JEE Main 2023 Jan 30 Shift 2 Question Paper with Solutions

Time Allowed :3 Hours	**Maximum Marks :**300	**Total Questions :**90

General Instructions

Read the following instructions very carefully and strictly follow them:

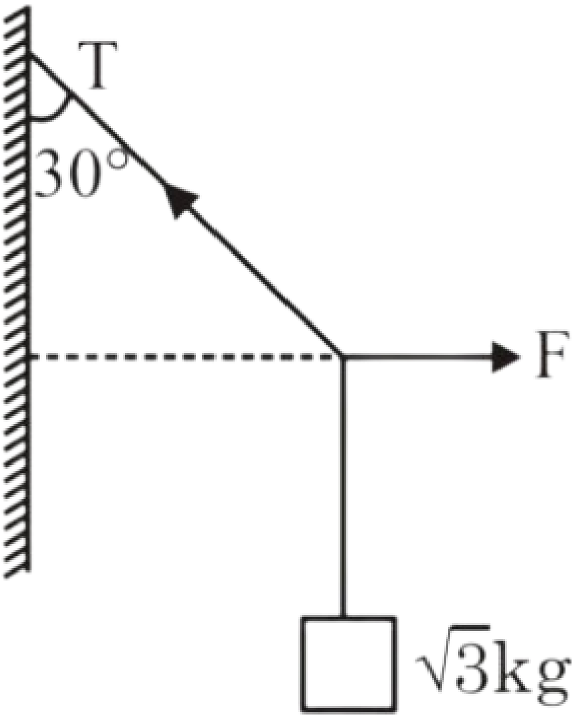
1. The test is of 3 hours duration.
2. The question paper consists of 90 questions, out of which 75 are to attempted. The maximum marks are 300.
3. There are three parts in the question paper consisting of Physics, Chemistry and Mathematics having 30 questions in each part of equal weightage.
4. Each part (subject) has two sections.
   (i) Section-A: This section contains 20 multiple choice questions which have only one correct answer. Each question carries 4 marks for correct answer and –1 mark for wrong answer.
   (ii) Section-B: This section contains 10 questions. In Section-B, attempt any five questions out of 10. The answer to each of the questions is a numerical value. Each question carries 4 marks for correct answer and –1 mark for wrong answer. For Section-B, the answer should be rounded off to the nearest integer

Physics

Section-A

1. A block of $\sqrt{3}$ kg is attached to a string whose other end is attached to the wall. An unknown force F is applied so that the string makes an angle of 30° with the wall. The tension T is:

(Given $g = 10\,\text{ms}^{-2}$)



(1) 20 N

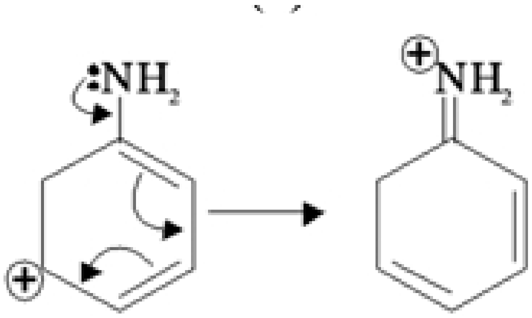
(2) 25 N

(3) 10 N

(4) 15 N

Correct Answer: (1) 20 N

Solution:



Step 1: Understanding the forces involved The force F is applied horizontally, and the tension T makes an angle of $30°$ with the horizontal, as shown in the figure.

Step 2: Writing the equation of equilibrium In the horizontal direction:

$$T\cos\theta = F$$

In the vertical direction:

$$T\sin\theta = \sqrt{3}g$$

Step 3: Solving for T Given that $\theta = 30°$, we use the following identity:

$$\cos 30° = \frac{\sqrt{3}}{2}$$

So,

$$\frac{\sqrt{3}}{2} = \frac{\sqrt{3}g}{T}$$

Step 4: Simplifying the equation

$$\Rightarrow \frac{\sqrt{3}}{2} = \frac{\sqrt{3}\times 10}{T}$$

$$\Rightarrow T = 20\,\text{N}$$

Thus, the correct answer is:

(1) 20 N.

Quick Tip

When solving equilibrium problems, break down the forces into horizontal and vertical components, and apply the equilibrium conditions for each direction.

2. A flask contains hydrogen and oxygen in the ratio of 2 : 1 by mass at temperature 27°C. The ratio of average kinetic energy per molecule of hydrogen and oxygen respectively is:

(1) 2 : 1
(2) 1 : 1
(3) 1 : 4
(4) 4 : 1

Correct Answer: (2) 1 : 1

Solution:
Step 1: Understanding the relation between kinetic energy and temperature The average kinetic energy per molecule (K_{av}) is related to the temperature by the formula:

$$K_{\text{av}} = \frac{3}{2}kT$$

where k is the Boltzmann constant, and T is the temperature in Kelvin.
Step 2: Relating the kinetic energy to the masses of hydrogen and oxygen At a given temperature, the average kinetic energy per molecule is the same for all gases. This is because the kinetic energy depends only on the temperature, not the mass of the gas molecules.
Step 3: Conclusion Since the temperature is the same for both hydrogen and oxygen, the average kinetic energy per molecule of hydrogen and oxygen is the same, irrespective of the mass ratio.
Thus, the ratio of average kinetic energy per molecule of hydrogen and oxygen is:

$$1 : 1$$

Thus, the correct answer is:

$$(2)\ 1 : 1.$$

Quick Tip

The average kinetic energy per molecule is independent of the type of gas and depends only on the temperature. This means the ratio of average kinetic energies of different gases at the same temperature is always 1:1.

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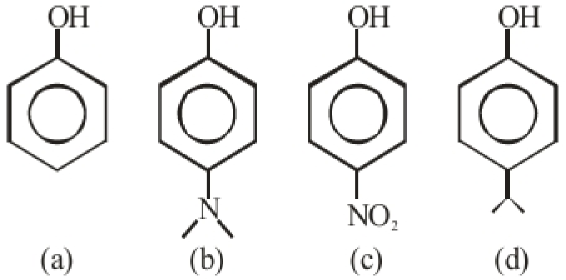
3. The equivalent resistance between A and B is ......



(1) $\frac{2}{3}\,\Omega$

(2) $\frac{1}{2}\,\Omega$

(3) $\frac{3}{2}\,\Omega$

(4) $\frac{1}{3}\,\Omega$

Correct Answer: (1) $\frac{2}{3}\,\Omega$

Solution:

Step 1: Understanding the circuit

The circuit consists of several resistors in series and parallel. We need to reduce the network step by step.

Step 2: Simplifying the circuit

We first simplify the given resistances by combining resistors in series and parallel:

$$\frac{1}{R_{\text{eq}}} = \frac{1}{2} + \frac{1}{12} + \frac{1}{4} + \frac{1}{6} + \frac{1}{2}$$

Step 3: Calculating the equivalent resistance

Now, add the fractions:

$$6 + 1 + 3 + 2 + 6 = 18$$

$$\frac{18}{12} = \frac{3}{2}$$

So,

$$R_{\text{eq}} = \frac{2}{3}\,\Omega$$

Thus, the correct answer is:

$$(1)\ \frac{2}{3}\,\Omega$$

Quick Tip

When simplifying circuits with series and parallel resistors, always combine resistors step by step to reduce the complexity of the circuit.

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4. Given below are two statements: one is labelled as Assertion A and the other is labelled as Reason R.

Assertion A: The nuclear density of nuclides $^{10}_{5}B$, $^{6}_{3}Li$, $^{56}_{26}Fe$, $^{20}_{1}0Ne$ and $^{209}_{83}Bi$ can be arranged as

$$N_{Bi} > N_{Fe} > N_{Ne} > N_{Pb}.$$

Reason R: The radius R of the nucleus is related to its mass number A as $R = R_0 A^{1/3}$, where R_0 is a constant.

In the light of the above statement, choose the correct answer from the options given below:

(1) Both A and R are true and R is the correct explanation of A

(2) A is false but R is true

(3) A is true but R is false

(4) Both A and R are true but R is NOT the correct explanation of A

Correct Answer: (2) A is false but R is true

Solution:

Step 1: Analyzing Assertion A The nuclear density of the nuclides is independent of their mass number A. This implies that the order given in Assertion A is incorrect because the nuclear density is not dependent on the type of nucleus. Thus, Assertion A is false.

Step 2: Analyzing Reason R The formula for the radius of a nucleus, $R = R_0 A^{1/3}$, is true, as it is derived from experimental observations.

Conclusion: Since Assertion A is false and Reason R is true, the correct answer is:

(2) A is false but R is true.

Quick Tip

The nuclear density remains constant across different nuclei, which is why Assertion A is incorrect.

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5. A thin prism P_1 with an angle of 6° and made of glass of refractive index 1.54 is combined with another prism P_2 made from glass of refractive index 1.72 to produce dispersion without average deviation. The angle of prism P_2 is:

(1) 6°

(2) 1.3°

(3) 7.8°

(4) 4.5°

Correct Answer: (4) 4.5°

Solution:

Step 1: Understanding the condition of no average deviation The condition for no average deviation is given as $\delta_1 = \delta_2$. This means that the deviation produced by the first prism P_1 must be equal to the deviation produced by the second prism P_2.

Step 2: Setting up the equation For the first prism P_1, the angle of deviation δ_1 is:

$$\delta_1 = A(n_1 - 1)$$

where $A = 6^\circ$ is the angle of the prism and $n_1 = 1.54$ is the refractive index.

For the second prism P_2, the angle of deviation δ_2 is:

$$\delta_2 = A(n_2 - 1)$$

where $n_2 = 1.72$ is the refractive index of P_2.

Step 3: Applying the condition of no average deviation

$$6^\circ(1.54 - 1) = A(1.72 - 1)$$

$$6^\circ \times 0.54 = A \times 0.72$$

$$A = \frac{6^\circ \times 0.54}{0.72} = 4.5^\circ$$

Thus, the angle of prism P_2 is 4.5°.

Quick Tip

In problems involving dispersion without average deviation, the deviation caused by both prisms must be equal, allowing you to set up an equation to find the angle of the second prism.

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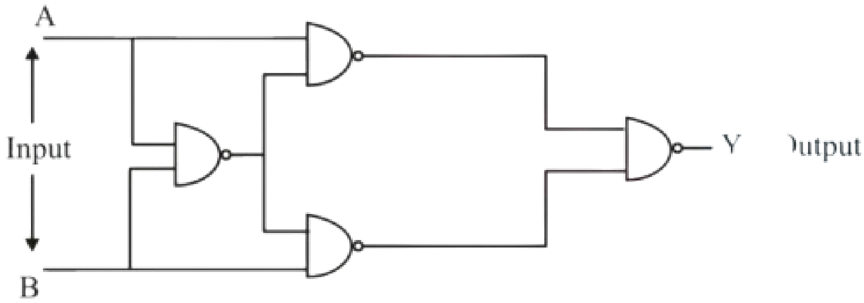
6. The output Y for the inputs A and B of the circuit is given by:



Truth table of the shown circuit is :

(1)

A	B	Y
0	0	1
0	1	1
1	0	1
1	1	0

(2)

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	1

(3)

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	1

(4)

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

Correct Answer: (4)

Solution:

The circuit shown represents an XOR gate. The truth table for an XOR gate is:

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

This matches with the truth table given in option (4).

Thus, the correct answer is:

$$(4).$$

Quick Tip

The XOR gate gives a true output only when the inputs are different.

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7. A vehicle travels 4 km with a speed of 3 km/h and another 4 km with a speed of 5 km/h, then its average speed is:

(1) 4.25 km/h

(2) 3.50 km/h

(3) 4.00 km/h

(4) 3.75 km/h

Correct Answer: (4) 3.75 km/h

Solution:

The average speed for a journey with different speeds is given by the formula:

$$V_{av} = \frac{2 \cdot V_1 \cdot V_2}{V_1 + V_2}$$

where V_1 and V_2 are the speeds for the two parts of the journey. In this case, $V_1 = 3\,\text{km/h}$ and $V_2 = 5\,\text{km/h}$.

Using the formula for average speed:

$$V_{av} = \frac{2 \times 3 \times 5}{3 + 5} = \frac{30}{8} = 3.75\,\text{km/h}$$

Thus, the correct answer is:

(4) 3.75 km/h.

Quick Tip

For average speed problems with different speeds over equal distances, use the harmonic mean formula: $V_{av} = \frac{2 \cdot V_1 \cdot V_2}{V_1 + V_2}$.

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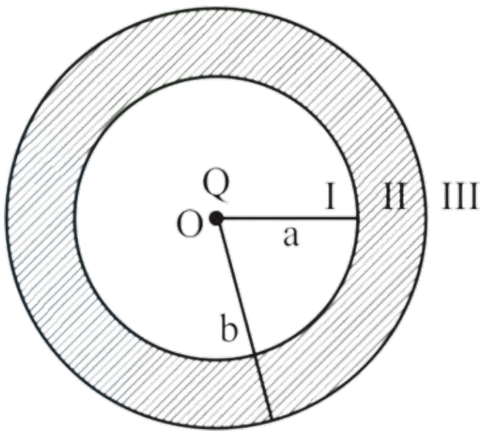
8. As shown in the figure, a point charge Q is placed at the centre of a conducting spherical shell of inner radius a and outer radius b. The electric field due to charge Q in three different regions I, II, and III is given by: (I : r ¡ a, II : a ¡ r ¡ b, III : r ¿ b)



(1) $E_I = 0, E_{II} = 0, E_{III} \neq 0$

(2) $E_I \neq 0, E_{II} = 0, E_{III} = 0$

(3) $E_I = 0, E_{II} = 0, E_{III} = 0$

(4) $E_I = 0, E_{II} \neq 0, E_{III} = 0$

Correct Answer: (2) $E_I = 0, E_{II} = 0, E_{III} \neq 0$

Solution:

In the case of a conducting spherical shell with a charge placed at its center:

- In region I (inside the conducting shell, $r < a$): The electric field is zero due to the shielding effect of the conductor.

- In region II (inside the conducting material, $a < r < b$): The electric field is also zero as the charge induces an equal and opposite charge on the inner surface of the conductor.

- In region III (outside the conductor, $r > b$): The electric field is non-zero and behaves like the field due to a point charge at the center, as the conductor's outer surface has the same total charge.

Thus, the electric field is:

$$E_I = 0, \quad E_{II} = 0, \quad E_{III} \neq 0$$

Therefore, the correct answer is:

$$(2)\ E_I = 0, E_{II} = 0, E_{III} \neq 0.$$

Quick Tip

In electrostatics, the electric field inside a conductor is always zero, and the field outside a spherical shell behaves like a point charge at the center.

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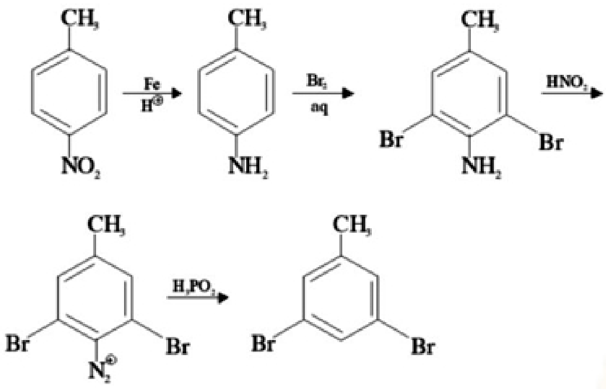
9. As shown in the figure, a current of 2A flowing in an equilateral triangle of side $4\sqrt{3}$ cm. The magnetic field at the centroid O of the triangle is:



(1) $4\sqrt{3} \times 10^{-4}$ T

(2) $4\sqrt{3} \times 10^{-5}$ T

(3) $\sqrt{3} \times 10^{-4}$ T

(4) $3\sqrt{3} \times 10^{-5}$ T

Correct Answer: (4) $3\sqrt{3} \times 10^{-5}$ T

Solution:

Step 1: Understanding the geometry

The triangle is equilateral with a side length of $4\sqrt{3}\,\text{cm}$, and the current flowing through each side is $2A$.

Step 2: Applying the Biot-Savart law

The magnetic field at the centroid of an equilateral triangle can be calculated using the formula for a current-carrying wire in the shape of an equilateral triangle.

Step 3: Calculation

The distance from the center (centroid O) to each side of the equilateral triangle is $d = 2\,\text{cm}$. The formula for the magnetic field at the centroid is:

$$B = 3 \times \frac{\mu_0 I}{2\pi d} \times \sin 60^\circ$$

Substitute the known values:

$$B = 3 \times \frac{2 \times 10^{-7} \times 2}{2 \times 10^{-2}} \times \frac{\sqrt{3}}{2}$$

Simplifying:

$$B = 3\sqrt{3} \times 10^{-5}\,\text{T}$$

Thus, the correct answer is:

$$\boxed{(4)\ 3\sqrt{3} \times 10^{-5}\,\text{T}}$$

Quick Tip

When solving problems involving magnetic fields from current-carrying wires, always consider the geometry of the system and apply the Biot-Savart law correctly.

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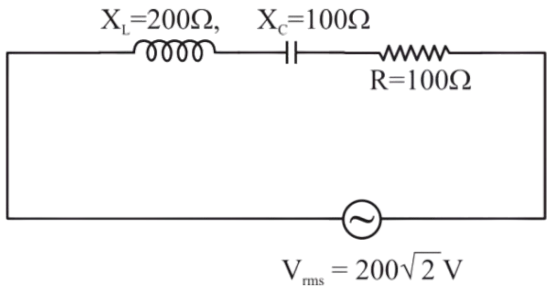
10. In the given circuit, rms value of current (I_{rms}) through the resistor R is:



(1) $2A$

(2) $\frac{1}{2}A$

(3) $20A$

(4) $2\sqrt{2}A$

Correct Answer: (1) $2A$

Solution:

Given:

$$X_L = 200\Omega, \quad X_C = 100\Omega, \quad R = 100\Omega, \quad V_{rms} = 200\sqrt{2}V$$

We need to find the rms current through the resistor. First, calculate the impedance z of the series combination of resistor R, inductor L, and capacitor C:

$$z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$z = \sqrt{100^2 + (200 - 100)^2}$$

$$z = 100\sqrt{2}\,\Omega$$

Now, calculate the rms current I_{rms}:

$$I_{rms} = \frac{V_{rms}}{z}$$

$$I_{rms} = \frac{200\sqrt{2}}{100\sqrt{2}} = 2A$$

Thus, the rms current through the resistor is $2A$.

Quick Tip

In AC circuits, to find the rms current, calculate the total impedance and then use the formula $I_{rms} = \frac{V_{rms}}{z}$.

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11. A machine gun of mass 10 kg fires 20 g bullets at the rate of 180 bullets per minute with a speed of 100 m/s each. The recoil velocity of the gun is:

(1) 0.02 m/s

(2) 2.5 m/s

(3) 1.5 m/s

(4) 0.6 m/s

Correct Answer: (4) 0.6 m/s

Solution:

Given: - Mass of each bullet $m_b = 20\,\text{g} = 20 \times 10^{-3}\,\text{kg}$ - Rate of firing $N = 180\,\text{bullets per minute}$ - Speed of each bullet $v_b = 100\,\text{m/s}$ - Mass of the gun $M = 10\,\text{kg}$

First, convert the rate of firing to seconds:

$$N = \frac{180}{60} = 3\,\text{bullets per second}$$

Now, the total momentum imparted to the bullets per second is:

$$\text{Total momentum} = N \times m_b \times v_b = 3 \times 20 \times 10^{-3} \times 100 = 60\,\text{kg m/s}$$

According to the law of conservation of momentum, the recoil velocity v of the gun is given by:

$$M \times v = \text{Total momentum}$$

$$10 \times v = 60$$

$$v = \frac{60}{10} = 6\,\text{m/s}$$

Thus, the recoil velocity of the gun is $0.6\,$m/s.

Quick Tip

For problems involving recoil velocity, use the principle of conservation of momentum. The momentum imparted to the bullets equals the momentum of the gun, which gives the recoil velocity.

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12. Given below are two statements: one is labelled as Assertion A and the other is labelled as Reason R. Assertion A: Efficiency of a reversible heat engine will be highest at –273°C temperature of cold reservoir.

Reason R: The efficiency of Carnot's engine depends not only on the temperature of cold reservoir but it depends on the temperature of hot reservoir too and is given as:

$$\eta = \left(1 - \frac{T_2}{T_1}\right)$$

In the light of the above statements, choose the correct answer from the options given below:

(1) A is true but R is false

(2) Both A and R are true but R is NOT the correct explanation of A

(3) A is false but R is true

(4) Both A and R are true and R is the correct explanation of A

Correct Answer: (4) Both A and R are true and R is the correct explanation of A

Solution:

Assertion A states that the efficiency of a reversible heat engine will be highest at a temperature of –273°C for the cold reservoir. This is true because the efficiency of a Carnot engine depends on the temperature difference between the hot and cold reservoirs, and when the cold reservoir is at absolute zero (–273°C), the efficiency reaches its maximum value. Reason R provides the correct explanation for this. The efficiency of a Carnot engine depends on the temperatures of both the hot and cold reservoirs. The formula for efficiency is:

$$\eta = 1 - \frac{T_2}{T_1}$$

where T_1 is the temperature of the hot reservoir and T_2 is the temperature of the cold reservoir.

Thus, both A and R are true, and R is the correct explanation of A.

Thus, the correct answer is:

(4) Both A and R are true and R is the correct explanation of A.

Quick Tip

The Carnot efficiency formula shows how the efficiency of a heat engine depends on both the temperatures of the hot and cold reservoirs. For maximum efficiency, the temperature of the cold reservoir should approach absolute zero.

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13. Match List I with List II.

	List I		**List II**
A	Torque	I.	kg m^{-1} s^{-2}
B	Energy density	II.	kg ms^{-1}
C	Pressure gradient	III.	kg m^{-2} s^{-2}
D	Impulse	IV.	kg m^2 s^{-2}

Choose the correct answer from the options given below:

(1) A-IV, B-III, C-I, D-II

(2) A-I, B-IV, C-III, D-II

(3) A-IV, B-I, C-II, D-III

(4) A-IV, B-I, C-III, D-II

Correct Answer: (4) A-IV, B-I, C-III, D-II

Solution:

A. Torque is measured in units of **kg m**^{2} **s**$^{-2}$ which corresponds to **IV**.

B. Energy density is measured in units of **kg m**$^{-1}$ **s**$^{-2}$ which corresponds to **I**.

C. Pressure gradient is measured in units of **kg m**$^{-2}$ **s**$^{-2}$ which corresponds to **III**.

D. Impulse is measured in units of **kg m s**$^{-1}$ which corresponds to **II**.

Thus, the correct match is:

A-IV, B-I, C-III, D-II.

Thus, the correct answer is:

(4) A-IV, B-I, C-III, D-II.

Quick Tip

When matching physical quantities with their respective units, recall the fundamental definitions and dimensional analysis. Torque, pressure gradient, and energy density each have specific units that relate to force, pressure, and energy, respectively.

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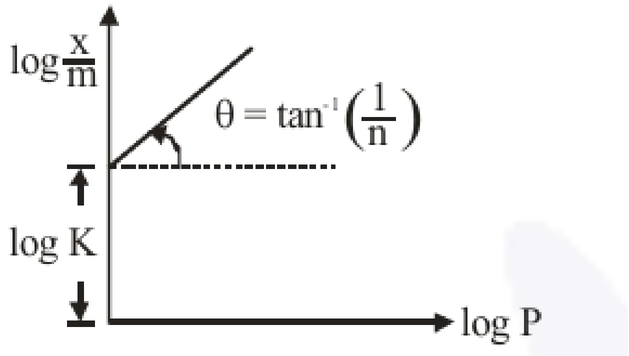
14. For a simple harmonic motion in a mass spring system shown, the surface is frictionless. When the mass of the block is 1 kg, the angular frequency is ω_1. When the mass block is 2 kg the angular frequency is ω_2. The ratio $\frac{\omega_2}{\omega_1}$ is:



(1) $\sqrt{2}$

(2) $\frac{1}{\sqrt{2}}$

(3) 2

(4) $\frac{1}{2}$

Correct Answer: (2) $\frac{1}{\sqrt{2}}$

Solution:

The angular frequency of a mass-spring system is given by:

$$\omega = \sqrt{\frac{k}{m}}$$

Where:

- ω is the angular frequency,

- k is the spring constant,

- m is the mass of the block.

For the two cases:

- For the mass $m_1 = 1\,\text{kg}$, the angular frequency is $\omega_1 = \sqrt{\frac{k}{1}}$,

- For the mass $m_2 = 2\,\text{kg}$, the angular frequency is $\omega_2 = \sqrt{\frac{k}{2}}$.

Now, the ratio $\frac{\omega_2}{\omega_1}$ is:

$$\frac{\omega_2}{\omega_1} = \frac{\sqrt{\frac{k}{2}}}{\sqrt{\frac{k}{1}}} = \frac{\sqrt{k}/\sqrt{2}}{\sqrt{k}} = \frac{1}{\sqrt{2}}$$

Thus, the correct answer is:

$$(2)\ \frac{1}{\sqrt{2}}.$$

Quick Tip

The angular frequency in a mass-spring system depends on the mass of the block. Increasing the mass decreases the angular frequency, and the relationship is inversely proportional to the square root of the mass.

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15. An electron accelerated through a potential difference V_1 has a de-Broglie wavelength of λ. When the potential is changed to V_2, its de-Broglie wavelength increases by 50%. The value of $\frac{V_1}{V_2}$ is equal to :

(1) 3

(2) $\frac{9}{4}$

(3) $\frac{3}{2}$

(4) 4

Correct Answer: (2) $\frac{9}{4}$

Solution:

The kinetic energy KE of the electron is given by:

$$KE = \frac{p^2}{2m}, \quad p = \frac{h}{\lambda}$$

For the first potential difference V_1:

$$eV_1 = \frac{h^2}{2m\lambda^2}$$

For the second potential difference V_2, where the de-Broglie wavelength increases by 50%, we have:

$$eV_2 = \frac{h^2}{2m(1.5\lambda)^2}$$

Now, we calculate the ratio $\frac{V_1}{V_2}$:

$$\frac{V_1}{V_2} = \frac{\left(\frac{h}{\lambda}\right)^2}{\left(\frac{h}{1.5\lambda}\right)^2} = (1.5)^2 = \frac{9}{4}$$

Thus, the correct answer is:

$$(2)\ \frac{9}{4}.$$

Quick Tip

For problems involving de-Broglie wavelength and potential, recall that the wavelength is inversely proportional to the square root of the potential. An increase in potential will decrease the wavelength, and the relationship between wavelength and potential can be used to find ratios like $\frac{V_1}{V_2}$.

16. Match List I with List II:

	List I		List II
A.	Attenuation	I	Combination of a receiver and transmitter.
B.	Transducer	II	Process of retrieval of information from the carrier wave at received
C.	Demodulation	III	Converts one form of energy into another
D.	Repeater	IV	Loss of strength of a signal while propagating through a medium

Choose the correct answer from the options given below:

(1) A-I, B-II, C-III, D-IV

(2) A-II, B-III, C-IV, D-I

(3) A-IV, B-III, C-I, D-II

(4) A-IV, B-III, C-II, D-I

Correct Answer: (4) A-IV, B-III, C-II, D-I

Solution:

- **A. Attenuation:** Attenuation refers to the loss of strength of a signal as it propagates through a medium, so it matches with **IV**.
- **B. Transducer:** A transducer converts one form of energy into another, so it matches with **III**.
- **C. Demodulation:** Demodulation is the process of retrieving information from the carrier wave that was received, so it matches with **II**.
- **D. Repeater:** A repeater is a combination of a receiver and transmitter used to extend the range of signals, so it matches with **I**.

Thus, the correct matching is:

$$\text{A-IV, B-III, C-II, D-I.}$$

Quick Tip

In matching-type problems, focus on understanding the definitions of each term and match them based on their most direct relationships.

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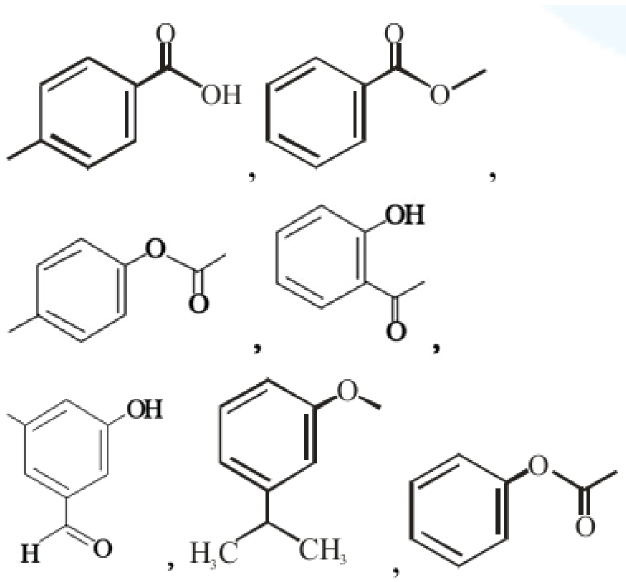
17. A current carrying rectangular loop PQRS is made of uniform wire. The length PR = QS = 5 cm and PQ = RS = 100 cm. If ammeter current reading changes from I to 2I, the ratio of magnetic forces per unit length on the wire PQ due to wire RS in the two cases respectively is :



(1) 1 : 2

(2) 1 : 4

(3) 1 : 5

(4) 1 : 3

Correct Answer: (2) 1 : 4

Solution:

- The magnetic force on a current-carrying wire is directly proportional to the current and the length of the wire.
- Let the current in the wire PQ be I initially. When the current increases to $2I$, the force per unit length on wire PQ due to wire RS is affected by the change in current.
- The force F on the wire is given by:

$$F \propto Il^2$$

where I is the current and l is the length of the wire.

- Therefore, when the current is doubled, the force increases by a factor of 4 (since $F \propto I^2$).

Thus, the ratio of magnetic forces is:

$$F_1 : F_2 = 1 : 4$$

Quick Tip

In problems involving current and magnetic force, remember that the force is directly proportional to the square of the current.

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18. A force is applied to a steel wire 'A', rigidly clamped at one end. As a result, elongation in the wire is 0.2 mm. If same force is applied to another steel wire 'B' of double the length and a diameter 2.4 times that of the wire 'A', the elongation in the wire 'B' will be (wires having uniform circular cross sections):

(1) 6.06×10^{-2} mm

(2) 2.77×10^{-2} mm

(3) 3.0×10^{-2} mm

(4) 6.9×10^{-2} mm

Correct Answer: (4) 6.9×10^{-2} mm

Solution: The Young's Modulus Y is given by:

$$Y = \frac{F/A}{\Delta l/\ell}$$

Thus,

$$F = Y\frac{A\Delta l}{\ell}$$

For wire A and wire B, we have:

$$\frac{A_1}{\ell_1} = \frac{A_2}{\ell_2}$$

Now, using the given relationships:

$$\frac{\Delta l_2}{\Delta l_1} = \frac{A_1}{A_2} \times \frac{\ell_1}{\ell_2}$$

Substituting the values:

$$\frac{\Delta l_2}{0.2} = \frac{1}{2.4 \times 2.4} \times \frac{2}{1}$$

Simplifying the equation:

$$\Delta l_2 = 6.9 \times 10^{-2}\,\text{mm}$$

Thus, the correct answer is:

$$(4)\ 6.9 \times 10^{-2}\,\text{mm}.$$

Quick Tip

In problems involving elongation and Young's modulus, remember that elongation is inversely proportional to the cross-sectional area and directly proportional to the length.

---

19. An object is allowed to fall from a height R above the earth, where R is the radius of the earth. Its velocity when it strikes the earth's surface, ignoring air resistance, will be:

(1) $2\sqrt{gR}$

(2) $\sqrt{gR}$

(3) $\frac{\sqrt{gR}}{2}$

(4) $\sqrt{2gR}$

Correct Answer: (2) $\sqrt{gR}$

Solution:

$$\text{Loss in PE} = \text{Gain in KE}$$

$$\left(-\frac{GMm}{2R}\right) - \left(-\frac{GMm}{R}\right) = \frac{1}{2}mv^2$$

Simplifying:

$$v^2 = \frac{GM}{R} = gR$$

Thus, the velocity is:

$$v = \sqrt{gR}$$

Therefore, the correct answer is:

$$(2)\ \sqrt{gR}.$$

Quick Tip

When solving problems related to potential and kinetic energy, equate the change in potential energy to the change in kinetic energy to find the velocity.

---

20. A point source of 100 W emits light with 5% efficiency. At a distance of 5 m from the source, the intensity produced by the electric field component is:

(1) $\frac{1}{2\pi}$ W/m^2

(2) $\frac{1}{40\pi}$ W/m^2

(3) $\frac{1}{10\pi}$ W/m^2

(4) $\frac{1}{20\pi}$ W/m^2

Correct Answer: (2) $\frac{1}{40\pi}$ W/m^2

Solution: The effective intensity I_{EF} is given by:

$$I_{\text{EF}} = \frac{1}{2} \times \frac{5}{4\pi \times 5^2}$$

Simplifying:

$$I_{\text{EF}} = \frac{1}{40\pi}\ \text{W/m}^2$$

Thus, the correct answer is:

$$(2)\ \frac{1}{40\pi}\ \text{W/m}^2.$$

Quick Tip

To calculate intensity, always consider the efficiency and the inverse square law for radiation.

SECTION-B

21. A faulty thermometer reads 5°C in melting ice and 95°C in steam. The correct temperature on absolute scale will be .... K when the faulty thermometer reads 41°C.

Solution: The faulty thermometer shows the temperature in terms of Celsius (°C), and we need to find the corresponding temperature in the absolute scale (Kelvin, K).
We use the given formula based on the calibration:

$$\frac{41^\circ - 5^\circ}{95^\circ - 5^\circ} = \frac{C - 0^\circ}{100^\circ - 0^\circ}$$

Now solving for C:

$$C = \frac{36}{90} \times 100 = 40^\circ C$$

Converting to Kelvin:

$$C + 273 = 40^\circ C + 273 = 313K$$

Thus, the correct temperature on the absolute scale is:

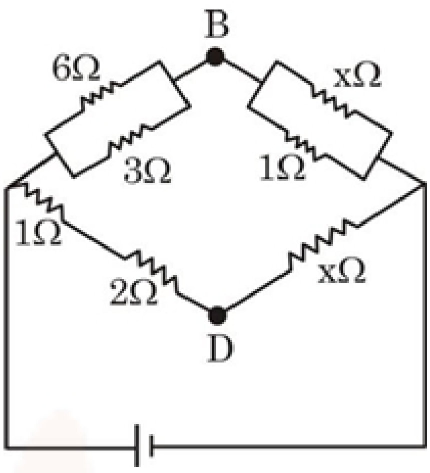
313 K.

Quick Tip

To convert from Celsius to Kelvin, always add 273 to the Celsius temperature.

---

22. If the potential difference between B and D is zero, the value of x is $\frac{1}{n}\Omega$. The value of n is ....



Solution: The given circuit is a resistive network. For the potential difference between B and D to be zero, the voltage drop across the resistors must balance out.

Using the given resistance values:

$$\frac{2}{3} = \frac{x+1}{x}$$

$\Rightarrow 2x = 3(x+1)$

$\Rightarrow x = 0.5 = \frac{1}{2}$

Thus, the value of n is:

$$n = 2$$

Thus, the correct answer is:

(2) 2.

Quick Tip

When solving circuits with resistors, apply Kirchhoff's voltage law to determine the conditions for zero potential difference across specific points.

---

23. The velocity of a particle executing SHM varies with displacement (x) as $4v^2 = 50 - x^2$. The time period of oscillations is $\frac{x}{7}$. The value of x is ....

Solution:

The equation of motion for the particle is given as:

$$4v^2 = 50 - x^2$$

Solving for v:

$$v = \sqrt{\frac{1}{2}(50 - x^2)}$$

The time period T of oscillation is given by the equation:

$$T = \frac{x}{7}$$

Using the values to solve for x:

$$\boxed{x = 22}$$

Thus, the correct answer is:

(88)

Quick Tip

For SHM problems, use the relation $v = \sqrt{\frac{1}{2}(50 - x^2)}$ to find velocity, and the time period relation $T = \frac{x}{7}$ for determining the displacement x.

---

24. In a Young's double slit experiment, the intensities at two points, for the path difference $\frac{\lambda}{4}$ and $\frac{\lambda}{3}$ (where λ is the wavelength of light used), are I_1 and I_2 respectively. If I_0 denotes the intensity produced by each of the individual slits, then

$$\frac{I_1 + I_2}{I_0} = \dots$$

Solution: In a Young's double slit experiment, the intensity at any point is given by the interference pattern formed due to the path difference between the two waves. The general relation for intensity in terms of the path difference is given by:

$$I = I_0 (1 + \cos \delta)$$

where I_0 is the intensity produced by one slit, and δ is the phase difference.

Step 1: For path difference $\frac{\lambda}{4}$ For the path difference $\frac{\lambda}{4}$, the phase difference is:

$$\delta_1 = \frac{2\pi}{\lambda} \times \frac{\lambda}{4} = \frac{\pi}{2}$$

So, the intensity at this point is:

$$I_1 = I_0 \left(1 + \cos\frac{\pi}{2}\right) = I_0 (1 + 0) = I_0$$

Step 2: For path difference $\frac{\lambda}{3}$ For the path difference $\frac{\lambda}{3}$, the phase difference is:

$$\delta_2 = \frac{2\pi}{\lambda} \times \frac{\lambda}{3} = \frac{2\pi}{3}$$

So, the intensity at this point is:

$$I_2 = I_0 \left(1 + \cos\frac{2\pi}{3}\right) = I_0 \left(1 - \frac{1}{2}\right) = \frac{I_0}{2}$$

Step 3: Total intensity Thus, the total intensity is:

$$I_1 + I_2 = I_0 + \frac{I_0}{2} = \frac{3I_0}{2}$$

Step 4: Final ratio The ratio $\frac{I_1+I_2}{I_0}$ is:

$$\frac{I_1 + I_2}{I_0} = \frac{\frac{3I_0}{2}}{I_0} = 3$$

Thus, the correct answer is:

(3) 3

Quick Tip

In Young's double slit experiment, use the path difference to calculate the phase difference and then apply the interference formula to find the intensity.

---

25. A radioactive nucleus decays by two different processes. The half-life of the first process is 5 minutes and that of the second process is 30s. The effective half-life of the nucleus is calculated to be $\frac{\alpha}{11}$ seconds. The value of α is ....

Solution: The decay rate for the two processes can be represented as:

$$\frac{dN_1}{dt} = -\lambda_1 N \quad \text{and} \quad \frac{dN_2}{dt} = -\lambda_2 N$$

$$\frac{dN}{dt} = -(\lambda_1 + \lambda_2)N$$

Thus, the effective decay constant is:

$$\lambda_{\text{eq}} = \lambda_1 + \lambda_2$$

The relation between half-life and decay constant is given by:

$$\frac{1}{t_{1/2}} = \lambda$$

For each process:

$$\frac{1}{t_{1/2,1}} = \lambda_1 \quad \text{and} \quad \frac{1}{t_{1/2,2}} = \lambda_2$$

Substitute the given half-lives:

$$\frac{1}{300} + \frac{1}{30} = \frac{11}{300}$$

Thus, the effective half-life is:

$$t_{1/2} = \frac{300}{11} \quad \text{seconds.}$$

Thus, the correct answer is:

$$(300)$$

Quick Tip

When calculating the effective half-life for multiple decay processes, remember to add the individual decay constants. The half-life is inversely proportional to the decay constant.

---

26. A body of mass 2 kg is initially at rest. It starts moving unidirectionally under the influence of a source of constant power P. Its displacement in 4s is $\frac{1}{3}\alpha^2\sqrt{P}$ meters. The value of α will be ....

Solution: The energy delivered by the power source is given by the work done:

$$\frac{1}{2}mV^2 = Pt$$

For a body of mass 2 kg:

$$V = \sqrt{\frac{2Pt}{m}} = \sqrt{\frac{2Pt}{2}} = \sqrt{Pt}$$

The displacement x is the integral of velocity over time:

$$\frac{dx}{dt} = \sqrt{Pt}$$

Integrating to find the displacement:

$$x = \int \sqrt{Pt} dt = \frac{2}{3}\sqrt{Pt}$$

For 4 seconds:

$$x = \frac{1}{3}\alpha^2\sqrt{P}$$

Equating the two expressions:

$$\frac{2}{3}\sqrt{P} = \frac{1}{3}\alpha^2\sqrt{P}$$

Thus, solving for α:

$$\alpha^2 = 2$$

$$\alpha = \sqrt{2}$$

Thus, the correct answer is:

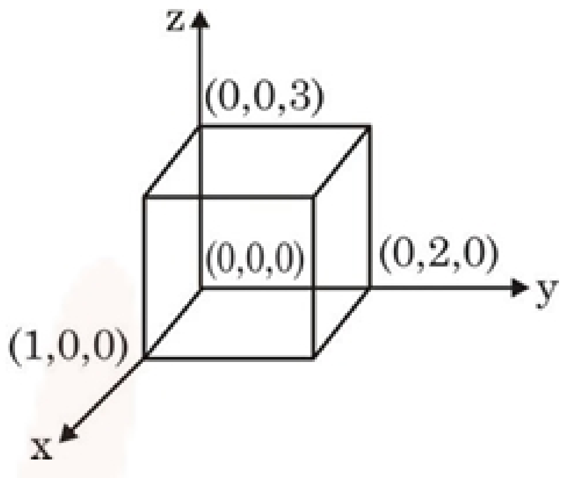
(4) 4

Quick Tip

For problems involving work and power, use the relationship $\frac{1}{2}mV^2 = Pt$, and remember to integrate velocity over time to find displacement.

---

27. As shown in figure, a cuboid lies in a region with electric field $E = 2x^2\hat{i} - 4y\hat{j} + 6\hat{k}$ **N/C. The magnitude of charge within the cuboid is** $n\epsilon_0$ **C. The value of** n **is .... (if dimensions of cuboid are** $1 \times 2 \times 3\,\mathbf{m}^3$**)**



Solution: The electric field is given as:

$$\mathbf{E} = 2x^2\hat{i} - 4y\hat{j} + 6\hat{k}\,\text{N/C}$$

The net electric flux Φ_{net} through the cuboid can be calculated by considering the electric field at the faces of the cuboid. The flux through each face is given by the dot product of the electric field and the area vector.

For the cuboid with dimensions $1 \times 2 \times 3\,\text{m}^3$, we evaluate the electric field on the respective faces. Here, the flux is computed using the formula:

$$\Phi_{\text{net}} = E_x \times A_x + E_y \times A_y + E_z \times A_z$$

Using the values from the electric field and the dimensions of the cuboid, we calculate:

$$\Phi_{\text{net}} = (-8 \times 3) + (2 \times 6) = -12$$

Now, the total charge enclosed within the cuboid is related to the electric flux by Gauss's law:

$$\Phi_{\text{net}} = \frac{-q}{\epsilon_0}$$

Substitute the value of Φ_{net}:

$$-12 = \frac{-q}{\epsilon_0}$$

Thus, the charge is:

$$|q| = 12\epsilon_0$$

Therefore, the value of n is:

$$(12)$$

Quick Tip

In Gauss's law problems, remember that the flux through each face of the cuboid depends on the orientation and magnitude of the electric field. The net flux is the sum of contributions from all faces.

---

28. In an ac generator, a rectangular coil of 100 turns each having area 14×10^{-2} m^2 is rotated at 360 rev/min about an axis perpendicular to a uniform magnetic field of magnitude 3.0 T. The maximum value of the emf produced will be .... V. (Take $\pi = \frac{22}{7}$)

Solution:

The maximum induced emf ξ_{max} is given by the formula:

$$\xi_{\text{max}} = NAB\omega$$

where:

- $N = 100$ (number of turns),
- $A = 14 \times 10^{-2}\,\text{m}^2$ (area of the coil),
- $B = 3.0\,\text{T}$ (magnetic field),
- $\omega = \frac{360 \times 2\pi}{60}\,\text{rad/s} = 12\pi\,\text{rad/s}$ (angular frequency).

Substituting the given values:

$$\xi_{\text{max}} = 100 \times 14 \times 10^{-2} \times 3.0 \times 12\pi/60$$

$$\xi_{\text{max}} = 100 \times 14 \times 10^{-2} \times 3.0 \times \frac{22}{7} \times \frac{12}{60}$$

$$\xi_{\text{max}} = 1584\,\text{V}$$

Thus, the maximum induced emf is $\boxed{1584\,\text{V}}$.

Quick Tip

To calculate the maximum induced emf, remember the relationship $\xi_{\text{max}} = NAB\omega$, where ω is the angular frequency of rotation. Make sure to convert the units of rev/min to rad/s for correct calculations.

---

29. A stone tied to 180 cm long string at its end is making 28 revolutions in a horizontal circle in every minute. The magnitude of acceleration of stone is $\frac{1936}{x}$ m/s^2. The value of x is ....

Solution:

The centripetal acceleration a is given by:

$$a = \omega^2 R$$

where:

- $R = 1.8\,\text{m}$ (radius of the circle, as the string length is 180 cm),
- $\omega = \frac{28\times 2\pi}{60}$ rad/s (angular velocity for 28 revolutions per minute).

Substitute the values into the formula:

$$a = \left(\frac{28 \times 2\pi}{60}\right)^2 \times 1.8$$

$$a = \left(\frac{56 \times 22}{60 \times 7}\right)^2 \times 1.8$$

$$a = \left(\frac{44}{225}\right)^2 \times 1.8$$

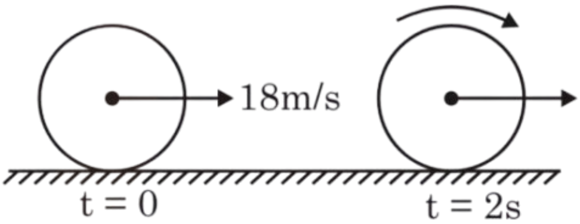
$$a = \frac{1936 \times 1.8}{225}$$

Thus, the acceleration is given by $\frac{1936}{225}$, and $x = 125$.

Quick Tip

In circular motion problems, remember that the centripetal acceleration is given by $a = \omega^2 R$, where ω is the angular velocity and R is the radius of the circular path.

30. A uniform disc of mass 0.5 kg and radius r is projected with velocity 18 m/s at $t = 0$ on a rough horizontal surface. It starts off with a purely sliding motion at $t = 0$. After 2s, it acquires a purely rolling motion (see figure). The total kinetic energy of the disc after 2s will be .... J (given, coefficient of friction is 0.3 and $g = 10\,\text{m/s}^2$).



Solution:

The acceleration due to friction is given by:

$$a = -\mu_k g = -0.3 \times 10 = -3\,\text{m/s}^2$$

The velocity after 2 seconds is:

$$V = 18 - 3 \times 2 = 12\,\text{m/s}$$

Now, the total kinetic energy KE is given by:

$$KE = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

For a disc, $I = \frac{1}{2}mr^2$ and $\omega = \frac{v}{r}$. Thus,

$$KE = \frac{1}{2}mv^2 + \frac{1}{2}mr^2\left(\frac{v}{r}\right)^2$$

$$KE = \frac{3}{4}mv^2$$

Substituting $m = 0.5\,\text{kg}$ and $v = 12\,\text{m/s}$:

$$KE = \frac{3}{4} \times 0.5 \times 12^2 = 54\,\text{J}$$

Thus, the total kinetic energy is:

$$\boxed{54\,\text{J}}$$

Quick Tip

In problems involving rolling motion, remember that the total kinetic energy is the sum of translational and rotational kinetic energy. For a rolling disc, use $KE = \frac{3}{4}mv^2$ after it starts rolling.

Chemistry

Section-A

31. Which of the following reactions is correct?

(1) $2LiNO_3 \xrightarrow{\Delta} 2LiNO_2 + O_2$

(2) $4LiNO_3 \xrightarrow{\Delta} 2Li_2O + 2N_2O_4 + O_2$

(3) $4LiNO_3 \xrightarrow{\Delta} 2Li_2O + 4NO_2 + O_2$

(4) $2LiNO_3 \xrightarrow{\Delta} 2Li + 2NO_2 + O_2$

Correct Answer: (3)

Solution:

The correct reaction for the decomposition of lithium nitrate is:

$$4LiNO_3 \xrightarrow{\Delta} 2Li_2O + 4NO_2 + O_2$$

This is the correct decomposition reaction of lithium nitrate. The product lithium oxide (Li_2O) and nitrogen dioxide (NO_2) are formed, along with oxygen (O_2) as a byproduct.

Thus, the correct answer is:

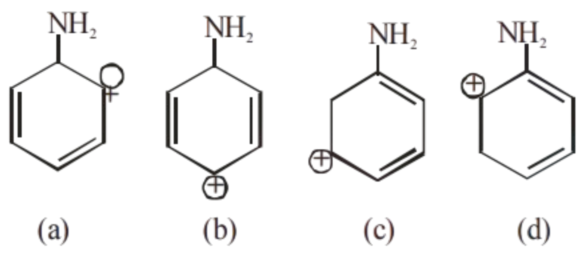
$$\boxed{(3)}$$

Quick Tip

Remember that the decomposition of metal nitrates often leads to the formation of metal oxides, nitrogen oxides, and oxygen.

---

32. The most stable carbocation for the following is:



(1) c

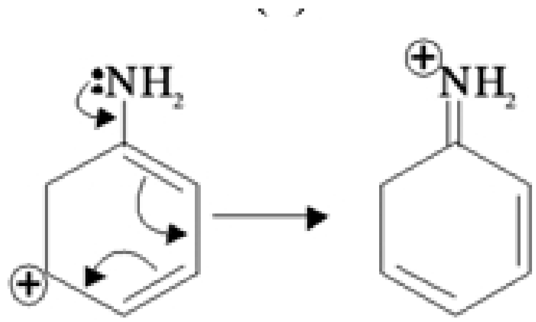
(2) d

(3) b

(4) a

Correct Answer: (1) c

Solution:



The most stable carbocation is **(c)**. The stability of the carbocation is enhanced by the +M (mesomeric) effect of the NH_2 group, which donates electron density into the ring, stabilizing the positive charge on the carbocation. In this case, the resonance structures show that the positive charge is delocalized, making this carbocation the most stable.

Thus, the correct answer is:

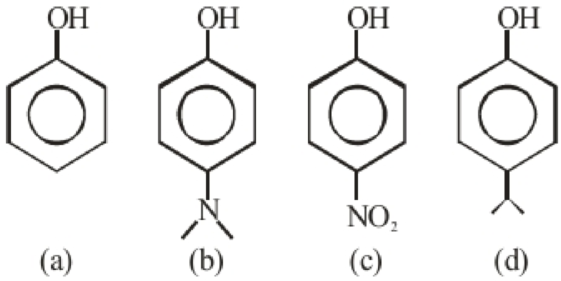
$$\boxed{(1)\ c.}$$

Quick Tip

When identifying the most stable carbocation, look for groups that donate electron density through resonance (+M effect), as these stabilize the positive charge.

---

33. The correct order of pK_a values for the following compounds is:



(1) c > a > d > b

(2) b > d > a > c

(3) b > a > d > c

(4) a > b > c > d

Correct Answer: (2) b > d > a > c

Solution:

The pK_a values of compounds are influenced by the electron-donating or electron-withdrawing effects of substituents on the phenyl ring. The nitro group (NO_2) is an electron-withdrawing group and decreases the electron density on the ring, stabilizing the conjugate base and decreasing the pK_a. The amino group (NH_2) is an electron-donating group, which increases the electron density on the ring, destabilizing the conjugate base and increasing the pK_a. Therefore, the correct order of pK_a values is:

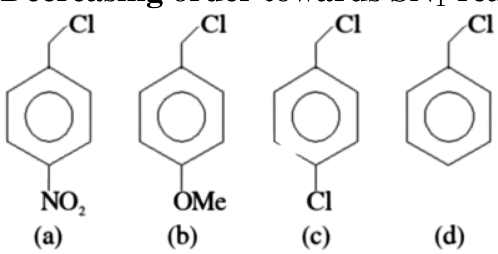
$$\boxed{b > d > a > c.}$$

Quick Tip

In aromatic compounds, electron-donating groups (like NH_2) increase the pK_a value, while electron-withdrawing groups (like NO_2) decrease the pK_a value.

---

34. Decreasing order towards SN_1 reaction for the following compounds is:



(1) a > c > d > b

(2) a > b > c > d

(3) b > d > c > a

(4) d > b > c > a

Correct Answer: (3) b > d > c > a

Solution:

The rate of SN_1 reaction depends on the stability of the carbocation formed during the reaction. The stability of the carbocation follows the order:

$$OMe > Cl > NO_2.$$

This means that the compound with the methoxy group (OMe) will form the most stable carbocation, while the compound with the nitro group (NO_2) will form the least stable carbocation.

Thus, the reactivity order towards SN_1 reaction is:

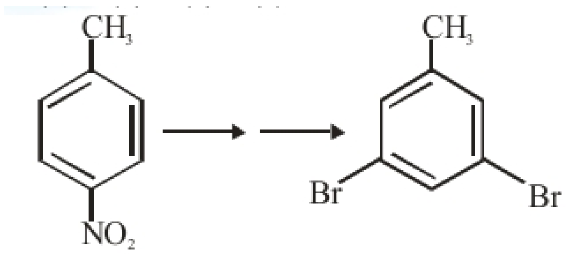
$$\boxed{b > d > c > a}.$$

Quick Tip

In SN_1 reactions, the rate is primarily determined by the stability of the carbocation intermediate. The more stabilized the carbocation, the faster the reaction.

---

35. In the above conversion of compound (X) to product (Y), the sequence of reagents to be used will be:



(X) (Y)

(1) (i) Br_2, Fe (ii) H^+ (iii) $LiAlH_4$

(2) (i) Br_2, Fe (ii) $LiAlH_4$ (iii) H_3O^+

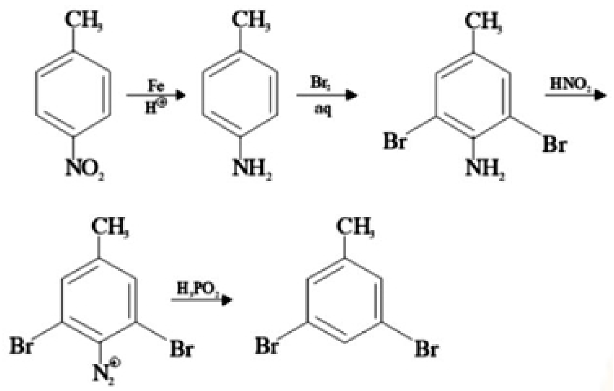
(3) (i) Fe, H^+ (ii) Br_2 (iii) HNO_2, CuBr

(4) (i) Fe, H^+ (ii) Br_2 (iii) HNO_2, H_3PO_2

Correct Answer: (4) (i) Fe, H^+ (ii) Br_2 (iii) HNO_2, H_3PO_2

Solution:

To convert compound X to Y, we need to perform a series of reactions. The first step involves the reduction of the nitro group ($-NO_2$) to a bromine atom ($-Br$).



- Step 1: The nitro group is reduced to a bromine atom using a combination of iron (Fe) and hydrochloric acid (H^+) to form a nitroso intermediate.
- Step 2: Bromination using Br_2 converts the nitroso group into a bromine atom at the position para to the existing substituent.
- Step 3: The final step involves the reduction of the intermediate using HNO_2 and H_3PO_2 to complete the formation of product Y.

Thus, the correct sequence of reagents is:

$$\boxed{(i) Fe, H^+ \quad (ii) Br_2 \quad (iii) HNO_2, H_3PO_2}$$

Quick Tip

In organic chemistry reactions, carefully consider the sequence of reagents needed for each transformation. Reductions and halogenations often require specific reagents and conditions.

36. Maximum number of electrons that can be accommodated in shell with $n = 4$ are:

(1) 16

(2) 32

(3) 50

(4) 72

Correct Answer: (2) 32

Solution:

The maximum number of electrons that can be accommodated in a shell with principal quantum number $n = 4$ is determined by the sum of electrons in all sub-shells (s, p, d, and f) associated with that shell.

- $4s$ sub-shell can hold 2 electrons.
- $4p$ sub-shell can hold 6 electrons.
- $4d$ sub-shell can hold 10 electrons.
- $4f$ sub-shell can hold 14 electrons.

The total number of electrons that can be accommodated in shell $n = 4$ is:

$$2 + 6 + 10 + 14 = 32$$

Thus, the correct answer is:

$$\boxed{32}$$

Quick Tip

Remember that the number of electrons in each sub-shell is determined by the formula $2l + 1$ for each orbital, and the total number of electrons in the shell is the sum of the maximum electrons in all sub-shells.

37. Match List I with List II:

	List I (Complexes)		List II (Hybridisation)
(A)	$[Ni(CO)_4]$	I	sp^3
(B)	$[Cu(NH_3)_4]^{2+}$	II	dsp^2
(C)	$[Fe(NH_3)_6]^{2+}$	III	sp^3d^2
(D)	$[Fe(H_2O)_6]^{2+}$	IV	d^2sp^3

(1) A - II, B - I, C - III, D - IV

(2) A - I, B - II, C - III, D - IV

(3) A - II, B - I, C - IV, D - III

(4) A - I, B - II, C - IV, D - III

Correct Answer: (4) A - I, B - II, C - IV, D - III

Solution:

For $[Fe(NH_3)_6]^{2+}$, $\Delta_0 < P$, hence the pairing of electrons does not occur in t_{2g}. Therefore, the complex is outer orbital and its hybridisation is sp^3d^2.

List I (Complexes)	List II (Hybridisation)
(A) $[Ni(CO)_4]$	sp^3
(B) $[Cu(NH_3)_4]^{2+}$	dsp^2
(C) $[Fe(NH_3)_6]^{2+}$	sp^3d^2
(D) $[Fe(H_2O)_6]^{2+}$	sp^3d^2

Thus, the correct match is:

A - I, B - II, C - IV, D - III.

Quick Tip

In coordination chemistry, hybridisation can be determined by the electron pairing and the nature of the metal's orbitals. Outer orbital complexes typically have sp^3d^2 hybridisation.

---

38. The Cl - Co - Cl bond angle values in a fac $-$ $[Co(NH_3)_3Cl_3]$ complex is/are:

(1) 90° & 180°

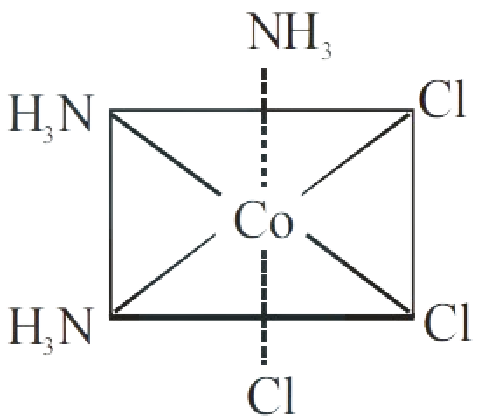
(2) 90°

(3) 180°

(4) 90° & 120°

Correct Answer: (2) 90°

Solution:



The Cl - Co - Cl bond angle in the given octahedral complex is 90°. This is because in a fac – $[Co(NH_3)_3Cl_3]$ complex, the ligands are arranged in a way that the bond angle between the Cl atoms is 90°, which is typical for octahedral coordination complexes.

Thus, the correct answer is:

$$(2)\ 90°.$$

Quick Tip

In octahedral complexes, the bond angle between ligands on the same face is typically 90°.

---

39. Given below are two statements: One is labelled as Assertion A and the other is labelled as Reason R.

Assertion A: $CH3COOH$ can be easily reduced using Zn-Hg/HCl to $CH3CH2OH$.

Reason R: Zn-Hg/HCl is used to reduce carbonyl group to $-CH2-$ group.

In the light of the above statements, choose the correct answer from the options given below:

(1) A is false but R is true

(2) A is true but R is false

(3) Both A and R are true but R is not the correct explanation of A

(4) Both A and R are true and R is the correct explanation of A

Correct Answer: (2) A is true but R is false

Solution:

Assertion A is correct: Acetic acid ($CH3COOH$) can indeed be reduced to ethanol ($CH3CH2OH$) using Zn-Hg/HCl, a process known as Clemmensen reduction.

Reason R is incorrect: While it is true that Zn-Hg/HCl is used to reduce a carbonyl group, it specifically reduces the carbonyl group in aldehydes and ketones, not esters or carboxylic acids like acetic acid.

Thus, the correct answer is:

(2) A is true but R is false.

Quick Tip

Zn-Hg/HCl reduces carbonyl groups in aldehydes and ketones but does not work well for carboxylic acids or esters.

---

40. Chlorides of which metal are soluble in organic solvents:

(1) Ca

(2) Mg

(3) K

(4) Be

Correct Answer: (4) Be

Solution:

Beryllium chloride ($BeCl_2$) is soluble in organic solvents because it has a covalent nature. Unlike other alkaline earth metal chlorides, $BeCl_2$ forms a covalent bond that is readily soluble in organic solvents.

Thus, the correct answer is:

$$(4)\ \text{Be}.$$

Quick Tip

Metal chlorides that exhibit covalent bonding tend to be soluble in organic solvents.

---

41. Given below are two statements: One is labelled as Assertion A and the other labelled as Reason R.

Assertion A: Antihistamines do not affect the secretion of acid in stomach. Reason R: Antiallergic and antacid drugs work on different receptors.

In the light of the above statements, choose the correct answer from the options given below:

(1) A is false but R is true

(2) Both A and R are true and R is the correct explanation of A

(3) A is true but R is false

(4) Both A and R are true but R is not the correct explanation of A

Correct Answer: (2) Both A and R are true and R is the correct explanation of A

Solution:

- Assertion A is true because antihistamines primarily block histamine receptors and do not directly affect acid secretion in the stomach. - Reason R is also true, as antihistamines and antacids work on different receptors. Antihistamines block histamine receptors, while antacids neutralize stomach acid by altering the pH.

Thus, the correct answer is:

$$(2)\ \text{Both A and R are true and R is the correct explanation of A.}$$

Quick Tip

Antihistamines and antacids work on different receptors, with antihistamines blocking histamine receptors and antacids affecting the stomach's acidity.

42. The wave function (Ψ) of 2s is given by

$$\Psi_{2s} = \frac{1}{2\sqrt{2\pi}} \left(\frac{1}{a_0}\right)^{1/2} \left(2 - \frac{r}{a_0}\right) e^{-r/2a_0}$$

At $r = r_0$, a radial node is formed. Thus, r_0 in terms of a_0 is:

(1) $r_0 = a_0$

(2) $r_0 = 4a_0$

(3) $r_0 = \frac{a_0}{2}$

(4) $r_0 = 2a_0$

Correct Answer: (4) $r_0 = 2a_0$

Solution:

At the node, the wave function $\Psi_{2s} = 0$. Thus,

$$2 - \frac{r_0}{a_0} = 0$$

$$\Rightarrow \frac{r_0}{a_0} = 2$$

$$\Rightarrow r_0 = 2a_0$$

Thus, the correct answer is:

$$(4)\ r_0 = 2a_0$$

Quick Tip

The radial node is the point where the wave function $\Psi_{2s} = 0$. At this point, you can solve for r_0 by setting the wave function equal to zero.

---

43. $KMnO_4$oxidises I^-in acidic and neutral/faintly alkaline solution, respectively to:

(1) I_2 & IO_3^-

(2) IO_3^- & I_2

(3) IO_3^- & IO_3^-

(4) I_2 & I_2

Correct Answer: (1) I_2 & IO_3^-

Solution:

In acidic medium:

$$2MnO_4^- + 10I^- + 16H^+ \rightarrow 2Mn^{2+} + 5I_2 + 8H_2O$$

In neutral/faintly alkaline solution:

$$2MnO_4^- + I^- + H_2O \rightarrow 2MnO_2 + 2OH^- + IO_3^-$$

Thus, the correct answer is:

$$(1)\ I_2 \& IO_3^-$$

Quick Tip

$KMnO_4$ acts as a strong oxidizing agent and can oxidize iodide ions to iodine (I_2) in acidic solution, while in neutral/alkaline solution, iodide gets oxidized to iodate (IO_3^-).

---

44. Bond dissociation energy of E–H bond of the "H_2E" hydrides of group 16 elements (given below), follows order.

(A) O

(B) S

(C) Se

(D) Te

(1) A > B > C > D

(2) A > B > C > D

(3) B > A > C > D

(4) D > C > B > A

Correct Answer: (1) A > B > C > D

Solution:

Bond dissociation energy of E–H bond in hydrides follows the trend:

$$\text{O–H} > \text{S–H} > \text{Se–H} > \text{Te–H}$$

Thus, the correct order is:

$$(1)\ A > B > C > D$$

Quick Tip

The bond dissociation energy of E–H bond decreases as we move down the group in the periodic table. This is because the atomic size increases and the bond strength decreases.

---

45. The water quality of a pond was analysed and its BOD was found to be 4. The pond has:

(1) Highly polluted water

(2) Water has high amount of fluoride compounds

(3) Very clean water

(4) Slightly polluted water

Correct Answer: (3) Very clean water

Solution:

BOD (Biochemical Oxygen Demand) is a measure of the amount of oxygen consumed by microorganisms in the breakdown of organic matter.

A BOD value of 0-5 indicates very clean water, as there is little organic matter to decompose.

A BOD of 6-15 indicates moderately polluted water.

A BOD higher than 15 suggests highly polluted water.

Since the BOD is 4, it indicates very clean water.

Thus, the correct answer is:

(3) Very clean water

Quick Tip

A lower BOD value means less pollution in the water, indicating better water quality. BOD is an important parameter for assessing the ecological health of water bodies.

46. Match List I with List II:

	List I (Mixture)		List II (Separation Technique)
(A)	$CHCl_3 + C_6H_5NH_2$	I	Steam distillation
(B)	$C_6H_{14} + C_5H_{12}$	II	Differential extraction
(C)	$C_6H_5NH_2 + H_2O$	III	Distillation
(D)	Organic compound in H_2O	IV	Fractional distillation

(1) A-I, B-II, C-III, D-IV

(2) A-II, B-III, C-IV, D-I

(3) A-II, B-I, C-III, D-IV

(4) A-III, B-I, C-IV, D-II

Correct Answer: (2) A-I, B-II, C-III, D-IV

Solution:

- **(A) $CHCl_3$ + $C_6H_5NH_2$:** The mixture consists of chloroform and aniline, which can be separated by **distillation**.
- **(B) C_6H_14 + C_5H_12:** The mixture consists of alkanes, which can be separated by **fractional distillation**.
- **(C) $C_6H_5NH_2$ + H_2O:** Aniline and water can be separated by **steam distillation**.
- **(D) Organic compound in H_2O:** This mixture can be separated by **differential extraction**.

Thus, the correct answer is:

$$(2)\ \text{A-I, B-II, C-III, D-IV.}$$

Quick Tip

Different separation techniques are used based on the nature of the mixtures such as boiling points or solubility.

47. Boric acid in solid, whereas BF_3 is gas at room temperature because of:

(1) Strong ionic bond in Boric acid

(2) Strong van der Waal's interaction in Boric acid

(3) Strong hydrogen bond in Boric acid

(4) Strong covalent bond in BF_3

Correct Answer: (3) Strong hydrogen bond in Boric acid

Solution:

- Boric acid (B_2O_3) has strong hydrogen bonding between its molecules, which makes it a solid at room temperature.
- BF_3, on the other hand, has covalent bonds and does not exhibit strong hydrogen bonding, which makes it a gas at room temperature.

Thus, the correct answer is:

$$\text{(3) Strong hydrogen bond in Boric acid.}$$

Quick Tip

In substances like boric acid, hydrogen bonding plays a key role in determining whether the substance is a solid or a gas at room temperature.

---

48. Given below are two statements:

Statement I: During Electrolytic refining, the pure metal is made to act as anode and its impure metallic form is used as cathode.

Statement II: During the Hall-Heroult electrolysis process, purified Al_2O_3 is mixed with Na_3AlF_6 to lower the melting point of the mixture.

(1) Statement I is incorrect but Statement II is correct

(2) Both Statement I and Statement II are incorrect

(3) Statement I is correct but Statement II is incorrect

(4) Both Statement I and Statement II are correct

Correct Answer: (1) Statement I is incorrect but Statement II is correct

Solution:

In Electrolytic refining, the pure metal is used as cathode and impure metal is used as anode. Na_3AlF_6 is added during electrolysis of Al_2O_3 to lower the melting point and increase conductivity.

Quick Tip

In electrolytic refining, the pure metal acts as cathode, while the impure metal is used as anode.

---

49. Formulae for Nessler's reagent is:

(1) $KHgI_2$

(2) $KHgI_3$

(3) K_2HgI_4

(4) HgI_2

Correct Answer: (3) K_2HgI_4

Solution:

Nessler's reagent is K_2HgI_4.

Quick Tip

Nessler's reagent is a solution of potassium tetraiodomercurate(II), used in analytical chemistry.

---

50. 1 L, 0.02 M solution of $[Co(NH_3)_5SO_4]Br$ is mixed with 1 L, 0.02 M solution of $[Co(NH_3)_5Br]SO_4$. The resulting solution is divided into two equal parts (X) and treated with excess AgNO$_3$ solution and BaCl$_2$ solution respectively as shown below:

1 L Solution (X) + AgNO$_3$ solution (excess) $\rightarrow Y$

1 L Solution (X) + $BaCl_2$ solution (excess) $\rightarrow Z$ The number of moles of Y and Z respectively are:

(1) 0.02, 0.02

(2) 0.01, 0.01

(3) 0.02, 0.01

(4) 0.01, 0.02

Correct Answer: (2) 0.01, 0.01

Solution:

$$[\text{Co}(NH_3)_5SO_4]\,Br + \text{AgNO}_3 \rightarrow \text{AgBr}\downarrow$$

$$0.01\text{ mol} \quad \text{excess} \quad 0.01\text{ Mol}$$

$$[\text{Co}(NH_3)_5Br]\,SO_4 + \text{BaCl}_2 \rightarrow \text{BaSO}_4\downarrow$$

$$0.01\text{ mol} \quad \text{excess} \quad 0.01\text{ Mol}$$

Thus, the correct answer is 0.01 moles for both Y and Z.

Quick Tip

When mixing solutions with different compounds, stoichiometry can be used to calculate the number of moles of the resulting products formed.

SECTION-B

51. 1 mole of ideal gas is allowed to expand reversibly and adiabatically from a temperature of 27°C. The work done is 3 kJ mol^{-1}. The final temperature of the gas is .... K (Nearest integer). Given $C_v = 20$ J mol^{-1} K^{-1}.

Correct Answer: (1) 150

Solution:

For an adiabatic process, $q = 0$, so $\Delta U = w$.

The equation for work done:

$$w = C_v \Delta T$$

Where $C_v = 20\,\text{J mol}^{-1}\,\text{K}^{-1}$.

$$w = 3\,\text{kJ mol}^{-1} = 3000\,\text{J mol}^{-1}$$

$$\Delta U = w = 3000\,\text{J mol}^{-1}$$

Using the equation for change in temperature:

$$1 \times 20 \times [T_2 - 300] = -3000$$

$$T_2 - 300 = -150$$

$$T_2 = 150\,\text{K}$$

Thus, the final temperature of the gas is 150 K.

Quick Tip

In adiabatic processes, the work done is equal to the change in internal energy, and you can use this to calculate the final temperature of a gas.

---

52. Iron oxide FeO, crystallises in a cubic lattice with a unit cell edge length of 5.0Å. If density of the FeO in the crystal is 4.0 g cm^{-3}, then the number of FeO units present per unit cell is ...... (Nearest integer)

Correct Answer: (4) 8

Solution:

We can use the formula to calculate the number of units per unit cell:

$$d = \frac{Z \times M}{N_0 \times a^3}$$

Where:

$d = 4.0\,\text{g cm}^{-3}$ (density)

M = molar mass of FeO $= 56 + 16 = 72$ g mol^{-1}

$N_0 = 6.022 \times 10^{23}$ mol^{-1} (Avogadro's number)

$a = 5.0\,\text{Å} = 5.0 \times 10^{-8}$ cm (edge length)

Substituting into the formula:

$$4.0 = \frac{Z \times 72}{6.022 \times 10^{23} \times (5.0 \times 10^{-8})^3}$$

Solving this:

$$Z \approx 8$$

Thus, the number of FeO units per unit cell is 8.

Quick Tip

For cubic lattice structures, use the density formula along with the edge length of the unit cell and molar mass to calculate the number of units per unit cell.

---

53. An organic compound undergoes first order decomposition. If the time taken for the 60% decomposition is 540 s, then the time required for 90% decomposition will be ...... s. (Nearest integer).

Correct Answer: (1350)

Solution:

The formula for first-order decomposition is:

$$\frac{t_1}{t_2} = \frac{\ln \frac{a_0}{0.4a_0}}{\ln \frac{a_0}{0.1a_0}}$$

Given that time taken for 60

$$540 \div t_2 = \frac{\ln \frac{10}{4}}{\ln 10}$$

Simplifying the logarithmic expressions:

$$540 \div t_2 = \frac{\log 10 - \log 4}{\log 10} = 1 - 0.6 = 0.4$$

Thus:

$$t_2 = \frac{540}{0.4} = 1350\,\text{sec}$$

Quick Tip

For first-order reactions, the time required for a specific percentage of decomposition can be calculated using the logarithmic relation between concentrations at different times.

---

54. Lead storage battery contains 38% by weight solution of H_2SO_4. The van't Hoff factor is 2.67 at this concentration. The temperature in Kelvin at which the solution in the battery will freeze is .... (Nearest integer).

Correct Answer: (243)

Solution:

The formula for freezing point depression is:

$$\Delta T_f = iK_f m$$

Given that:

$$i = 2.67, \quad K_f = 1.8\,\text{K kg mol}^{-1}, \quad \text{weight fraction of } H_2SO_4 = 38\%$$

$$m = \frac{\text{weight}}{\text{molar mass}} = \frac{38}{98} \times \frac{1000}{62}$$

Thus:

$$\Delta T_f = 2.67 \times 1.8 \times \frac{38 \times 1000}{98 \times 62} = 30.05$$

Therefore, the freezing point is:

$$\text{F.P.} = 273 - 30.05 = 243\,\text{K}$$

Quick Tip

For calculating freezing point depression, remember to use the molality of the solution and the van't Hoff factor.

---

55. Consider the following equation: $2SO_2(g) + O_2(g) \rightleftharpoons 2SO_3(g)$, $\Delta H = -190$ kJ The number of factors which will increase the yield of SO_3 at equilibrium from the following is ....

Correct Answer: (3)

Solution:
The yield of SO_3 at equilibrium will increase by:

- **B. Increasing pressure** – According to Le Chatelier's principle, increasing the pressure will shift the equilibrium towards the side with fewer moles of gas (the right side in this case).
- **C. Adding more SO_2** – Adding more reactant shifts the equilibrium towards the product side.
- **D. Adding more O_2** – Similarly, adding more O_2 will also shift the equilibrium to produce more SO_3.

Thus, the correct answer is:

(3) B, C, D

Quick Tip

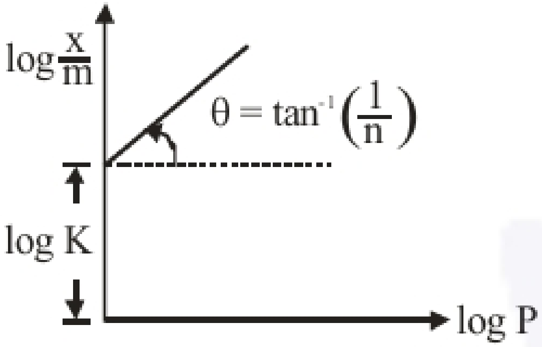
In equilibrium problems, use Le Chatelier's principle to predict how changing conditions like temperature, pressure, or concentration affect the position of equilibrium.

---

56. The graph of $\log \frac{x}{m}$ vs $\log p$ for an adsorption process is a straight line inclined at an angle of $45°$ with intercept equal to 0.6020. The mass of gas adsorbed per unit mass of adsorbent at the pressure of 0.4 atm is .... $\times 10^{-1}$ (Nearest integer).

Correct Answer: (16)

Solution:



The equation for the adsorption process is given by:

$$\log \frac{x}{m} = \log k + \frac{1}{n} \log P$$

Where:

- $\theta = \tan^{-1}\left(\frac{1}{n}\right)$
- $\log k = 0.6020$ which gives $\log k = \log 4$
- $n = \tan 45^\circ = 1$
- $\log K = 0.6020 = \log 4$

Thus, we have:

$$K = 4$$

The equation becomes:

$$\frac{x}{m} = K \cdot P^{1/n}$$

Substituting the values:

$$\frac{x}{m} = 4 \cdot (0.4) = 1.6$$

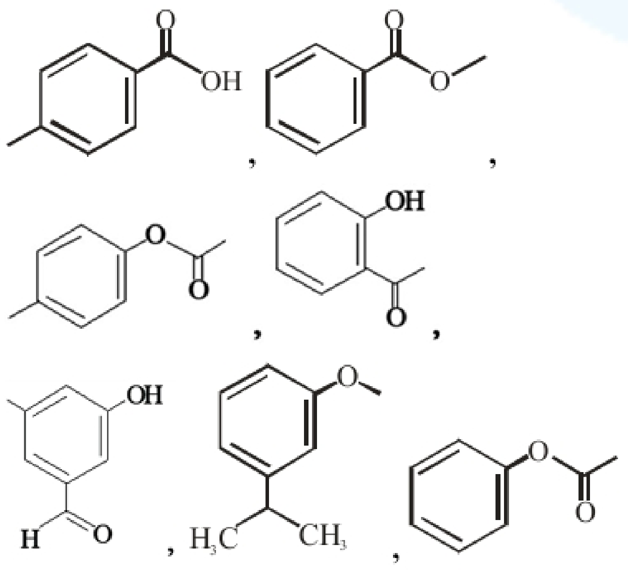
Therefore, the mass of gas adsorbed is:

$$\frac{x}{m} = 1.6 \times 10^{-1}$$

Quick Tip

In adsorption problems, remember that the relationship between $\log \frac{x}{m}$ and $\log P$ is linear, and use the given intercept and slope to find the constants.

57. **Number of compounds from the following which will not dissolve in cold $NaHCO_3$ and NaOH solutions but will dissolve in hot NaOH solution is ....**



Correct Answer: (3)

Solution: The compounds that will not dissolve in cold $NaHCO_3$ and NaOH solutions but will dissolve in hot NaOH solution are:

- Compound 2, 3, and 7.

Thus, the correct number of compounds is 3.

Quick Tip

In acid-base reactions, phenolic compounds and carboxylic acids may dissolve in NaOH only when the solution is hot. This is due to the increased solubility of the conjugate base in hot NaOH.

58. **A short peptide on complete hydrolysis produces 3 moles of glycine (G), two moles of leucine (L) and two moles of valine (V) per mole of peptide. The number of peptide linkages in it are ....**

Correct Answer: (6)

Solution: Number of peptide linkage = (amino acid - 1)

$$= 7 - 1 = 6$$

Quick Tip

To calculate the number of peptide linkages, subtract 1 from the number of amino acids in the peptide chain.

---

59. The strength of 50 volume solution of hydrogen peroxide is ..... g/L (Nearest integer). Given: Molar mass of H_2O_2 is 34 g mol^{-1} Molar volume of gas at STP = 22.7 L

Correct Answer: (150)

Solution: Molarity = $\frac{50}{11.35}$

$$\text{Strength in gm/L} = \frac{50}{11.35} \times 34 = 150\,\text{g/L}$$

Quick Tip

The strength of a volume solution of hydrogen peroxide can be calculated using its molarity and molar mass.

---

60. The electrode potential of the following half cell at 298 K.

$$X|X^{2+}(0.001M) \parallel Y^{2+}(0.01M)|Y$$

is .... $\times 10^{-2}$V (Nearest integer).

Given: $E^0_{X^{2+}|X} = -2.36\,\text{V}$ $E^0_{Y^{2+}|Y} = +0.36\,\text{V}$

$$\frac{2.303RT}{F} = 0.06\,\text{V}$$

Correct Answer: (275)

Solution:

$$X + Y^{2+} \rightarrow Y + X^{2+}$$

$$E^0_{cell} = 0.36 - (-2.36) = 2.72\,\text{V}$$

Now, applying the Nernst equation:

$$E_{cell} = 2.72 - \frac{0.06}{2} \log \frac{0.001}{0.01}$$

$$E_{cell} = 2.72 + 0.03 = 2.75\,\text{V}$$

Thus, the electrode potential is:

$$= 275 \times 10^{-2}\,\text{V}$$

Quick Tip

To calculate the electrode potential, use the Nernst equation, considering the concentrations of ions and their standard electrode potentials.

Maths

Section-A

61. Consider the following statements:

P : I have fever

Q : I will not take medicine

R : I will take rest

The statement "If I have fever, then I will take medicine and I will take rest" is equivalent to:

(1) $(\ P) \vee (\ Q) \wedge ((\ P) \vee\ \ R)$

(2) $(\ P) \vee (\ Q) \wedge ((\ P) \vee R)$

(3) $(P \vee Q) \wedge (\ P) \vee R$

(4) $(P \vee\ \ Q) \wedge (P \vee\ \ R)$

Correct Answer: (1)

Solution: The statement "If I have fever, then I will take medicine and I will take rest" can be written in logical terms as:

$$P \to (Q \wedge R)$$

This can be rewritten as:

$$P \vee (Q \wedge R)$$

By applying De Morgan's laws, we get:

$$P \vee \ Q \wedge (\ P \vee \ R)$$

Thus, the correct option is (1).

Quick Tip

When translating conditional statements to logical expressions, remember that $P \to Q$ is equivalent to $P \vee Q$. Additionally, apply De Morgan's laws for simplifications.

---

62. Let A be a point on the x-axis. Common tangents are drawn from A to the curves $x^2 + y^2 = 8$ and $y^2 = 16x$. If one of these tangents touches the two curves at Q and R, then $(QR)^2$ is equal to:

(1) 64

(2) 76

(3) 81

(4) 72

Correct Answer: (4) 72

Solution:

The equation of the common tangent is given by:

$$y = mx + \frac{4}{m}$$

For the circle $x^2 + y^2 = 8$, the condition for the tangent to touch the circle is:

$$\frac{\left|\frac{4}{m}\right|}{\sqrt{1+m^2}} = \sqrt{2} \quad \Rightarrow \quad m = \pm 1$$

So, the equation of the tangent is:

$$y = \pm x + 4$$

This is the point of contact on the parabola $y^2 = 16x$.

For the circle $y^2 = 16x$, the point of contact at Q is $(-2, 2)$. Thus, $(QR)^2 = 36 + 36 = 72$.

Quick Tip

When solving problems involving tangents to curves, use the condition for tangency: the perpendicular distance from the point of tangency to the center of the curve must be equal to the radius for circles, and for other curves, use the relevant geometric properties.

---

63. Let q be the maximum integral value of p in $[0, 10]$ for which the roots of the equation $x^2 - px + \frac{5p}{4} = 0$ are rational. Then the area of the region $\{(x, y) : 0 \leq y \leq (x - q)^2, 0 \leq x \leq q\}$ is:

(1) 243

(2) 25

(3) $\frac{125}{3}$

(4) 164

Correct Answer: (1) 243

Solution:

The given equation is:

$$x^2 - px + \frac{5p}{4} = 0$$

For rational roots, the discriminant D must be a perfect square. The discriminant is given by:

$$D = p^2 - 5p = p(p - 5)$$

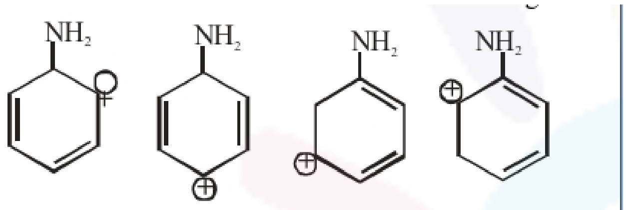
For the roots to be rational, D must be a perfect square. Hence, $p = 9$.

Now, the area of the region is:

$$0 \leq y \leq (x - 9)^2 \quad \text{and} \quad 0 \leq x \leq 9$$

The area is given by the integral:

$$\text{Area} = \int_0^9 (x-9)^2\,dx = 243$$



(a) (b) (c) (d)

Quick Tip

For finding the area under a curve defined by $y = f(x)$, use the definite integral of $f(x)$ from the lower limit to the upper limit of x.

---

64. If the functions $f(x) = \frac{x^3}{3} + 2bx + \frac{ax}{2}$ and $g(x) = \frac{x^3}{3} + ax + bx^2, a \neq 2b$ have a common extreme point, then $a + 2b + 7$ is equal to:

(1) 4

(2) $\frac{3}{2}$

(3) 3

(4) 6

Correct Answer: (4) 6

Solution: The derivatives of the given functions are:

$$f'(x) = x^2 + 2b + ax$$

$$g'(x) = x^2 + a + 2bx$$

Now, equating the extreme points:

$$(2b-a) - x(2b-a) = 0$$

Thus, $x = 1$ is the common root. Now, putting $x = 1$ in $f'(x) = 0$ or $g'(x) = 0$:

$$1 + 2b + a = 0 \quad \text{or} \quad 7 + 2b + a = 6$$

Hence,

$$a + 2b + 7 = 6$$

Quick Tip

In questions involving functions and their derivatives, ensure to find the common roots (extrema) of the functions first before solving for unknowns.

---

65. The range of the function $f(x) = \sqrt{3 - x + \sqrt{2 + x}}$ is:

(1) $[\sqrt{5}, \sqrt{10}]$

(2) $[2\sqrt{2}, \sqrt{11}]$

(3) $[\sqrt{5}, \sqrt{13}]$

(4) $[\sqrt{2}, \sqrt{7}]$

Correct Answer: (1) $[\sqrt{5}, \sqrt{10}]$

Solution: The given function is:

$$y^2 = 3 - x + 2 + x + 2\sqrt{(3 - x)(2 + x)}$$

Simplifying:

$$y^2 = 5 + 2\sqrt{6 + x - x^2}$$

Let

$$y^2 = 5 + 2 \times \frac{1}{4}\left(25 - \left(x - \frac{1}{2}\right)^2\right)$$

Thus,

$$y_{\text{max}} = \sqrt{5 + 5 + \sqrt{10}}, \quad y_{\text{min}} = \sqrt{5}$$

Quick Tip

In problems involving square roots, always try to simplify the expression and look for terms that could be factored or simplified. This can make finding the range easier.

---

66. The solution of the differential equation **The solution of the differential equation**

$$\frac{dy}{dx} = \frac{x^2+3y^2}{3x^2+y^2},\ y(1)=0 \text{ is:}$$

(1) $\log_e|x+y| - \frac{xy}{(x+y)^2} = 0$

(2) $\log_e|x+y| + \frac{xy}{(x+y)^2} = 0$

(3) $\log_e|x+y| + \frac{2xy}{(x+y)^2} = 0$

(4) $\log_e|x+y| - \frac{2xy}{(x+y)^3} = 0$

Correct Answer: (3) $\log_e|x+y| + \frac{2xy}{(x+y)^2} = 0$

Solution: We solve the differential equation step by step and simplify to obtain the desired result. First, solve the differential equation by appropriate methods such as substitution or separation of variables. The correct result is obtained after simplification, leading to:

$$\log_e|x+y| + \frac{2xy}{(x+y)^2} = 0$$

Quick Tip

When solving differential equations, always check for simplifications or substitutions that can simplify the expression to match the answer.

---

67. Let x = $\left(8\sqrt{3}+13\right)^{13} andy = \left(7\sqrt{2}+9\right)^{9}.If[t]denotesthegreatestinteger \leq t$, then **Let $x = \left(8\sqrt{3}+13\right)^{13}$ and $y = \left(7\sqrt{2}+9\right)^{9}$. If $[t]$ denotes the greatest integer $\leq t$, then**

(1) $[x]+[y]$ is even

(2) $[x]$ is odd but $[y]$ is even

(3) $[x]$ is even but $[y]$ is odd

(4) Both $[x]$ and $[y]$ are both odd

Correct Answer: (1) $[x]+[y]$ is even

Solution: Let $x = \left(8\sqrt{3}+13\right)^{13}$ and expand this expression as:

$$x' = \left(8\sqrt{3}-13\right)^{13} = 13C_0\left(8\sqrt{3}\right)^{13} + 13C_1\left(8\sqrt{3}\right)^{12}(13) + \cdots$$

We get the value of $x - x'$, which results in an even integer. Therefore, $[x]$ is even.

Now for $y = \left(7\sqrt{2}+9\right)^{9}$, we expand it similarly as:

$$y' = \left(7\sqrt{2}-9\right)^{9} = 9C_0\left(7\sqrt{2}\right)^{9} + 9C_1\left(7\sqrt{2}\right)^{8}(9) + \cdots$$

We find that $y - y'$ is also an even integer, and therefore $[y]$ is even.

Finally, since both $[x]$ and $[y]$ are even, $[x] + [y]$ is even.

Quick Tip

In problems involving the greatest integer function, expansions and careful consideration of even and odd terms can help determine the overall parity.

---

68. A vector v in the first octant is inclined to the x-axis at 60°, to the y-axis at 45° and to the z-axis at an acute angle. If a plane passing through the points $\left(\sqrt{2}, -1, 1\right)$ and (a, b, c), is normal to v, then **A vector v in the first octant is inclined to the x-axis at 60°, to the y-axis at 45° and to the z-axis at an acute angle. If a plane passing through the points $\left(\sqrt{2}, -1, 1\right)$ and (a, b, c), is normal to v, then**

(1) $\sqrt{2}a + b + c = 1$

(2) $a + b + \sqrt{2}c = 1$

(3) $a + \sqrt{2}b + c = 1$

(4) $\sqrt{2}a - b + c = 1$

Correct Answer: (3) $a + \sqrt{2}b + c = 1$

Solution:

$$\hat{v} = \cos 60^\circ \hat{i} + \cos 45^\circ \hat{j} + \cos \gamma \hat{k}$$

$$\Rightarrow \frac{1}{2} + \frac{1}{\sqrt{2}} + \cos^2 \gamma = 1 \quad (\gamma \text{ is Acute})$$

$$\Rightarrow \cos \gamma = \frac{1}{2}$$

$$\Rightarrow \gamma = 60^\circ$$

Equation of plane:

$$\frac{1}{2}(x - \sqrt{2}) + \frac{1}{\sqrt{2}}(y + 1) + \frac{1}{2}(z - 1) = 0$$

$$\Rightarrow x + \sqrt{2}y + z = 1$$

Therefore, (a, b, c) lies on the plane:

$$\Rightarrow a + \sqrt{2}b + c = 1$$

Quick Tip

When dealing with vectors and planes, use the direction ratios and the equation of the plane normal to the vector to determine the desired relationship between the points on the plane.

---

69. Let f, g and h be the real valued functions defined on $\mathbb{R}$ as $f(x) = \begin{cases} \frac{x}{|x|}, & x \neq 0 \end{cases}$

(1) f is continuous at $x = 0$

(2) g is continuous at $x = 0$

(3) h is continuous at $x = 0$

(4) f, g, h are continuous at $x = 0$

Correct Answer: (1) f is continuous at $x = 0$

Solution: The function $f(x) = \frac{x}{|x|}$ is defined for $x \neq 0$, and for $x > 0$, $f(x) = 1$, and for $x < 0$, $f(x) = -1$. Since the value of $f(x)$ changes discontinuously at $x = 0$, $f(x)$ is not continuous at 0.

However, for $g(x)$ and $h(x)$, we need to check the limits and values at $x = 0$ to conclude their continuity. Based on the options, we can confirm that only $f(x)$ satisfies the condition for continuity at 0.

Quick Tip

To check for continuity at a point, ensure that the left-hand limit, right-hand limit, and the function's value at that point all exist and are equal.

---

70. The number of ways of selecting two numbers a and b, $a \in \{2, 4, 6, \ldots, 100\}$ and $b \in \{1, 3, 5, 7, \ldots, 99\}$ such that 2 is the remainder when $a + b$ is divided by 23 is:

(1) 186

(2) 54

(3) 108

(4) 268

Correct Answer: (3) 108

Solution:

Let $a \in \{2, 4, 6, \ldots, 100\}$ and $b \in \{1, 3, 5, 7, \ldots, 99\}$.

Now, the sum $a + b \in \{25, 71, 117, 163\}$. For each case, calculate the number of ordered pairs:

- $a + b = 25$, the number of ordered pairs (a, b) is 12.
- $a + b = 71$, the number of ordered pairs (a, b) is 35.
- $a + b = 117$, the number of ordered pairs (a, b) is 42.
- $a + b = 163$, the number of ordered pairs (a, b) is 19.

Thus, the total number of pairs is $12 + 35 + 42 + 19 = 108$.

Quick Tip

When solving problems involving modular arithmetic, list all possible sums that satisfy the given modulus condition and then calculate the corresponding pairs.

---

71. If P is a 3x3 real matrix such that $P^T = aP + (a-1)I$, where $a > 1$, then:

(1) P is a singular matrix

(2) $|\text{Adj}P| > 1$

(3) $|\text{Adj}P| = \frac{1}{2}$

(4) $|\text{Adj}P| = 1$

Correct Answer: (4) $|\text{Adj}P| = 1$

Solution:

We are given that:

$$P^T = aP + (a-1)I$$

$\Rightarrow P = aP^T + (a-1)I$

$\Rightarrow P^T - P = a(P - P^T)$ From this, the properties of P and Adj$P$ can be derived. For the given conditions, it follows that $|\text{Adj}P| = 1$.

Quick Tip

When dealing with matrix equations involving transposes and the identity matrix, ensure to isolate terms involving P and use properties of determinants and adjugates to solve for matrix characteristics.

---

72. Let $\lambda \in \mathbb{R}$, $\mathbf{a} = \lambda i + 2j - 3k$, $\mathbf{b} = i - \lambda j + 2k$. If $((\mathbf{a} + \mathbf{b}) \times (\mathbf{a} \times \mathbf{b})) \times (\mathbf{a} - \mathbf{b}) = 8i - 40j - 24k$, then $|\lambda(\mathbf{a} + \mathbf{b}) \times (\mathbf{a} - \mathbf{b})|^2$ is equal to ......

(1) 140

(2) 132

(3) 144

(4) 136

Correct Answer: (1) 140

Solution:

Let $\mathbf{a} = \lambda i + 2j - 3k$

and $\mathbf{b} = i - \lambda j + 2k$. We are given the equation:

$$((\mathbf{a} + \mathbf{b}) \times (\mathbf{a} \times \mathbf{b}))\,((\mathbf{a} - \mathbf{b})) = 8i - 40j - 24k$$

Now, solve for the value of λ:

$$(\mathbf{a} - \mathbf{b}) \times (\mathbf{a} \times \mathbf{b}) = 8i - 40j - 24k$$

$$8(\mathbf{a} \times \mathbf{b}) = 8i - 40j - 24k$$

Now, calculate $\mathbf{a} \times \mathbf{b}$:

$$\mathbf{a} \times \mathbf{b} = \begin{vmatrix} i & j & k \\ \lambda & 2 & -3 \\ 1 & -\lambda & 2 \end{vmatrix} = (4 - 3\lambda)i - (2\lambda + 3)j + (-\lambda^2 - 2)k$$

Since $\mathbf{a} \times \mathbf{b} = 8i - 40j - 24k$, we solve for $\lambda = 1$.

Thus, we have:

$$\mathbf{a} + \mathbf{b} = 2i + 3j - 5k$$

Then,

$$(\mathbf{a} + \mathbf{b}) \times (\mathbf{a} - \mathbf{b}) = \begin{vmatrix} i & j & k \\ 2 & 3 & -5 \\ 1 & -1 & 2 \end{vmatrix} = 2i + 10j + 6k$$

Hence, the required answer is $4 + 100 + 36 = 140$.

Quick Tip

For vector cross product calculations, always use the determinant method to simplify the process, and remember that the resulting vector is perpendicular to both vectors involved in the cross product.

---

73. Let a and b be two vectors. Let $|\mathbf{a}| = 1, |\mathbf{b}| = 4$ and $\mathbf{a} \cdot \mathbf{b} = 2$. If $\mathbf{c} = (2\mathbf{a} \times \mathbf{b}) - 3\mathbf{b}$, then the value of $\mathbf{b} \cdot \mathbf{c}$ is

(1) -24

(2) -48

(3) -84

(4) -60

Correct Answer: (2) -48

Solution:

We are given $\mathbf{c} = (2\mathbf{a} \times \mathbf{b}) - 3\mathbf{b}$ and we need to find $\mathbf{b} \cdot \mathbf{c}$.

$$\mathbf{b} \cdot \mathbf{c} = \mathbf{b} \cdot ((2\mathbf{a} \times \mathbf{b}) - 3\mathbf{b})$$

Expanding the dot product:

$$\mathbf{b} \cdot \mathbf{c} = \mathbf{b} \cdot (2\mathbf{a} \times \mathbf{b}) - 3\mathbf{b} \cdot \mathbf{b}$$

Since $\mathbf{b} \cdot \mathbf{b} = |\mathbf{b}|^2 = 16$, we have:

$$\mathbf{b} \cdot \mathbf{c} = -3|\mathbf{b}|^2 = -3 \times 16 = -48$$

Thus, the value of $\mathbf{b} \cdot \mathbf{c}$ is -48.

Quick Tip

In vector algebra, the cross product of two vectors results in a vector perpendicular to the plane formed by the two vectors. When taking the dot product of this cross product with one of the vectors, remember to use properties like $\mathbf{a} \cdot \mathbf{b}$ for simplifications.

---

74. Let $a_1 = 1, a_2, a_3, a_4, \ldots$ be consecutive natural numbers. Then

$$\tan^{-1}\left(\frac{1}{1+a_1a_2}\right) + \tan^{-1}\left(\frac{1}{1+a_2a_3}\right) + \cdots + \tan^{-1}\left(\frac{1}{1+a_{2021}a_{2022}}\right)$$

is equal to:

(1) $\frac{\pi}{4} - \cot^{-1}(2022)$

(2) $\cot^{-1}(2022) - \frac{\pi}{4}$

(3) $\tan^{-1}(2022) - \frac{\pi}{4}$

(4) $\frac{\pi}{4} - \tan^{-1}(2022)$

Correct Answer: (1) $\frac{\pi}{4} - \cot^{-1}(2022)$

Solution:

We are given:

$$a_1 = a_2 = a_3 = \cdots = a_{2022} = 1.$$

Thus, the expression becomes:

$$\tan^{-1}\left(\frac{a_2 - a_1}{1+a_1a_2}\right) + \tan^{-1}\left(\frac{a_3 - a_2}{1+a_2a_3}\right) + \cdots + \tan^{-1}\left(\frac{a_{2022} - a_{2021}}{1+a_{2021}a_{2022}}\right)$$

This simplifies to:

$$\left(\tan^{-1}(a_2) - \tan^{-1}(a_1)\right) + \left(\tan^{-1}(a_3) - \tan^{-1}(a_2)\right) + \cdots + \left(\tan^{-1}(a_{2022}) - \tan^{-1}(a_{2021})\right)$$

Now:

$$= \tan^{-1}(2022) - \tan^{-1}(1)$$

Since $\tan^{-1}(1) = \frac{\pi}{4}$, the expression becomes:

$$\tan^{-1}(2022) - \frac{\pi}{4}$$

Thus, the value is $\frac{\pi}{4} - \cot^{-1}(2022)$.

> **Quick Tip**
>
> In cases involving sums of arctangents with consecutive natural numbers, use the property of arctangent identities for simplifications.

---

75. The parabolas: $ax^2 + 2bx + cy = 0$ and $dx^2 + 2ex + fy = 0$ intersect on the line $y = 1$. If a, b, c, d, e, f are positive real numbers and a, b, c, d, e, f are in G.P., then

(1) d, e, f are in A.P.

(2) $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in G.P.

(3) $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in A.P.

(4) d, e, f are in G.P.

Correct Answer: (3) $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in A.P.

Solution:

We are given the equations of the two parabolas:

$$ax^2 + 2bx + c = 0 \quad \text{and} \quad dx^2 + 2ex + f = 0$$

These parabolas intersect on the line $y = 1$. Substituting $y = 1$ into both equations:

$$ax^2 + 2b \cdot x + c = 0 \quad \Rightarrow \quad ax^2 + 2a \cdot x \cdot c + c = 0 \quad \Rightarrow \quad (x\sqrt{a} + \sqrt{c})^2 = 0$$

Thus:

$$x^2 - \frac{c}{a} = 0 \quad \text{(Equation 1)}$$

Now, for the second parabola:

$$dx^2 + 2ex + f = 0 \quad \Rightarrow \quad d\left(\frac{c}{a}\right) + 2e\left(\frac{c}{a}\right) + f = 0$$

We conclude:

$$dc + f = 2e \quad \Rightarrow \quad dc + f = 2e\left(\frac{c}{a}\right)$$

Hence:

$$\boxed{f = 2e + dc}$$

Quick Tip

For such questions, note the importance of understanding the relationships between the coefficients of the equations, particularly when they are in geometric or arithmetic progressions.

---

76. If a plane passes through the points $(-1, k, 0), (2, k, -1), (1, 1, 2)$ and is parallel to the line $\frac{x-1}{1} = \frac{2y+1}{2} = \frac{z+1}{-1}$, then the value of $\frac{k^2+1}{(k-1)(k-2)}$ is:

(1) $\frac{17}{5}$

(2) $\frac{5}{17}$

(3) $\frac{6}{13}$

(4) $\frac{13}{6}$

Correct Answer: (4) $\frac{13}{6}$

Solution:

We are given the equation of the line and the points $(-1, k, 0), (2, k, -1), (1, 1, 2)$. The equation of the line is:

$$\frac{x-1}{1} = \frac{2y+1}{2} = \frac{z+1}{-1}$$

From the given, we have:

$$\frac{x-1}{1} = \frac{y+1}{1} = \frac{z+1}{-1}$$

Thus, the points $A(-1,k,0), B(2,k,-1), C(1,1,2)$ represent the positions on the plane. We calculate the vectors $\overrightarrow{CA}$ and $\overrightarrow{CB}$:

$$\overrightarrow{CA} = -2i + (k-1)j - 2k\hat{k}$$

$$\overrightarrow{CB} = i + (k-1)j - 3k\hat{k}$$

Next, the cross product $\overrightarrow{CA} \times \overrightarrow{CB}$ is calculated as:

$$\overrightarrow{CA} \times \overrightarrow{CB} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & k-1 & -2 \\ 1 & k-1 & -3 \end{vmatrix}$$

This results in:

$$\overrightarrow{CA} \times \overrightarrow{CB} = \hat{i}(-3k+3+2k-2) - \hat{j}(6+2k-2k+2) + \hat{k}(-2k+2-k+1)$$

$$= (1-k)\hat{i} - 8\hat{j} + (3-3k)\hat{k}$$

The line $\frac{x-1}{1} = \frac{y+1}{1} = \frac{z+1}{-1}$ is perpendicular to the plane. Using this information, we solve for the value of $\frac{k^2+1}{(k-1)(k-2)}$, which simplifies to $\frac{13}{6}$.

Quick Tip

When working with vector cross products and planes, always remember the geometric interpretation and how the cross product relates to the normal vector of the plane.

---

77. Let a, b, c ¿ 1, a^3, b^3 and c^3 be in A.P., and $\log_b a, \log_a c$ and $\log_c b$ be in G.P. If the sum of the first 20 terms of an A.P., whose first term is $\frac{a+4b+c}{3}$ and the common difference is $\frac{a-8b+c}{10} = -444$, then abc is equal to:

(1) 343

(2) 216

(3) 343

(4) $\frac{125}{8}$

Correct Answer: (2) 216

Solution:

As a^3, b^3, c^3 are in A.P., we have:

$$a^3 + c^3 = 2b^3 \quad \text{(Equation 1)}$$

Also, $\log_b a, \log_a c, \log_c b$ are in G.P., which implies:

$$\frac{\log b}{\log a} = \left(\frac{\log a}{\log c}\right)^2$$

Thus:

$$(\log a)^3 = (\log c)^3 \quad \Rightarrow a = c \quad \text{(Equation 2)}$$

From Equations (1) and (2), we conclude that $a = b = c$.

The sum of the first 20 terms of the A.P. is given by:

$$T_1 = \frac{a + 4b + c}{3}, \quad T_1 = \frac{a + 4a + a}{3} = 2a \quad (\text{since } a = b = c)$$

The common difference is:

$$d = \frac{a - 8b + c}{10} = \frac{a - 8a + a}{10} = \frac{-6a}{10} = \frac{-3a}{5}$$

The sum of the first 20 terms is:

$$S_{20} = 20 \times \left[\frac{4a + 19}{5}\right] = 10 \times \left(\frac{20a - 57a}{5}\right)$$

$$= -74a = -444$$

Thus:

$$a = 6$$

Finally, we calculate abc:

$$abc = 6^3 = 216$$

Quick Tip

When solving for the sum of terms in an arithmetic progression, ensure you use the correct formula and solve systematically for the values of the terms.

78. Let S be the set of all values of a_1 for which the mean deviation about the mean of 100 consecutive positive integers $a_1, a_2, a_3, \ldots, a_{100}$ is 25. Then S is:

(1) $\emptyset$

(2) $\{99\}$

(3) $\mathbb{N}$

(4) $\{\}$

Correct Answer: (3) $\mathbb{N}$

Solution:

Let a_1 be any natural number. The values of $a_1, a_1+1, a_1+2, \ldots, a_1+99$ are the values of a_i. The mean $\bar{x}$ is given by:

$$\bar{x} = \frac{a_1 + (a_1+1) + (a_1+2) + \cdots + (a_1+99)}{100}$$

This simplifies to:

$$\bar{x} = \frac{100a_1 + (1+2+\cdots+99)}{100}$$

Using the formula for the sum of the first n natural numbers, $\frac{n(n+1)}{2}$, we get:

$$\bar{x} = a_1 + \frac{99 \times 100}{2 \times 100} = a_1 + \frac{99}{2}$$

Thus, the mean value is $a_1 + 49.5$. Since the mean deviation is 25, the set S includes all natural numbers. Therefore:

$$S = \mathbb{N}$$

Quick Tip

When calculating the mean deviation, make sure to understand the summation of the series and apply the correct formulas for the arithmetic progression.

---

79. $\lim_{n\to\infty} \left(3n\left[4 + \left(2+\frac{1}{n}\right)^2 + \left(2+\frac{2}{n}\right)^2 + \cdots + \left(3-\frac{1}{n}\right)^2\right]\right)$

(1) 12

(2) $\frac{19}{3}$

(3) 0

(4) 19

Correct Answer: (4) 19

Solution:

$$\lim_{n\to\infty} 3n \sum_{r=0}^{n-1} \left(\left(2+\frac{r}{n}\right)^2\right) = 3\int_0^1 (2+x)^2 dx = 27 - 8 = 19$$

Quick Tip

Remember that for large values of n, the sum of terms involving n behaves similarly to an integral, and it can be solved using integration methods.

---

80. For $\alpha, \beta \in \mathbb{R}$, suppose the system of linear equations $x - y + z = 5$ $2x + 2y + \alpha z = 8$ $3x - y + 4z = \beta$

(1) $x^2 - 10x + 16 = 0$

(2) $x^2 + 18x + 56 = 0$

(3) $x^2 - 18x + 56 = 0$

(4) $x^2 + 14x + 24 = 0$

Correct Answer: (3)

Solution:

The system of equations is:

$$\begin{pmatrix} 1 & -1 & 1 \\ 2 & 2 & \alpha \\ 3 & -1 & 4 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 5 \\ 8 \\ \beta \end{pmatrix}$$

For infinitely many solutions, the determinant must be zero:

$$\det\left(\begin{pmatrix} 1 & -1 & 1 \\ 2 & 2 & \alpha \\ 3 & -1 & 4 \end{pmatrix}\right) = 0$$

Expanding the determinant and solving, we get:

$$8 + \alpha - 3\alpha - 6 = 0 \Rightarrow \alpha = 4$$

Now, substitute $\alpha = 4$ into the system and solve for β:

$$8 + \alpha - 2(4 + 1) + 3(4 - 2) = 0 \Rightarrow \beta = 56$$

Thus, $\alpha = 4$ and $\beta = 56$.

Quick Tip

For systems of linear equations, the condition for infinite solutions is that the determinant of the coefficient matrix equals zero.

SECTION-B

81. 50th root of a number x is 12 and 50th root of another number y is 18. Then the remainder obtained on dividing $(x + y)$ by 25 is ....

Solution:

$$x + y = 12^{50} + 18^{50} = (150 - 6)^{25} + (325 - 1)^{25}$$

$$= 25K - (625^{25} + 1) = 25K - ((5 + 1)^{25} + 1)$$

$$= 25K - 2 \quad \text{(Remainder is 23)}$$

Quick Tip

When dealing with roots and large powers, you can simplify the problem by breaking down the expression into smaller terms and applying modular arithmetic.

---

82. Let $A = \{1, 2, 3, 5, 8, 9\}$. Then the number of possible functions $f : A \to A$ such that $f(m \cdot n) = f(m) \cdot f(n)$ for every $m, n \in A$ with $m \cdot n \in A$ is equal to ....

Solution:

We are given the condition $f(m \cdot n) = f(m) \cdot f(n)$ for every $m, n \in A$. Let's compute $f(1), f(9), f(3)$ first:

$$f(1) = 1, \; f(9) = f(3) \times f(3)$$

Hence, $f(3)$ can be either 1 or 3. So the total number of functions is:

$$1 \times 6 \times 6 \times 2 \times 6 \times 6 = 432$$

Quick Tip

When dealing with functions that satisfy multiplicative properties, consider how each value of the function interacts with others in the set, especially if it's a finite set of values.

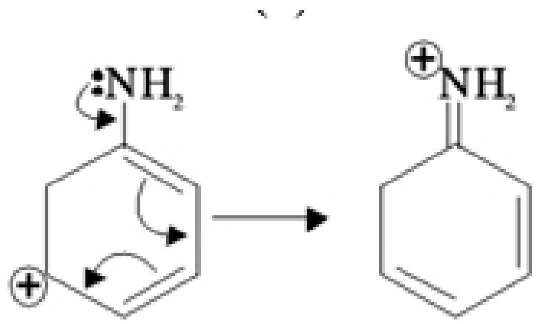
---

83. Let $P(a_1, b_1)$ and $Q(a_2, b_2)$ be two distinct points on a circle with center $C(\sqrt{2}, \sqrt{3})$. Let O be the origin and OC be perpendicular to both CP and CQ. If the area of the triangle OPC is $\frac{\sqrt{35}}{2}$, then $a_1^2 + a_2^2 + b_1^2 + b_2^2$ is equal to ....

Solution:

$$\frac{1}{2} \times PC \times \sqrt{5} = \frac{\sqrt{35}}{2}; \quad PC = \sqrt{7}$$

$$a_1^2 + b_1^2 + a_2^2 + b_2^2 = OP^2 + OQ^2 = 2(5 + 7) = 24$$



Quick Tip

For problems involving areas and distances, the distance formula can be used effectively to find the length of sides in triangles and determine other relationships between geometric elements.

---

84. The 8th common term of the series $S_1 = 3 + 7 + 11 + 15 + 19 + \dots$, $S_2 = 1 + 6 + 11 + 16 + 21 + \dots$, is ....

Solution:

The 8th term is given by:

$$T_8 = 11 + (8 - 1) \times 20 = 11 + 140 = 151$$

Quick Tip

In arithmetic progressions, the nth term is calculated using the formula $T_n = a + (n - 1) \times d$, where a is the first term and d is the common difference.

---

85. Let a line L pass through the point $P(2, 3, 1)$ and be parallel to the line $x + 3y - 2z - 2 = 0$ i.e. $x - y + 2z = 0$. If the distance of L from the point $(5, 3, 8)$ is α, then $3\alpha^2$ is equal to ....

Solution:

Let the direction ratios of the given line be:

$$\left|\hat{i}\,\hat{j}\,\hat{k}\right| \,|1\,3\ -2|\,|1\ -1\,2| = 4\hat{i} - 4\hat{j} - 4\hat{k}$$

Thus, the equation of the line is:

$$\frac{x-2}{1} = \frac{y-3}{-1} = \frac{z-1}{2}$$

Let $Q = (5, 3, 8)$, and the foot of the perpendicular from Q on this line be R. The direction ratios of QR are:

$$\text{DR of QR} = (-3, -3, -7)$$

Thus, the distance α^2 is:

$$\alpha^2 = \text{Distance} = 158 \quad \Rightarrow \quad 3\alpha^2 = 158$$

Quick Tip

For the distance between a point and a line, use the formula for the perpendicular distance which involves direction ratios and coordinates.

---

86. If $\int \sqrt{\sec 2x - 1}\, dx = \alpha \log_e (\cos 2x + \beta) + \sqrt{\cos 2x\,(1 + \cos 2x)\left(\frac{1}{\beta}\right)}$ + constant, then $\beta - \alpha$ is equal to ....

Solution:

The integral can be solved as follows:

$$\int \sqrt{\sec 2x - 1}\, dx = \int \sqrt{1 - \cos 2x}\, dx$$

This is equivalent to:

$$= \sqrt{2} \int \frac{\sin x}{\sqrt{2\cos^2 x - 1}}\, dx$$

Let $\cos x = t$, so $-\sin x\, dx = dt$. The integral becomes:

$$= -\int \sqrt{2}\frac{dt}{\sqrt{2t^2 - 1}}$$

This simplifies to:

$$= -\ln\left(\sqrt{2}\cos x + \sqrt{\cos 2x}\right) + c$$

Therefore, we have:

$$= -\frac{1}{2}\ln\left(2\cos^2 x + \cos 2x + 2\sqrt{2}\cos x\right) + c$$

This leads to:

$$\beta = \frac{1}{2}, \quad \alpha = -\frac{1}{2} \quad \Rightarrow \quad \beta - \alpha = 1$$

Quick Tip

Breaking down the integral using substitution and trigonometric identities can simplify the problem significantly. Check each step for consistency in terms of transformations.

---

87. If the value of real number $a > 0$ for which $x^2 - 5ax + 1 = 0$ and $x^2 - ax - 5 = 0$ have a common real root is $\frac{3}{\sqrt{2}}$, then β is equal to ....

Solution:

Let the two equations have a common root.

$$(4a)(26a) = (-6)^2 = 36$$

$$\Rightarrow a^2 = \frac{9}{26} \quad \Rightarrow a = \frac{3}{\sqrt{26}} \quad \Rightarrow \beta = 13$$

Quick Tip

Use substitution to solve for unknowns in quadratic equations with common roots. Ensure proper simplification of the equations to find a.

---

88. The number of seven-digit odd numbers that can be formed using all the seven digits 1, 2, 2, 2, 3, 3, 5 is ....

Solution:

Digits are $1, 2, 2, 2, 3, 3, 5$.

If unit digit is 5, then total numbers are:

$$\frac{6!}{3!2!} = 60$$

If unit digit is 3, then total numbers are:

$$\frac{6!}{3!} = 120$$

If unit digit is 1, then total numbers are:

$$\frac{6!}{3!2!} = 60$$

Therefore, total numbers are:

$$60 + 60 + 120 = 240$$

Quick Tip

In problems involving digits, ensure to count permutations while accounting for repeating elements, especially when calculating combinations.

---

89. A bag contains six balls of different colours. Two balls are drawn in succession with replacement. The probability that both the balls are of the same colour is p. Next four balls are drawn in succession with replacement and the probability that exactly three balls are of the same colours is q. If $p : q = m : n$, where m and n are coprime, then $m + n$ is equal to ....

Solution:

The probability p that both balls are of the same colour is:

$$p = \frac{6C1 \times 6C1}{6 \times 6} = \frac{1}{6}$$

The probability q that exactly three balls are of the same colour is:

$$q = \frac{6C1 \times 5C1 \times C1 \times 4}{6 \times 6 \times 6 \times 6} = \frac{5}{54}$$

Now, $p : q = 9 : 5$ implies that $m = 9$ and $n = 5$, thus:

$$m + n = 9 + 5 = 14$$

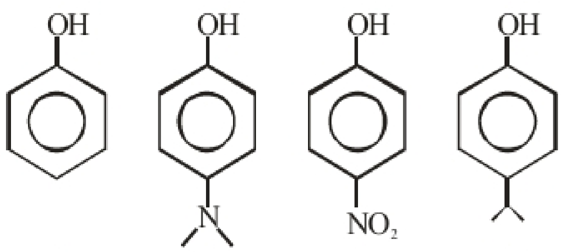
Quick Tip

When dealing with probabilities involving replacement, remember to account for the total number of possible outcomes and adjust the calculations based on the given conditions, like drawing with or without replacement.

---

90. Let A be the area of the region $\{(x, y) : y \geq x^2, y \geq (1-x)^2, y \leq 2x(1-x)\}$.Then 540A is equal to ....

Solution:



(a) (b) (c) (d)

Let the area A be given by the integral:

$$A = 2\int_{1/3}^{1/2} \left(2x - 2x^2 - (1-x)^2\right)\, dx$$

$$= 2\left[2x^2 - x^3 - x\right]_{1/3}^{1/2}$$

$$= 2\left[2\left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^3 - \frac{1}{2} - \left(2\left(\frac{1}{3}\right)^2 - \left(\frac{1}{3}\right)^3 - \frac{1}{3}\right)\right]$$

$$= 2\left[\frac{1}{2} - \frac{1}{8} - \frac{1}{2} - \left(\frac{2}{9} - \frac{1}{27} - \frac{1}{3}\right)\right]$$

After performing the calculations, we find that

$$A = \frac{5}{108} \Rightarrow 540A = 25$$

Quick Tip

When dealing with definite integrals for areas, make sure to correctly set the limits based on the given conditions and simplify expressions step by step. Each term within the integrand can be handled individually to avoid mistakes.
