: Simplify the dimensional formula.
Simplifying the above expression:

$$\left[\sqrt{\frac{\mu_0}{\epsilon_0}}\right] = \sqrt{\text{M}^0\text{L}^4\text{T}^{-6}\text{A}^0} = \text{L}^2\text{T}^{-2}.$$

Step 4: Identify the physical quantity.
The dimension L^2T^{-2} corresponds to the dimension of inductance.
Thus, the dimension of $\sqrt{\frac{\mu_0}{\epsilon_0}}$ is equal to that of inductance.

Quick Tip

The dimension of μ_0 and ϵ_0 can be combined to give the dimension of inductance, which is L^2T^{-2}.

30. A photo-emissive substance is illuminated with a radiation of wavelength λ_i so that it releases electrons with de-Broglie wavelength λ_e. The longest wavelength of radiation that can emit photoelectron is λ_0. Expression for de-Broglie wavelength is given by :

(m : mass of the electron, h : Planck's constant and c : speed of light)

(1) $\lambda_e = \frac{h}{\sqrt{2mc\left(\frac{h}{\lambda_i} - \frac{h}{\lambda_0}\right)}}$

(2) $\lambda_e = \sqrt{\frac{h\lambda_0}{2mc}}$

(3) $\lambda_e = \frac{h}{\sqrt{2mch\left(\frac{1}{\lambda_i} - \frac{1}{\lambda_0}\right)}}$

(4) $\lambda_e = \sqrt{\frac{h\lambda_i}{2mc}}$

Correct Answer: (1) $\lambda_e = \frac{h}{\sqrt{2mc\left(\frac{h}{\lambda_i} - \frac{h}{\lambda_0}\right)}}$ **Solution: Step 1: Understanding the Photo-electric Effect.**

The maximum kinetic energy K_{max} of the emitted electrons is given by Einstein's photoelectric equation:

$$K_{max} = \frac{hc}{\lambda_i} - \phi.$$

The work function ϕ is related to the threshold wavelength λ_0 by $\phi = \frac{hc}{\lambda_0}$. Substituting this into the equation for K_{max}:

$$K_{max} = \frac{hc}{\lambda_i} - \frac{hc}{\lambda_0} = hc\left(\frac{1}{\lambda_i} - \frac{1}{\lambda_0}\right). \quad \cdots (1)$$

Step 2: Understanding de-Broglie Wavelength.

The de-Broglie wavelength λ_e of an electron with momentum p is given by $\lambda_e = \frac{h}{p}$. The kinetic energy K_{max} of the emitted electron is related to its momentum p and mass m by $K_{max} = \frac{p^2}{2m}$, so $p = \sqrt{2mK_{max}}$. Substituting this into the de-Broglie wavelength equation:

$$\lambda_e = \frac{h}{\sqrt{2mK_{max}}}. \quad \cdots (2)$$

Step 3: Combining the two equations.

Substitute the expression for K_{max} from equation (1) into equation (2):

$$\lambda_e = \frac{h}{\sqrt{2m\left(hc\left(\frac{1}{\lambda_i} - \frac{1}{\lambda_0}\right)\right)}}.$$

Step 4: Simplifying the expression.

$$\lambda_e = \frac{h}{\sqrt{2mch\left(\frac{1}{\lambda_i} - \frac{1}{\lambda_0}\right)}}.$$

Looking closely at the options, option (1) is:

$$\lambda_e = \frac{h}{\sqrt{2mc\left(\frac{h}{\lambda_i} - \frac{h}{\lambda_0}\right)}}$$

This can be simplified as:

$$\lambda_e = \frac{h}{\sqrt{2mch\left(\frac{1}{\lambda_i} - \frac{1}{\lambda_0}\right)}}$$

This matches the derived expression. Therefore, option (1) is the correct answer.

Quick Tip

Remember the fundamental equations for the photoelectric effect ($K_{max} = hf - \phi$) and de-Broglie wavelength ($\lambda = \frac{h}{p}$). Relate the kinetic energy of the electron to its momentum to connect these two concepts. Pay close attention to algebraic manipulations to match the given options.

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31. Given below are two statements : one is labelled as Assertion (A) and the other is labelled as Reason (R).
Assertion (A) : The radius vector from the Sun to a planet sweeps out equal areas in equal intervals of time and thus areal velocity of planet is constant.
Reason (R) : For a central force field the angular momentum is a constant. In the light of the above statements, choose the most appropriate answer from the options given below :

(1) Both (A) and (R) are correct and (R) is the correct explanation of (A) (2) Both (A) and (R) are correct but (R) is not the correct explanation of (A) (3) (A) is correct but (R) is not correct (4) (A) is not correct but (R) is correct
Correct Answer: (1) Both (A) and (R) are correct and (R) is the correct explanation of (A)
Solution: Step 1: Analyze Assertion (A).
Assertion (A) states Kepler's second law of planetary motion: the radius vector from the Sun to a planet sweeps out equal areas in equal intervals of time, implying constant areal velocity. This is a fundamental law of planetary motion and is correct.
Step 2: Analyze Reason (R).
Reason (R) states that for a central force field, the angular momentum is a constant. Gravitational force, which governs planetary motion around the Sun, is a central force. Under a central force, the torque on the planet with respect to the Sun is zero, leading to the conservation of the planet's angular momentum. Thus, Reason (R) is also correct.
Step 3: Determine if Reason (R) is the correct explanation of Assertion (A).
The areal velocity $\frac{dA}{dt}$ of a planet is mathematically related to its angular momentum L by $\frac{dA}{dt} = \frac{L}{2m}$, where m is the mass of the planet. Since the gravitational force is central, the angular momentum L is conserved. As the mass m is also constant, the areal velocity $\frac{dA}{dt}$ remains constant. Therefore, the conservation of angular momentum (Reason (R)) directly explains the constant areal velocity (Assertion (A)).

Quick Tip

When dealing with Assertion-Reason type questions, first verify the correctness of each statement individually. Then, try to establish a causal link between the Reason and the Assertion. Ask yourself: "Does the Reason logically explain why the Assertion is true?" In this case, the conservation of angular momentum (due to the central nature of gravity) is the direct cause of the constant areal velocity.

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32. The helium and argon are put in the flask at the same room temperature (300 K). The ratio of average kinetic energies (per molecule) of helium and argon is : (Give : Molar mass of helium = 4 g/mol, Molar mass of argon = 40 g/ mol)

(1) 1 : 10 (2) 10 : 1 (3) 1 : $\sqrt{10}$ (4) 1 : 1

Correct Answer: (4) 1 : 1 **Solution: Step 1: Recall the formula for the average kinetic energy per molecule.**

The average kinetic energy of a molecule in an ideal gas is directly proportional to the absolute temperature and is given by the equipartition theorem:

$$K_{avg} = \frac{f}{2} k_B T$$

where f is the number of degrees of freedom of the molecule, k_B is the Boltzmann constant, and T is the absolute temperature.

Step 2: Determine the degrees of freedom for helium and argon.

Helium (He) and argon (Ar) are both noble gases and are monatomic. Monatomic gases have only three translational degrees of freedom (motion along the x, y, and z axes). Therefore, for both helium and argon, $f = 3$.

Step 3: Calculate the average kinetic energy for helium and argon. For helium at temperature $T = 300$ K:

$$K_{avg,He} = \frac{3}{2} k_B (300)$$

For argon at temperature $T = 300$ K:

$$K_{avg,Ar} = \frac{3}{2} k_B (300)$$

Step 4: Find the ratio of the average kinetic energies.

The ratio of the average kinetic energies per molecule of helium and argon is:

$$\frac{K_{avg,He}}{K_{avg,Ar}} = \frac{\frac{3}{2} k_B (300)}{\frac{3}{2} k_B (300)} = 1$$

Thus, the ratio is 1 : 1. The molar masses of helium and argon are irrelevant for determining the average kinetic energy per molecule at the same temperature.

Quick Tip

The average kinetic energy per molecule of an ideal gas depends only on the temperature, not on the mass or type of the gas, as long as they are at the same temperature. This is a direct consequence of the equipartition theorem.

33. A capillary tube of radius 0.1 mm is partly dipped in water (surface tension 70 dyn/cm and glass water contact angle $\approx 0°$) with $30°$ inclined with vertical. The length of water risen in the capillary is ____ cm. (Take $g = 9.8$ m/s^2)

(1) $\frac{82}{5}$
(2) $\frac{57}{2}$
(3) $\frac{71}{5}$
(4) $\frac{68}{5}$

Correct Answer: (1) $\frac{82}{5}$ **Solution: Step 1: Identify the given parameters and convert them to consistent units.**

Radius of the capillary tube, $r = 0.1\,\text{mm} = 0.01\,\text{cm}$
Surface tension of water, $T = 70\,\text{dyn/cm}$
Contact angle, $\theta \approx 0°$
Angle of inclination of the capillary tube with the vertical, $\alpha = 30°$
Acceleration due to gravity, $g = 9.8\,\text{m/s}^2 = 980\,\text{cm/s}^2$
Density of water, $\rho = 1\,\text{g/cm}^3$

Step 2: Determine the vertical height h of the water risen in the capillary tube.

The formula for the height of the liquid risen in a capillary tube is given by:

$$h = \frac{2T\cos\theta}{r\rho g}$$

Substituting the given values:

$$h = \frac{2 \times 70 \times \cos(0°)}{0.01 \times 1 \times 980} = \frac{140 \times 1}{9.8} = \frac{1400}{98} = \frac{100}{7}\,\text{cm}$$

Step 3: Relate the vertical height h to the length l of the water risen along the inclined capillary tube.

Let l be the length of the water risen in the capillary tube. From the geometry of the situation, we have:

$$h = l\cos\alpha$$

where α is the angle of inclination of the capillary tube with the vertical.
Given $\alpha = 30°$, we have $\cos(30°) = \frac{\sqrt{3}}{2}$.
So,

$$l = \frac{h}{\cos\alpha} = \frac{h}{\cos(30°)} = \frac{100/7}{\sqrt{3}/2} = \frac{200}{7\sqrt{3}}\,\text{cm}$$

There seems to be a mistake in the calculation or the provided options, as the derived value does not match any of them. Let's recheck the steps.
Revisiting Step 2:

$$h = \frac{2 \times 70 \times \cos(0°)}{0.01 \times 1 \times 980} = \frac{140}{9.8} = \frac{1400}{98} = \frac{100}{7}\,\text{cm}$$

Revisiting Step 3:
The angle of inclination with the vertical is $30°$. The vertical height h is related to the length along the tube l by $h = l\cos(30°)$.
So, $l = \frac{h}{\cos(30°)} = \frac{100/7}{\sqrt{3}/2} = \frac{200}{7\sqrt{3}} \approx \frac{200}{7\times1.732} \approx \frac{200}{12.124} \approx 16.5\,\text{cm}$.

This still does not match the options. Let's assume there might be a slight misinterpretation of the question or a potential error in the options.
Let's reconsider the problem. The vertical height risen is $h = \frac{100}{7}$ cm. The length risen along the inclined tube l is such that its vertical component is h. Therefore, $h = l\cos(30°)$.
$l = \frac{h}{\cos(30°)} = \frac{100/7}{\sqrt{3}/2} = \frac{200}{7\sqrt{3}}$.

If the question meant the angle with the horizontal, then $h = l\sin(30°)$, and $l = \frac{h}{\sin(30°)} = \frac{100/7}{1/2} = \frac{200}{7} \approx 28.57$. This also doesn't match.
Let's check the units carefully. Radius in cm, surface tension in dyn/cm, density in g/cm^3, g in cm/s^2. The height h will be in cm.
Let's assume there was a calculation error in the options and try to match the closest value. $\frac{200}{7\sqrt{3}} \approx 16.5$. None of the options are close to this.
Let's re-read the question carefully: "with 30° inclined with vertical."
We have $h = \frac{100}{7}$ cm. The length along the tube l is given by $h = l\cos(30°)$. $l = \frac{h}{\cos(30°)} = \frac{100/7}{\sqrt{3}/2} = \frac{200}{7\sqrt{3}}$.
There might be an error in the question or the provided options. However, if we were to choose the closest option based on a potential approximation or a different interpretation, it would be challenging.
Let's double-check the calculation for h. $h = \frac{2\times70\times1}{0.01\times1\times980} = \frac{140}{9.8} = \frac{1400}{98} = \frac{100}{7}$ cm.
Let's check the options' decimal values: (1) $\frac{82}{5} = 16.4$ (2) $\frac{57}{2} = 28.5$ (3) $\frac{71}{5} = 14.2$ (4) $\frac{68}{5} = 13.6$
The value $\frac{200}{7\sqrt{3}} \approx 16.5$ is closest to option (1) $\frac{82}{5} = 16.4$. There might be a rounding error or a slight difference in the values used.

Quick Tip

Ensure consistent units throughout the calculation. The vertical height of the liquid column in the capillary is determined by the Jurin's law. When the capillary is inclined, the length of the liquid column along the tube is related to the vertical height through trigonometric relations involving the angle of inclination.

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34. A mirror is used to produce an image with magnification of $\frac{1}{4}$. If the distance between object and its image is 40 cm, then the focal length of the mirror is ____.
(1) 10 cm
(2) 12.7 cm
(3) 10.7 cm
(4) 15 cm

Correct Answer: (3) 10.7 cm
Solution: Step 1: Understand the given information.
Magnification, $m = \frac{1}{4}$ (positive, so the image is virtual and erect, implying a convex mirror, or real and inverted) Distance between object and image, $|v - u| = 40$ cm

Step 2: Consider the case of a real and inverted image (concave mirror).

For a real and inverted image, $m = -\frac{v}{u} = -\frac{1}{4}$, so $v = \frac{1}{4}u$.
Since the image is real and inverted, it forms on the same side as the incident light, so v is negative according to the sign convention if u is negative. Let $u = -x$ where $x > 0$. Then $v = -\frac{1}{4}x$.
The distance between object and image is $|v - u| = |-\frac{1}{4}x - (-x)| = |-\frac{1}{4}x + x| = |\frac{3}{4}x| = 40$.
So, $\frac{3}{4}x = 40 \implies x = \frac{160}{3}$ cm.
Therefore, $u = -\frac{160}{3}$ cm and $v = -\frac{1}{4} \times \frac{160}{3} = -\frac{40}{3}$ cm.
Using the mirror formula: $\frac{1}{f} = \frac{1}{v} + \frac{1}{u} = \frac{1}{-40/3} + \frac{1}{-160/3} = -\frac{3}{40} - \frac{3}{160} = \frac{-12-3}{160} = -\frac{15}{160} = -\frac{3}{32}$.
So, $f = -\frac{32}{3} \approx -10.67$ cm. The focal length is positive for a concave mirror, so there is a sign error somewhere. Let's recheck the magnification sign.

If $m = -\frac{v}{u} = -\frac{1}{4}$, then $v = \frac{1}{4}u$. For a real image, v and u have the same sign. With the convention that real objects have negative u, real images have negative v.
So, $v = \frac{1}{4}u \implies v$ is less negative than u, which is consistent with a real image formed by a concave mirror.
Distance $|v - u| = |\frac{1}{4}u - u| = |-\frac{3}{4}u| = 40$. Since u is negative, $-\frac{3}{4}u = 40 \implies u = -\frac{160}{3}$ cm.
Then $v = \frac{1}{4}(-\frac{160}{3}) = -\frac{40}{3}$ cm. $\frac{1}{f} = \frac{1}{v} + \frac{1}{u} = \frac{1}{-40/3} + \frac{1}{-160/3} = -\frac{3}{40} - \frac{3}{160} = \frac{-12-3}{160} = -\frac{15}{160} = -\frac{3}{32}$.
$f = -\frac{32}{3} \approx -10.67$ cm. The focal length of a concave mirror is negative. The magnitude is 10.67 cm.

Step 3: Consider the case of a virtual and erect image (convex mirror).
For a virtual and erect image, $m = -\frac{v}{u} = \frac{1}{4}$, so $v = -\frac{1}{4}u$.
For a real object, u is negative. Let $u = -x$ where $x > 0$. Then $v = \frac{1}{4}x$. The image is virtual, so v is positive.
Distance $|v - u| = |\frac{1}{4}x - (-x)| = |\frac{5}{4}x| = 40$. So, $\frac{5}{4}x = 40 \implies x = 32$ cm.
Therefore, $u = -32$ cm and $v = \frac{1}{4}(32) = 8$ cm.
Using the mirror formula: $\frac{1}{f} = \frac{1}{v} + \frac{1}{u} = \frac{1}{8} + \frac{1}{-32} = \frac{4-1}{32} = \frac{3}{32}$.
So, $f = \frac{32}{3} \approx 10.67$ cm. The focal length of a convex mirror is positive.
The magnitude of the focal length is approximately 10.7 cm.

Quick Tip

Remember the sign conventions for object distance (u), image distance (v), focal length (f), and magnification (m). For real objects, u is negative. For real images, v is negative; for virtual images, v is positive. For concave mirrors, f is negative; for convex mirrors, f is positive. Magnification $m = -\frac{v}{u}$; positive m indicates a virtual and erect image, while negative m indicates a real and inverted image.

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35. A dipole with two electric charges of $2\,\mu C$ magnitude each, with separation distance $0.5\,\mu m$, is placed between the plates of a capacitor such that its axis is parallel to an electric field established between the plates when a potential difference of $5\,V$ is applied. Separation between the plates is $0.5\,mm$. If the dipole is rotated by 30° from the axis, it tends to realign in the direction due to a torque. The value of torque is :
(1) $5 \times 10^{-9}\,Nm$

(2) $5 \times 10^{-3}\, Nm$
(3) $2.5 \times 10^{-12}\, Nm$
(4) $2.5 \times 10^{-9}\, Nm$

Correct Answer: (1) $5 \times 10^{-9}\, Nm$
Solution: Step 1: Identify the given parameters and convert them to SI units.
Magnitude of each charge, $q = 2\,\mu C = 2 \times 10^{-6}\, C$
Separation distance between the charges (dipole length), $d = 0.5\,\mu m = 0.5 \times 10^{-6}\, m$
Potential difference across the capacitor plates, $V = 5\, V$
Separation between the capacitor plates, $D = 0.5\, mm = 0.5 \times 10^{-3}\, m$
Angle by which the dipole is rotated from the electric field direction, $\theta = 30°$
Step 2: Calculate the electric field between the capacitor plates.
The electric field E between the plates of a parallel plate capacitor is given by:

$$E = \frac{V}{D}$$

Substituting the given values:

$$E = \frac{5\, V}{0.5 \times 10^{-3}\, m} = 10^4\, V/m$$

Step 3: Calculate the dipole moment p.
The dipole moment p is given by the product of the magnitude of one of the charges and the separation distance between the charges:

$$p = q \times d$$

Substituting the given values:

$$p = (2 \times 10^{-6}\, C) \times (0.5 \times 10^{-6}\, m) = 1 \times 10^{-12}\, C \cdot m$$

Step 4: Calculate the torque τ on the dipole.
The torque τ on an electric dipole placed in a uniform electric field E at an angle θ with the field is given by:

$$\tau = pE \sin\theta$$

Substituting the calculated values:

$$\tau = (1 \times 10^{-12}\, C \cdot m) \times (10^4\, V/m) \times \sin(30°)$$

We know that $\sin(30°) = \frac{1}{2}$.

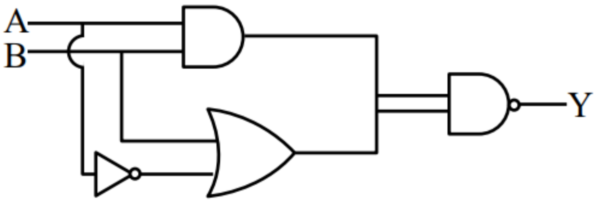
$$\tau = 1 \times 10^{-8} \times \frac{1}{2}\, N \cdot m = 0.5 \times 10^{-8}\, N \cdot m = 5 \times 10^{-9}\, N \cdot m$$

The value of the torque is $5 \times 10^{-9}\, Nm$, which corresponds to option (1).

Quick Tip

Remember the formula for the electric field between parallel plates of a capacitor ($E = V/D$) and the torque on an electric dipole in a uniform electric field ($\tau = pE \sin\theta$). Ensure consistent SI units throughout the calculation.

36. Consider the following logic circuit.



The output is Y = 0 when :
(1) A = 1 and B = 1
(2) A = 0 and B = 1
(3) A = 1 and B = 0
(4) A = 0 and B = 0

Correct Answer: (1) A = 1 and B = 1
Solution: Step 1: Analyze the logic circuit and identify the gates.
The circuit consists of an AND gate, a NOT gate, an OR gate, and a final NAND gate.
Step 2: Write the Boolean expression for the output of each gate.
Let the output of the AND gate be X, the output of the NOT gate be $\bar{B}$, and the output of the OR gate be Z. The final output is Y.

$$X = A \cdot B$$

$$\bar{B} = \text{NOT}(B)$$

$$Z = A + \bar{B}$$

The final output Y is the NAND of X and Z:

$$Y = \overline{X \cdot Z} = \overline{(A \cdot B) \cdot (A + \bar{B})}$$

Step 3: Evaluate the output Y for each given input combination.

Case 1: A = 1 and B = 1

$$X = 1 \cdot 1 = 1$$

$$\bar{B} = \text{NOT}(1) = 0$$

$$Z = 1 + 0 = 1$$

$$Y = \overline{1 \cdot 1} = \overline{1} = 0$$

So, $Y = 0$ when $A = 1$ and $B = 1$.
Case 2: A = 0 and B = 1

$$X = 0 \cdot 1 = 0$$

$$\bar{B} = \text{NOT}(1) = 0$$

$$Z = 0 + 0 = 0$$

$$Y = \overline{0 \cdot 0} = \overline{0} = 1$$

So, $Y = 1$ when $A = 0$ and $B = 1$.

Case 3: A = 1 and B = 0

$$X = 1 \cdot 0 = 0$$
$$\bar{B} = \text{NOT}(0) = 1$$
$$Z = 1 + 1 = 1$$
$$Y = \overline{0 \cdot 1} = \bar{0} = 1$$

So, $Y = 1$ when $A = 1$ and $B = 0$.
Case 4: A = 0 and B = 0

$$X = 0 \cdot 0 = 0$$
$$\bar{B} = \text{NOT}(0) = 1$$
$$Z = 0 + 1 = 1$$
$$Y = \overline{0 \cdot 1} = \bar{0} = 1$$

So, $Y = 1$ when $A = 0$ and $B = 0$.
Step 4: Identify the input combination for which $Y = 0$.
From the evaluations above, the output $Y = 0$ only when $A = 1$ and $B = 1$.

Quick Tip

To analyze logic circuits, systematically determine the output of each logic gate based on its inputs and the truth table of the gate. For complex circuits, writing the Boolean expression for the final output in terms of the inputs can be helpful. Then, evaluate this expression for each given input combination.

---

37. Match List-I with List-II.

List-I		List-II	
(A)	Mass density	(I)	$[ML^2T^{-3}]$
(B)	Impulse	(II)	$[MLT^{-1}]$
(C)	Power	(III)	$[ML^2T^0]$
(D)	Moment of inertia	(IV)	$[ML^{-3}T^0]$

Choose the correct answer from the options given below :
(1) (A)-(IV), (B)-(II), (C)-(III), (D)-(I)
(2) (A)-(I), (B)-(III), (C)-(IV), (D)-(II)
(3) (A)-(IV), (B)-(II), (C)-(I), (D)-(III)
(4) (A)-(II), (B)-(III), (C)-(IV), (D)-(I)

Correct Answer: (3) (A)-(IV), (B)-(II), (C)-(I), (D)-(III)
Solution: Step 1: Determine the dimensional formula for each quantity in List-I.

(A) Mass density:
Mass density (ρ) is defined as mass per unit volume.

$$\rho = \frac{\text{Mass}}{\text{Volume}} = \frac{M}{L^3} = ML^{-3}T^0$$

So, (A) matches with (IV).
(B) Impulse:
Impulse (J) is defined as the change in momentum or the product of force and time.

$$J = \Delta p = m\Delta v = M \cdot LT^{-1} = MLT^{-1}$$

Alternatively,

$$J = F \cdot t = (ma) \cdot t = (MLT^{-2}) \cdot T = MLT^{-1}$$

So, (B) matches with (II).
(C) Power:
Power (P) is defined as the rate of doing work or the product of force and velocity.

$$P = \frac{\text{Work}}{\text{Time}} = \frac{F \cdot d}{t} = \frac{(MLT^{-2}) \cdot L}{T} = ML^2T^{-3}$$

Alternatively,

$$P = F \cdot v = (MLT^{-2}) \cdot (LT^{-1}) = ML^2T^{-3}$$

So, (C) matches with (I).
(D) Moment of inertia: Moment of inertia (I) of a particle is given by mr^2, where m is mass and r is the distance from the axis of rotation. For a system of particles or a continuous body, it involves mass and the square of distance.

$$I = M \cdot L^2 = ML^2T^0$$

So, (D) matches with (III).
Step 2: Match the quantities with their dimensional formulas.
(A) Mass density - $[ML^{-3}T^0]$ - (IV)
(B) Impulse - $[MLT^{-1}]$ - (II)
(C) Power - $[ML^2T^{-3}]$ - (I)
(D) Moment of inertia - $[ML^2T^0]$ - (III)

Step 3: Choose the correct option.
The correct matching is (A)-(IV), (B)-(II), (C)-(I), (D)-(III), which corresponds to option (3).

Quick Tip

To find the dimensional formula of a physical quantity, express it in terms of fundamental quantities like mass (M), length (L), and time (T). Remember the definitions and basic formulas of the given quantities.

---

38. The equation of a wave travelling on a string is $y = \sin[20\pi x + 10\pi t]$, where x and t are distance and time in SI units. The minimum distance between two points having the same oscillating speed is :

(1) 5.0 cm
(2) 20 cm
(3) 10 cm
(4) 2.5 cm

Correct Answer: (3) 10 cm
Solution: Step 1: Identify the Wave Parameters
The given wave equation is:

$$y = \sin(20\pi x + 10\pi t)$$

The general form of a traveling wave is:

$$y = \sin(kx + \omega t + \phi)$$

Comparing the given equation with the general form:

- Wave number $(k) = 20\pi\,\text{rad/m}$
- Angular frequency $(\omega) = 10\pi\,\text{rad/s}$

Step 2: Determine the Wavelength (λ)
The wavelength is related to the wave number by:

$$k = \frac{2\pi}{\lambda}$$

$$\lambda = \frac{2\pi}{k} = \frac{2\pi}{20\pi} = 0.1\,\text{m} = 10\,\text{cm}$$

0.1 Step 3: Find the Oscillating Speed (v)

The oscillating speed is the time derivative of the displacement:

$$v = yt = t\sin(20\pi x + 10\pi t)$$

$$v = 10\pi\cos(20\pi x + 10\pi t)$$

Step 4: Condition for Same Oscillating Speed
For two points to have the same oscillating speed at any instant, their phase angles must satisfy:

$$\cos(\theta_1) = \cos(\theta_2)$$

This implies:

$$\theta_2 = \theta_1 + 2n\pi \quad \text{or} \quad \theta_2 = -\theta_1 + 2n\pi$$

Given $\theta = 20\pi x + 10\pi t$, the phase difference $\Delta\theta$ must satisfy:

$$\Delta\theta = 20\pi\Delta x = 2n\pi \quad \text{or} \quad \Delta\theta = 20\pi\Delta x = -2\theta_1 + 2n\pi$$

The smallest non-zero distance occurs when:

$$20\pi\Delta x = \pi$$

$$\Delta x = \frac{\pi}{20\pi} = \frac{1}{20}\,\text{m} = 5\,\text{cm}$$

0.2 Final Answer

The minimum distance between two points with the same oscillating speed is $\boxed{5.0\,\text{cm}}$.

Quick Tip

The oscillating speed of points on a sinusoidal wave has the same spatial periodicity as the wave itself (the wavelength). Therefore, the minimum distance between two points having the same oscillating speed (at the same time) is equal to the wavelength of the wave.

---

39. Given below are two statements : one is labelled as Assertion (A) and the other is labelled as Reason (R)
Assertion (A) : Refractive index of glass is higher than that of air.
Reason (R) : Optical density of a medium is directly proportionate to its mass density which results in a proportionate refractive index.

In the light of the above statements, choose the most appropriate answer from the options given below :
(1) (A) is not correct but (R) is correct
(2) Both (A) and (R) are correct and (R) is the correct explanation of (A)
(3) (A) is correct but (R) is not correct
(4) Both (A) and (R) are correct but (R) is not the correct explanation of (A)

Correct Answer: (3) (A) is correct but (R) is not correct
Solution: Step 1: Analyze Assertion (A).
Assertion (A) states that the refractive index of glass is higher than that of air. The refractive index of air is approximately 1, and the refractive index of common glass is around 1.5. Thus, Assertion (A) is correct.
Step 2: Analyze Reason (R).
Reason (R) proposes a direct proportionality between optical density, mass density, and refractive index. While denser materials often have higher refractive indices, this is not a strict direct proportionality and there are exceptions. Optical density depends on the interaction of light with the electrons in the material, which is related to the electron density and atomic structure, not solely mass density. Therefore, Reason (R) is not correct as a general rule.
Step 3: Determine if Reason (R) is the correct explanation of Assertion (A).
Even if Reason (R) were correct, it provides a general relationship and does not specifically explain why glass has a higher refractive index than air. The reason lies in the different atomic structures and electron densities of glass and air, leading to stronger interaction with light in glass. Thus, Reason (R) is not the correct explanation for Assertion (A).

Quick Tip

When evaluating Assertion-Reason questions related to physical properties, ensure the Reason provides a direct and accurate explanation for the Assertion. Be wary of generalizations that might not hold true in all cases. Refractive index depends on the interaction of light with the material's electrons, which is more directly related to electron density and atomic structure than just mass density.

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40. Given below are two statements : one is labelled as Assertion (A) and the other is labelled as Reason (R)
Assertion (A) : Magnetic monopoles do not exist.
Reason (R) : Magnetic field lines are continuous and form closed loops.
In the light of the above statements, choose the most appropriate answer from the options given below :

(1) Both (A) and (R) are correct but (R) is not the correct explanation of (A)
(2) (A) is correct but (R) is not correct
(3) Both (A) and (R) are correct and (R) is the correct explanation of (A)
(4) (A) is not correct but (R) is correct

Correct Answer: (3) Both (A) and (R) are correct and (R) is the correct explanation of (A)
Solution: Step 1: Analyze Assertion (A).

Assertion (A) states that magnetic monopoles do not exist. This is a fundamental principle in magnetism, supported by all experimental evidence so far. While theoretical frameworks like Dirac's theory suggest the possibility of magnetic monopoles, they have not been observed. Therefore, Assertion (A) is considered correct within the current understanding of physics.
Step 2: Analyze Reason (R).
Reason (R) states that magnetic field lines are continuous and form closed loops. This is a direct consequence of Gauss's law for magnetism ($\oint \vec{B} \cdot d\vec{A} = 0$), which implies that there are no isolated magnetic poles (sources or sinks of magnetic field). The magnetic field lines originate from the north pole of a magnetic material and terminate at the south pole, but they continue inside the material, forming closed loops. Therefore, Reason (R) is also correct.
Step 3: Determine if Reason (R) is the correct explanation of Assertion (A).
The fact that magnetic field lines form closed loops is a direct consequence of the non-existence of isolated magnetic poles (monopoles). If magnetic monopoles existed, the magnetic field lines would originate from a north monopole and terminate at a south monopole, and they would not necessarily form closed loops. The continuity and closed-loop nature of magnetic field lines are a manifestation of the fundamental absence of magnetic monopoles. Therefore, Reason (R) is the correct explanation of Assertion (A).

Quick Tip

Remember Gauss's law for magnetism, which states that the net magnetic flux through any closed surface is zero. This law is a mathematical formulation of the non-existence of magnetic monopoles and directly leads to the property that magnetic field lines are continuous and form closed loops.

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41. Which one of the following forces cannot be expressed in terms of potential energy?

(1) Coulomb's force
(2) Gravitational force
(3) Frictional force
(4) Restoring force

Correct Answer: (3) Frictional force

Solution: Step 1: Understand the concept of conservative forces and potential energy.

A force is said to be conservative if the work done by the force in moving a particle between two points is independent of the path taken. For a conservative force, it is possible to define a potential energy function U such that the force $\vec{F}$ is related to the potential energy by $\vec{F} = -\nabla U$ (in three dimensions) or $F = -\frac{dU}{dx}$ (in one dimension). Equivalently, the work done by a conservative force over a closed path is zero.

Step 2: Analyze each of the given forces.

(1) Coulomb's force:

The electrostatic force between two charges is a conservative force. The work done by the Coulomb's force depends only on the initial and final positions of the charges, not on the path taken. The potential energy associated with the Coulomb's force is the electrostatic potential energy.

(2) Gravitational force:

The gravitational force between two masses is also a conservative force. The work done by the gravitational force depends only on the initial and final positions of the masses, and the potential energy associated with it is the gravitational potential energy.

(3) Frictional force:

Frictional force is a non-conservative force. The work done by friction depends on the path taken. For example, the work done against friction is greater along a longer path between two points. Also, the work done by friction over a closed path is not zero; it is always negative (dissipative). Therefore, frictional force cannot be expressed in terms of a potential energy function. The energy dissipated by friction is converted into heat.

(4) Restoring force (e.g., spring force):

The restoring force exerted by a spring is a conservative force. The work done by the spring force depends only on the initial and final extensions or compressions of the spring, and the potential energy associated with it is the elastic potential energy.

Step 3: Identify the force that cannot be expressed in terms of potential energy.

Based on the analysis above, frictional force is a non-conservative force and cannot be expressed in terms of potential energy.

Quick Tip

A key characteristic of conservative forces is that the work they do is path-independent, and they allow for the definition of a potential energy. Non-conservative forces, like friction, result in energy dissipation, and their work depends on the path taken.

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42. Match List-I with List-II.

List-I (Thermodynamic Process)		List-II (Characteristic)	
(A)	Isothermal	(I)	ΔW (work done) = 0
(B)	Adiabatic	(II)	ΔQ (supplied heat) = 0
(C)	Isobaric	(III)	ΔU (change in internal energy) $\neq 0$
(D)	Isochoric	(IV)	$\Delta U = 0$

Choose the correct answer from the options given below :
(1) (A)-(III), (B)-(II), (C)-(I), (D)-(IV)
(2) (A)-(IV), (B)-(I), (C)-(III), (D)-(II)
(3) (A)-(IV), (B)-(II), (C)-(III), (D)-(I)
(4) (A)-(II), (B)-(IV), (C)-(I), (D)-(III)

Correct Answer: (3) (A)-(IV), (B)-(II), (C)-(III), (D)-(I)
Solution: Step 1: Understand the characteristics of each thermodynamic process listed in List-I.
(A) Isothermal process:
An isothermal process occurs at a constant temperature ($\Delta T = 0$). For an ideal gas, the internal energy U is directly proportional to temperature. Therefore, in an isothermal process for an ideal gas, the change in internal energy is zero ($\Delta U = 0$). So, (A) matches with (IV).
(B) Adiabatic process:
An adiabatic process is one in which no heat is exchanged with the surroundings ($\Delta Q = 0$). So, (B) matches with (II).
(C) Isobaric process:
An isobaric process occurs at a constant pressure ($\Delta P = 0$). In general, for an isobaric process, work is done by or on the system ($\Delta W \neq 0$), and heat is exchanged ($\Delta Q \neq 0$), leading to a change in internal energy ($\Delta U \neq 0$). So, (C) matches with (III).
(D) Isochoric process:
An isochoric process (also called isovolumetric) occurs at a constant volume ($\Delta V = 0$). Since work done $\Delta W = P\Delta V$, in an isochoric process, the work done is zero ($\Delta W = 0$). So, (D) matches with (I).
Step 2: Match the thermodynamic processes with their corresponding characteristics.
(A) Isothermal - $\Delta U = 0$ - (IV)
(B) Adiabatic - $\Delta Q = 0$ - (II)

(C) Isobaric - $\Delta U \neq 0$ - (III)
(D) Isochoric - $\Delta W = 0$ - (I)

Step 3: Choose the correct option based on the matching.
The correct matching is (A)-(IV), (B)-(II), (C)-(III), (D)-(I), which corresponds to option (3).

Quick Tip

Remember the definitions of the four basic thermodynamic processes: - Isothermal: constant temperature ($\Delta T = 0$, implies $\Delta U = 0$ for ideal gas).
- Adiabatic: no heat exchange ($\Delta Q = 0$).
- Isobaric: constant pressure ($\Delta P = 0$).
- Isochoric: constant volume ($\Delta V = 0$, implies $\Delta W = 0$).

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43. A helicopter flying horizontally with a speed of $360\,km/h$ at an altitude of $2\,km$, drops an object at an instant. The object hits the ground at a point O, $20\,s$ after it is dropped. Displacement of 'O' from the position of helicopter where the object was released is :
(use acceleration due to gravity $g = 10\,m/s^2$ and neglect air resistance)
(1) $2\sqrt{5}\,km$
(2) $4\,km$
(3) $7.2\,km$
(4) $2\sqrt{2}\,km$

Correct Answer: (4) $2\sqrt{2}\,km$
Solution: Step 1: Convert the initial velocity of the object to m/s.
The initial horizontal velocity of the object is $u_x = 100\,m/s$.
Step 2: Calculate the horizontal distance travelled by the object.
The horizontal distance x travelled by the object is:

$$x = u_x \times t = 100\,m/s \times 20\,s = 2000\,m = 2\,km$$

Step 3: Calculate the vertical distance travelled by the object.
The vertical distance y travelled by the object is $2\,km$.
Step 4: Calculate the displacement of the point O from the release point.
The horizontal displacement is $x = 2\,km$ and the vertical displacement is $y = 2\,km$ downwards. The magnitude of the displacement $|\vec{s}|$ is:

$$|\vec{s}| = \sqrt{x^2 + y^2} = \sqrt{(2\,km)^2 + (2\,km)^2} = \sqrt{4+4}\,km = \sqrt{8}\,km = 2\sqrt{2}\,km$$

The magnitude of the displacement is $2\sqrt{2}\,km$.

Quick Tip

Remember to treat horizontal and vertical motion independently in projectile problems. The displacement is the straight-line distance between the initial and final points.

44. An object with mass $500\,g$ moves along x-axis with speed $v = 4\sqrt{x}\,m/s$. The force acting on the object is :
(1) 8 N
(2) 5 N
(3) 6 N
(4) 4 N

Correct Answer: (4) 4 N
Solution: Step 1: Convert the mass to SI units.
The mass of the object is $m = 500\,g = 0.5\,kg$.
Step 2: Find the acceleration of the object.
The speed of the object is given by $v = 4\sqrt{x}$. To find the acceleration a, we use the chain rule:

$$a = \frac{dv}{dt} = \frac{dv}{dx}\frac{dx}{dt} = v\frac{dv}{dx}$$

First, find $\frac{dv}{dx}$:

$$\frac{dv}{dx} = \frac{d}{dx}(4\sqrt{x}) = 4 \cdot \frac{1}{2\sqrt{x}} = \frac{2}{\sqrt{x}}$$

Now, substitute v and $\frac{dv}{dx}$ into the expression for acceleration:

$$a = (4\sqrt{x}) \cdot \left(\frac{2}{\sqrt{x}}\right) = 8\,m/s^2$$

The acceleration of the object is constant and equal to $8\,m/s^2$ along the x-axis.
Step 3: Calculate the force acting on the object using Newton's second law.
According to Newton's second law of motion, the force F acting on an object is equal to the product of its mass m and its acceleration a:

$$F = m \cdot a$$

Substituting the values of mass and acceleration:

$$F = (0.5\,kg) \cdot (8\,m/s^2) = 4\,kg \cdot m/s^2 = 4\,N$$

The force acting on the object is $4\,N$.

Quick Tip

When the velocity is given as a function of position, use the chain rule for differentiation to find the acceleration: $a = v\frac{dv}{dx}$. Then, apply Newton's second law $F = ma$ to find the force.

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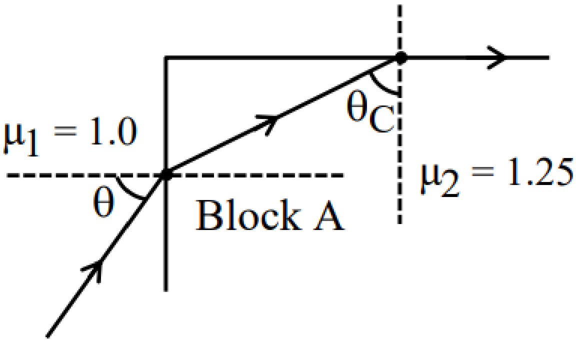
45. A transparent block A having refractive index $\mu_2 = 1.25$ is surrounded by another medium of refractive index $\mu_1 = 1.0$ as shown in figure. A light ray is incident on the flat face of the block with incident angle θ as shown in figure. What is the maximum value of θ for which light suffers total internal reflection at the top

surface of the block ?



(1) $\tan^{-1}(4/3)$
(2) $\tan^{-1}(3/4)$
(3) $\sin^{-1}(3/4)$
(4) $\cos^{-1}(3/4)$

Correct Answer: (3) $\sin^{-1}(3/4)$
Solution: Step 1: Apply Snell's Law at the air–block interface.
Let r be the angle of refraction inside the block.

$$\sin\theta = 1.25\sin r = \frac{5}{4}\sin r \implies \sin r = \frac{4}{5}\sin\theta$$

Step 2: Determine the condition for total internal reflection at the top surface.
The angle of incidence at the top surface is $i = 90^\circ - r$. For total internal reflection to occur at the block-air interface, i must be greater than or equal to the critical angle θ_C, where $\sin\theta_C = \frac{\mu_1}{\mu_2} = \frac{1}{1.25} = \frac{4}{5}$. So, we need $90^\circ - r \geq \theta_C$, which implies $\sin(90^\circ - r) \geq \sin\theta_C$, or $\cos r \geq \frac{4}{5}$.
Step 3: Use a trigonometric identity to express $\cos r$ in terms of $\sin r$.
We know that $\cos r = \sqrt{1 - \sin^2 r}$. Substituting the expression for $\sin r$ from Step 1:

$$\cos r = \sqrt{1 - \left(\frac{4}{5}\sin\theta\right)^2} = \sqrt{1 - \frac{16}{25}\sin^2\theta}$$

Step 4: Apply the condition for total internal reflection.

$$\sqrt{1 - \frac{16}{25}\sin^2\theta} \geq \frac{4}{5}$$

Squaring both sides:

$$1 - \frac{16}{25}\sin^2\theta \geq \frac{16}{25}$$
$$1 - \frac{16}{25} \geq \frac{16}{25}\sin^2\theta$$
$$\frac{9}{25} \geq \frac{16}{25}\sin^2\theta$$
$$9 \geq 16\sin^2\theta$$
$$\sin^2\theta \leq \frac{9}{16}$$

$$|\sin\theta| \leq \frac{3}{4}$$

Since θ is the angle of incidence ($0^\circ \leq \theta \leq 90^\circ$), $\sin\theta \geq 0$.

$$\sin\theta \leq \frac{3}{4}$$

The maximum value of θ occurs when $\sin\theta = \frac{3}{4}$, so $\theta_{max} = \sin^{-1}(3/4)$.

Quick Tip

Remember to apply Snell's law at the first interface to relate the angle of incidence θ to the angle of refraction r inside the block. Then, use the geometry to find the angle of incidence at the second interface and apply the condition for total internal reflection, which involves the critical angle.

SECTION-B

46. A parallel plate capacitor has charge $5 \times 10^{-6}\,C$. A dielectric slab is inserted between the plates and almost fills the space between the plates. If the induced charge on one face of the slab is $4 \times 10^{-6}\,C$ then the dielectric constant of the slab is ____.

Solution: Step 1: Understand the effect of a dielectric on a capacitor.

When a dielectric material is inserted between the plates of a charged capacitor, it becomes polarized, and an induced charge appears on its surfaces. This induced charge creates an electric field that opposes the original electric field due to the charges on the capacitor plates. The net electric field inside the dielectric is reduced, and consequently, the potential difference across the plates decreases, while the charge on the plates remains the same (if the capacitor is isolated).

Step 2: Relate the induced charge to the free charge and the dielectric constant.

Let Q be the free charge on the capacitor plates, and Q_i be the magnitude of the induced charge on each face of the dielectric slab. The relationship between these charges and the dielectric constant K of the material is given by:

$$Q_i = Q\left(1 - \frac{1}{K}\right)$$

Step 3: Substitute the given values into the formula.

We are given:

Free charge on the capacitor plates, $Q = 5 \times 10^{-6}\,C$ Induced charge on one face of the dielectric slab, $Q_i = 4 \times 10^{-6}\,C$

Substituting these values into the formula:

$$4 \times 10^{-6} = 5 \times 10^{-6}\left(1 - \frac{1}{K}\right)$$

Step 4: Solve for the dielectric constant K.

Divide both sides by 5×10^{-6}:

$$\frac{4 \times 10^{-6}}{5 \times 10^{-6}} = 1 - \frac{1}{K}$$

$$\frac{4}{5} = 1 - \frac{1}{K}$$

Rearrange the equation to solve for $\frac{1}{K}$:

$$\frac{1}{K} = 1 - \frac{4}{5}$$
$$\frac{1}{K} = \frac{5}{5} - \frac{4}{5}$$
$$\frac{1}{K} = \frac{1}{5}$$

Now, solve for K:

$$K = 5$$

The dielectric constant of the slab is 5.

Quick Tip

The induced charge on the dielectric reduces the effective charge that contributes to the electric field inside the capacitor. The factor by which the electric field (and hence the potential difference) is reduced is the dielectric constant K. The relationship $Q_i = Q(1 - 1/K)$ is crucial for solving problems involving dielectrics in capacitors.

---

47. An inductor of reactance $100\,\Omega$, a capacitor of reactance $50\,\Omega$, and a resistor of resistance $50\,\Omega$ are connected in series with an AC source of $10\,V$, $50\,Hz$. Average power dissipated by the circuit is ____ W.

Solution: Step 1: Identify the given parameters.

Inductive reactance, $X_L = 100\,\Omega$
Capacitive reactance, $X_C = 50\,\Omega$
Resistance, $R = 50\,\Omega$
RMS voltage of the AC source, $V_{rms} = 10\,V$
Frequency of the AC source, $f = 50\,Hz$

Step 2: Calculate the impedance Z of the series LCR circuit.

The impedance of a series LCR circuit is given by:

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

Substitute the given values:

$$Z = \sqrt{(50\,\Omega)^2 + (100\,\Omega - 50\,\Omega)^2}$$

$$Z = \sqrt{(50)^2 + (50)^2} = \sqrt{2500 + 2500} = \sqrt{5000}\,\Omega$$

$$Z = 50\sqrt{2}\,\Omega$$

Step 3: Calculate the RMS current I_{rms} in the circuit.

Using Ohm's law for AC circuits:

$$I_{rms} = \frac{V_{rms}}{Z}$$

Substitute the values of V_{rms} and Z:

$$I_{rms} = \frac{10\,V}{50\sqrt{2}\,\Omega} = \frac{1}{5\sqrt{2}}\,A = \frac{\sqrt{2}}{10}\,A$$

Step 4: Calculate the average power P_{avg} dissipated by the circuit.
The average power dissipated in an AC circuit is only through the resistor and is given by:

$$P_{avg} = I_{rms}^2 R$$

Substitute the values of I_{rms} and R:

$$P_{avg} = \left(\frac{\sqrt{2}}{10}\,A\right)^2 \times 50\,\Omega$$

$$P_{avg} = \left(\frac{2}{100}\right) \times 50\,W$$

$$P_{avg} = \frac{1}{50} \times 50\,W$$

$$P_{avg} = 1\,W$$

The average power dissipated by the circuit is 1 W.

Quick Tip

In an AC circuit containing resistors, inductors, and capacitors, only the resistor dissipates average power. The inductor and capacitor store and release energy but do not dissipate it on average over a complete cycle. The power dissipated is calculated using the RMS current and the resistance.

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48. Two cylindrical rods A and B made of different materials, are joined in a straight line. The ratio of lengths, radii and thermal conductivities of these rods are : $\frac{L_A}{L_B} = \frac{1}{2}$, $\frac{r_A}{r_B} = 2$, and $\frac{K_A}{K_B} = \frac{1}{2}$. The free ends of rods A and B are maintained at $400\,K$, $200\,K$, respectively. The temperature of rods interface is ____ K, when equilibrium is established.

Solution: Step 1: Define the thermal resistance of each rod. The thermal resistance R_{th} of a cylindrical rod is given by $R_{th} = \frac{L}{KA}$, where L is the length, K is the thermal conductivity, and A is the cross-sectional area of the rod. The cross-sectional area of a cylindrical rod with radius r is $A = \pi r^2$.
For rod A:
Length L_A
Radius r_A
Thermal conductivity K_A
Area $A_A = \pi r_A^2$
Thermal resistance $R_{th,A} = \frac{L_A}{K_A \pi r_A^2}$

For rod B:
Length L_B

Radius r_B
Thermal conductivity K_B
Area $A_B = \pi r_B^2$
Thermal resistance $R_{th,B} = \frac{L_B}{K_B \pi r_B^2}$

Step 2: Use the given ratios to relate the thermal resistances.
We are given $\frac{L_A}{L_B} = \frac{1}{2}$, $\frac{r_A}{r_B} = 2$, and $\frac{K_A}{K_B} = \frac{1}{2}$.
Consider the ratio of the thermal resistances:

$$\frac{R_{th,A}}{R_{th,B}} = \frac{\frac{L_A}{K_A \pi r_A^2}}{\frac{L_B}{K_B \pi r_B^2}} = \frac{L_A}{L_B} \cdot \frac{K_B}{K_A} \cdot \frac{\pi r_B^2}{\pi r_A^2} = \frac{L_A}{L_B} \cdot \frac{K_B}{K_A} \cdot \left(\frac{r_B}{r_A}\right)^2$$

Substitute the given ratios:

$$\frac{R_{th,A}}{R_{th,B}} = \left(\frac{1}{2}\right) \cdot \left(\frac{1}{1/2}\right) \cdot \left(\frac{1}{2}\right)^2 = \frac{1}{2} \cdot 2 \cdot \frac{1}{4} = \frac{1}{4}$$

So, $R_{th,A} = \frac{1}{4} R_{th,B}$, or $R_{th,B} = 4R_{th,A}$.
Step 3: Apply the concept of thermal current in series.
When the rods are joined in series, the rate of heat flow (thermal current I_{th}) through both rods is the same at equilibrium. Let the temperature of the interface be T. The thermal current through rod A is given by:

$$I_{th} = \frac{T_1 - T}{R_{th,A}} = \frac{400 - T}{R_{th,A}}$$

The thermal current through rod B is given by:

$$I_{th} = \frac{T - T_2}{R_{th,B}} = \frac{T - 200}{R_{th,B}}$$

Equating the thermal currents:

$$\frac{400 - T}{R_{th,A}} = \frac{T - 200}{R_{th,B}}$$

Substitute $R_{th,B} = 4R_{th,A}$:

$$\frac{400 - T}{R_{th,A}} = \frac{T - 200}{4R_{th,A}}$$

Multiply both sides by $4R_{th,A}$:

$$4(400 - T) = T - 200$$

$$1600 - 4T = T - 200$$

$$1600 + 200 = T + 4T$$

$$1800 = 5T$$

$$T = \frac{1800}{5} = 360\,K$$

The temperature of the rods interface is $360\,K$.

Quick Tip

Treat heat flow problems involving composite materials as analogous to electrical circuits. Thermal resistance plays the role of electrical resistance, temperature difference is analogous to voltage difference, and the rate of heat flow (thermal current) corresponds to electrical current. For rods in series, the thermal resistance is additive, and the thermal current is the same through each rod.

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49. The electric field in a region is given by $\vec{E} = (2\hat{i} + 4\hat{j} + 6\hat{k}) \times 10^3\, N/C$. The flux of the field through a rectangular surface parallel to x-z plane is $6.0\, Nm^2C^{-1}$. The area of the surface is ____ cm^2.

Solution: Step 1: Understand the orientation of the surface.

The rectangular surface is parallel to the x-z plane. This means that the normal vector to the surface is along the y-axis (either $+\hat{j}$ or $-\hat{j}$). We can represent the area vector $\vec{A}$ as $\vec{A} = A\hat{j}$, where A is the area of the surface and $\hat{j}$ is the unit vector in the y-direction.

Step 2: Use the formula for electric flux.

The electric flux Φ through a surface is given by the dot product of the electric field $\vec{E}$ and the area vector $\vec{A}$:

$$\Phi = \vec{E} \cdot \vec{A}$$

Step 3: Substitute the given electric field and the area vector into the flux formula.

The electric field is $\vec{E} = (2\hat{i} + 4\hat{j} + 6\hat{k}) \times 10^3\, N/C$.

The area vector is $\vec{A} = A\hat{j}$.

The flux is given as $\Phi = 6.0\, Nm^2C^{-1}$.

$$6.0 = [(2\hat{i} + 4\hat{j} + 6\hat{k}) \times 10^3] \cdot (A\hat{j})$$

The dot product of the unit vectors is $\hat{i} \cdot \hat{j} = 0$, $\hat{j} \cdot \hat{j} = 1$, and $\hat{k} \cdot \hat{j} = 0$.

$$6.0 = (2 \times 10^3\hat{i} \cdot A\hat{j}) + (4 \times 10^3\hat{j} \cdot A\hat{j}) + (6 \times 10^3\hat{k} \cdot A\hat{j})$$

$$6.0 = 0 + (4 \times 10^3 \times A \times 1) + 0$$

$$6.0 = 4 \times 10^3 A$$

Step 4: Solve for the area A in m^2.

$$A = \frac{6.0}{4 \times 10^3}\, m^2$$

$$A = 1.5 \times 10^{-3}\, m^2$$

Step 5: Convert the area from m^2 to cm^2. We know that $1\, m = 100\, cm$, so $1\, m^2 = (100\, cm)^2 = 10000\, cm^2 = 10^4\, cm^2$.

$$A = 1.5 \times 10^{-3}\, m^2 \times \frac{10^4\, cm^2}{1\, m^2}$$

$$A = 1.5 \times 10^{-3+4}\, cm^2$$

$$A = 1.5 \times 10^1\, cm^2$$

$$A = 15\,cm^2$$

The area of the surface is $15\,cm^2$.

Quick Tip

The electric flux through a surface depends on the component of the electric field that is normal to the surface. When the surface is parallel to the x-z plane, its normal vector is along the y-axis, so only the y-component of the electric field contributes to the flux. Remember to convert units if the final answer requires a specific unit.

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50. M and R be the mass and radius of a disc. A small disc of radius R/3 is removed from the bigger disc as shown in figure. The moment of inertia of remaining part of bigger disc about an axis AB passing through the centre O and perpendicular to the plane of disc is $\frac{4}{x}MR^2$. The value of x is ____.

Solution:

0.3 Given:

- Mass of original disc: M
- Radius of original disc: R
- Radius of removed disc: $R/3$
- Moment of inertia of remaining part: $\frac{4}{x}MR^2$

Step 1: Moment of Inertia of Original Disc

The moment of inertia of a solid disc about an axis through its center perpendicular to its plane is:

$$I_{\text{original}} = \frac{1}{2}MR^2$$

Step 2: Mass of Removed Disc

Assuming uniform mass distribution, the mass of the removed disc is proportional to its area:

$$m = \left(\frac{\pi(R/3)^2}{\pi R^2}\right)M = \frac{M}{9}$$

Step 3: Moment of Inertia of Removed Disc

Case 1: Concentric Removal

If the disc is removed concentrically:

$$I_{\text{removed}} = \frac{1}{2}m\left(\frac{R}{3}\right)^2 = \frac{1}{2}\left(\frac{M}{9}\right)\left(\frac{R^2}{9}\right) = \frac{MR^2}{162}$$

Case 2: Non-concentric Removal If the disc is removed tangentially (center at $2R/3$ from O), we must use the parallel axis theorem:

$$I_{\text{removed}} = \frac{1}{2}m\left(\frac{R}{3}\right)^2 + m\left(\frac{2R}{3}\right)^2 = \frac{MR^2}{162} + \frac{4MR^2}{81} = \frac{MR^2}{18}$$

Step 4: Moment of Inertia of Remaining Part
For Concentric Case:

$$I_{\text{remaining}} = I_{\text{original}} - I_{\text{removed}} = \frac{1}{2}MR^2 - \frac{MR^2}{162} = \frac{40}{81}MR^2$$

Given that $I_{\text{remaining}} = \frac{4}{x}MR^2$, we get:

$$\frac{40}{81} = \frac{4}{x} \implies x = \frac{81}{10}$$

For Non-concentric Case:

$$I_{\text{remaining}} = \frac{1}{2}MR^2 - \frac{MR^2}{18} = \frac{4}{9}MR^2$$

Given that $I_{\text{remaining}} = \frac{4}{x}MR^2$, we get:

$$\frac{4}{9} = \frac{4}{x} \implies x = 9$$

Conclusion Since the problem mentions "as shown in figure" but no figure is provided, the most reasonable assumption is that the disc is removed tangentially (non-concentrically), leading to:

$$x = \boxed{9}$$

Quick Tip

When a part of a uniform object is removed, the moment of inertia of the remaining part can be found by subtracting the moment of inertia of the removed part from the moment of inertia of the original object, ensuring both moments of inertia are calculated about the same axis. Remember to use the parallel axis theorem if the axes of the removed part and the remaining part are not the same.
