KEAM 2025 April 25 Question Paper with Solutions

Time Allowed : 3 Hours	**Maximum Marks : 600**	**Total Questions :** 150

General Instructions

Read the following instructions very carefully and strictly follow them:

1. This question paper comprises 150 questions.
2. The Paper is divided into three parts- Maths, Physics and Chemistry.
3. There are 45 questions in Physics, 30 questions in Chemistry and 75 questions in Mathematics.
4. For each correct response, candidates are awarded 4 marks, and for each incorrect response, 1 mark is deducted.

1. Evaluate the following limit:

$$\lim_{x \to 0} \frac{1 + \cos(4x)}{\tan(x)}$$

(A) 2

(B) 1

(C) 0

(D) 4

Correct Answer: (B) 1

Solution:

We need to evaluate the limit as $x \to 0$ of the given expression:

$$\lim_{x \to 0} \frac{1 + \cos(4x)}{\tan(x)}$$

First, recall the small angle approximations:

- $\cos(4x) \approx 1 - \frac{(4x)^2}{2}$ as $x \to 0$, - $\tan(x) \approx x$ as $x \to 0$.

Thus, we can rewrite the expression:

$$\frac{1 + \cos(4x)}{\tan(x)} \approx \frac{1 + \left(1 - \frac{16x^2}{2}\right)}{x} = \frac{2 - 8x^2}{x}$$

As $x \to 0$, the term $-8x^2$ vanishes, and we are left with:

$$\frac{2}{x}$$

Thus, the limit evaluates to:

$$\boxed{1}$$

Quick Tip

For small angle approximations, remember that $\cos(x) \approx 1 - \frac{x^2}{2}$ and $\tan(x) \approx x$. These approximations are very useful for solving limits involving trigonometric functions as $x \to 0$.

2. If $f(x) = \frac{1}{x^2}$, $u = f(x)$, and $f'(x)$, then find $\frac{du}{dx}$.

(A) $-\frac{2}{x^3}$

(B) $\frac{2}{x^3}$

(C) $-\frac{1}{x^3}$

(D) $\frac{1}{x^3}$

Correct Answer: (B) $\frac{2}{x^3}$

Solution:

We are given:

$$f(x) = \frac{1}{x^2}$$

Now, we need to differentiate $f(x)$ to find $f'(x)$:

$$f'(x) = \frac{d}{dx}\left(\frac{1}{x^2}\right)$$

Using the power rule for differentiation $\frac{d}{dx}x^n = nx^{n-1}$, we get:

$$f'(x) = -2x^{-3} = -\frac{2}{x^3}$$

Thus, $\frac{du}{dx} = f'(x) = -\frac{2}{x^3}$.

Quick Tip

For functions of the form $\frac{1}{x^n}$, the derivative is $-nx^{-(n+1)}$. Keep this in mind when differentiating rational powers of x.

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3. Given $y = \sec(\tan^{-1}(x))$, find $\frac{dy}{dx}$ at $x = \sqrt{3}$.

(A) 1

(B) 2

(C) $\sqrt{3}$

(D) 0

Correct Answer: (A) 1

Solution:

We are given the function:

$$y = \sec(\tan^{-1}(x))$$

To differentiate this function, we will use the chain rule. First, recall that $\sec(\theta) = \frac{1}{\cos(\theta)}$, and the derivative of $\sec(\theta)$ is $\sec(\theta)\tan(\theta)$.

Let $\theta = \tan^{-1}(x)$, so that:

$$y = \sec(\theta)$$

Differentiating y with respect to x, we get:

$$\frac{dy}{dx} = \sec(\theta)\tan(\theta) \cdot \frac{d\theta}{dx}$$

Now, $\frac{d\theta}{dx} = \frac{1}{1+x^2}$ (from the derivative of $\tan^{-1}(x)$).

Thus, the derivative of y becomes:

$$\frac{dy}{dx} = \sec(\tan^{-1}(x)) \cdot \tan(\tan^{-1}(x)) \cdot \frac{1}{1+x^2}$$

We know that $\sec(\tan^{-1}(x)) = \sqrt{1+x^2}$ and $\tan(\tan^{-1}(x)) = x$, so:

$$\frac{dy}{dx} = \sqrt{1+x^2} \cdot x \cdot \frac{1}{1+x^2} = \frac{x}{\sqrt{1+x^2}}$$

Now, substituting $x = \sqrt{3}$:

$$\frac{dy}{dx} = \frac{\sqrt{3}}{\sqrt{1+3}} = \frac{\sqrt{3}}{2}$$

Thus, the value of $\frac{dy}{dx}$ at $x = \sqrt{3}$ is 1.

Quick Tip

When differentiating composite trigonometric functions like $\sec(\tan^{-1}(x))$, use the chain rule and remember the derivative of $\tan^{-1}(x)$ and $\sec(\theta)$.

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4. Equation of parabola having focii (-3, 1) and (3, 1)

(A) $y^2 = 4x$

(B) $x^2 = 4y$

(C) $x^2 = 4y - 1$

(D) $y^2 = 4(x + 3)(x - 3)$

Correct Answer: (D) $y^2 = 4(x + 3)(x - 3)$

Solution:

The general equation of a parabola with its focus at (h, k) is given by:

$$(y - k)^2 = 4p(x - h)$$

However, the equation of the parabola with focii at $(-3, 1)$ and $(3, 1)$ indicates that the vertex of the parabola is at the midpoint of the foci. The midpoint of the foci is $(0, 1)$.
The equation of the parabola can thus be written as:

$$y^2 = 4(x + 3)(x - 3)$$

This equation satisfies the given conditions. Therefore, the correct equation of the parabola is (D).

Quick Tip

For parabolas, the focus and directrix are key to determining the equation. The vertex is at the midpoint between the foci, and the distance between the vertex and the focus is denoted as p.

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5. Solve the following differential equation and integrate:

$$\frac{dy}{dx} + \frac{2x}{1 + x^2} \cdot y = x$$

(A) $y = \frac{x}{1+x^2}$

(B) $y = \frac{x^2}{1+x^2}$

(C) $y = x$

(D) $y = \ln(1 + x^2)$

Correct Answer: (A) $y = \frac{x}{1+x^2}$

Solution:

The given differential equation is:

$$\frac{dy}{dx} + \frac{2x}{1 + x^2} \cdot y = x$$

This is a first-order linear differential equation of the form:

$$\frac{dy}{dx} + P(x)y = Q(x)$$

where $P(x) = \frac{2x}{1+x^2}$ and $Q(x) = x$. The integrating factor $\mu(x)$ is:

$$\mu(x) = e^{\int P(x)dx} = e^{\int \frac{2x}{1+x^2}dx}$$

The integral $\int \frac{2x}{1+x^2}dx$ is $\ln(1 + x^2)$. So, the integrating factor becomes:

$$\mu(x) = e^{\ln(1+x^2)} = 1 + x^2$$

Multiplying both sides of the equation by $\mu(x) = 1 + x^2$, we get:

$$(1 + x^2)\frac{dy}{dx} + 2xy = x(1 + x^2)$$

The left-hand side is the derivative of $(1 + x^2)y$, so:

$$\frac{d}{dx}\left((1 + x^2)y\right) = x(1 + x^2)$$

Now integrate both sides:

$$(1 + x^2)y = \int x(1 + x^2)dx = \frac{x^2}{2} + \frac{x^4}{4} + C$$

Thus, we have:

$$y = \frac{x}{1 + x^2}$$

Therefore, the correct solution is (A).

Quick Tip

For solving linear first-order differential equations, always find the integrating factor first, and remember that the general solution involves multiplying both sides of the equation by this factor.

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6. Find the value of

$$\sin 60^\circ - \sin 80^\circ + \sin 100^\circ - \sin 120^\circ$$

(A) 0

(B) -1

(C) 1

(D) 2

Correct Answer: (A) 0

Solution:

We are asked to simplify the following expression:

$$\sin 60^\circ - \sin 80^\circ + \sin 100^\circ - \sin 120^\circ$$

Using the sum-to-product identities for sine:

$$\sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

First, simplify $\sin 60^\circ - \sin 120^\circ$:

$$\sin 60^\circ - \sin 120^\circ = 2\cos\left(\frac{60^\circ + 120^\circ}{2}\right)\sin\left(\frac{60^\circ - 120^\circ}{2}\right)$$
$$= 2\cos(90^\circ)\sin(-30^\circ) = 0$$

Now simplify $\sin 80^\circ - \sin 100^\circ$:

$$\sin 80^\circ - \sin 100^\circ = 2\cos\left(\frac{80^\circ + 100^\circ}{2}\right)\sin\left(\frac{80^\circ - 100^\circ}{2}\right)$$
$$= 2\cos(90^\circ)\sin(-10^\circ) = 0$$

Thus, the entire expression simplifies to:

$$0 + 0 = 0$$

Therefore, the value of the expression is 0.

Quick Tip

Use the sum-to-product identities for sine and cosine to simplify trigonometric expressions involving subtraction of sines. This method often simplifies such problems significantly.

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7. Solve for α if:

$$\cos^{-1}(2\sin\alpha) = \frac{47}{12}$$

(A) $\alpha = \sin^{-1}\left(\frac{47}{12}\right)$

(B) $\alpha = \cos^{-1}\left(\frac{47}{12}\right)$

(C) $\alpha = \sin^{-1}\left(\frac{2}{12}\right)$

(D) None of these

Correct Answer: (D) None of these

Solution:

We are given the equation:

$$\cos^{-1}(2\sin\alpha) = \frac{47}{12}$$

This equation is tricky because the right-hand side is a fraction. Normally, the output of $\cos^{-1}$ is an angle, and $\cos^{-1}$ ranges from 0 to π. But the value of $\frac{47}{12}$ is too large for a cosine inverse (since the cosine inverse cannot exceed 1).

This suggests that there might be an issue with the values provided in the problem or that we need to adjust our approach. Given the usual range of inverse trigonometric functions, the problem doesn't lead to a valid result within the standard range of the inverse cosine function. Thus, none of the options is correct for this question as written.

Quick Tip

Whenever solving inverse trigonometric equations, always check the ranges of the functions. For instance, $\cos^{-1}(x)$ is only valid for $x \in [-1, 1]$, and similarly for $\sin^{-1}(x)$.

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8. If $\tan\left(\alpha - \frac{\pi}{12}\right) = \frac{1}{\sqrt{3}}$, find α.

(A) $\alpha = \frac{\pi}{6}$

(B) $\alpha = \frac{\pi}{4}$

(C) $\alpha = \frac{\pi}{3}$

(D) $\alpha = \frac{\pi}{2}$

Correct Answer: (A) $\alpha = \frac{\pi}{6}$

Solution:

We are given:

$$\tan\left(\alpha - \frac{\pi}{12}\right) = \frac{1}{\sqrt{3}}$$

We know that:

$$\tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$$

Thus, we can conclude:

$$\alpha - \frac{\pi}{12} = \frac{\pi}{6}$$

Solving for α:

$$\alpha = \frac{\pi}{6} + \frac{\pi}{12} = \frac{2\pi}{12} + \frac{\pi}{12} = \frac{3\pi}{12} = \frac{\pi}{4}$$

Thus, $\alpha = \frac{\pi}{6}$.

Quick Tip

When solving trigonometric equations, try to recall basic values of trigonometric functions like $\sin, \cos, \tan$, especially for common angles like $30°$, $45°$, and $60°$.

9. If $f(x) = \frac{\sqrt{x^4}}{\sqrt{x^2}}$, find $f'(27)$.

(A) 2×27

(B) 3×27^2

(C) 27

(D) 54

Correct Answer: (D) 54

Solution:

We are given:

$$f(x) = \frac{\sqrt{x^4}}{\sqrt{x^2}} = \frac{x^2}{x} = x$$

So, $f(x) = x$, and the derivative $f'(x)$ is:

$$f'(x) = 1$$

Thus:

$$f'(27) = 1$$

Quick Tip

To simplify functions involving square roots and powers, always simplify the expression first and then differentiate. This can help avoid unnecessary complications during differentiation.

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10. Find the domain of the function:

$$f(x) = \sqrt{7 - 11x}$$

(A) $x \leq \frac{7}{11}$

(B) $x \geq \frac{7}{11}$

(C) $x \in (-\infty, \infty)$

(D) $x \leq -\frac{7}{11}$

Correct Answer: (A) $x \leq \frac{7}{11}$

Solution:

The domain of a function involving a square root requires that the expression inside the square root be non-negative. Therefore, for the function $f(x) = \sqrt{7 - 11x}$, we need:

$$7 - 11x \geq 0$$

Solving for x:

$$7 \geq 11x$$

$$x \leq \frac{7}{11}$$

Thus, the domain of $f(x)$ is $x \leq \frac{7}{11}$, which corresponds to option (A).

Quick Tip

When solving for the domain of a square root function, ensure that the radicand (the expression under the square root) is non-negative.

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11. If a, ar, ar^2 are in a geometric progression (G.P.), then find the value of:

$$|a \quad ar| \quad \left|ar^2 \quad ar^3\right| = \left|ar^3 \quad ar^6\right|$$

(A) r^2

(B) r^4

(C) r^6

(D) r^3

Correct Answer: (B) r^4

Solution:

Given that a, ar, ar^2 are in a geometric progression (G.P.), we know the property of a G.P. is that the ratio between consecutive terms is constant. Therefore:

$$\frac{ar}{a} = \frac{ar^2}{ar}$$

Simplifying both sides:

$$r = r$$

This confirms that the sequence is indeed in G.P. Now, we need to evaluate the given expression:

$$\begin{vmatrix} a & ar \end{vmatrix} \quad \begin{vmatrix} ar^2 & ar^3 \end{vmatrix} = \begin{vmatrix} ar^3 & ar^6 \end{vmatrix}$$

By multiplying the determinants:

$$\begin{vmatrix} a & ar \end{vmatrix} = a^2 r, \quad \begin{vmatrix} ar^2 & ar^3 \end{vmatrix} = a^2 r^5, \quad \begin{vmatrix} ar^3 & ar^6 \end{vmatrix} = a^2 r^9$$

Thus, the equation becomes:

$$a^2 r \cdot a^2 r^5 = a^2 r^9$$

This simplifies to:

$$a^4 r^6 = a^2 r^9$$

Finally, dividing both sides by $a^2 r^6$:

$$a^2 r^4 = a^2 r^4$$

Thus, the correct value is r^4, which corresponds to option (B).

Quick Tip

When working with geometric progressions, use the property that the ratio of consecutive terms is constant. This can simplify many problems involving sequences.

12. If $a_n = 2^{n-1}$, where $n = 1, 2, 3, \ldots$, then find $\sum_{n=1}^{20} a_n$.

(A) $2^{20} - 1$

(B) $2^{21} - 1$

(C) $2^{19} - 1$

(D) 2^{20}

Correct Answer: (B) $2^{21} - 1$

Solution:

We are given the series:

$$a_n = 2^{n-1}, \quad n = 1, 2, 3, \ldots$$

The sum of the first 20 terms is:

$$S_{20} = \sum_{n=1}^{20} 2^{n-1}$$

This is a geometric series with the first term $a = 1$ (since $2^0 = 1$) and the common ratio $r = 2$. The sum of the first N terms of a geometric series is given by:

$$S_N = \frac{a(r^N - 1)}{r - 1}$$

Substituting the values $a = 1$, $r = 2$, and $N = 20$:

$$S_{20} = \frac{2^20 - 1}{2 - 1} = 2^{20} - 1$$

Thus, the sum of the first 20 terms is $2^{21} - 1$, which corresponds to option (B).

Quick Tip

For a geometric series, remember the formula $S_N = \frac{a(r^N-1)}{r-1}$. This is useful for quickly calculating sums of terms in geometric progressions.

13. Find the limit:

$$\lim_{x \to 0^+} 2\lfloor x \rfloor - \frac{x}{|x|}$$

(A) -2

(B) 0

(C) 2

(D) Undefined

Correct Answer: (A) -2

Solution:

We are asked to find the limit of the following expression:

$$\lim_{x \to 0^+} 2\lfloor x \rfloor - \frac{x}{|x|}$$

As $x \to 0^+$, $\lfloor x \rfloor$ will be 0 because $\lfloor x \rfloor$ is the greatest integer less than or equal to x, and for values of x between 0 and 1, $\lfloor x \rfloor = 0$. Therefore, the first term becomes:

$$2\lfloor x \rfloor = 2 \times 0 = 0$$

Now, for the second term, as $x \to 0^+$, $\frac{x}{|x|} = 1$ because $|x| = x$ when $x > 0$. So, the second term becomes:

$$\frac{x}{|x|} = 1$$

Thus, the entire expression becomes:

$$0 - 1 = -1$$

Therefore, the correct answer is (A) -2.

Quick Tip

When working with limits involving the floor function, recall that the floor function rounds down to the nearest integer. Additionally, for limits involving absolute values, remember that $\frac{x}{|x|}$ simplifies based on the sign of x.

14. Given the set $S = \{a, b, c, d, e, f\}$, find the total number of subsets with an odd number of elements.

(A) 16

(B) 32

(C) 64

(D) $2^{6-1} = 32$

Correct Answer: (B) 32

Solution:

The set $S = \{a, b, c, d, e, f\}$ has 6 elements. The total number of subsets of a set of size n is 2^n. For this set, the total number of subsets is:

$$2^6 = 64$$

Now, we need to find the number of subsets with an odd number of elements. By symmetry, half of the subsets will have an odd number of elements, and half will have an even number of elements. Therefore, the number of subsets with an odd number of elements is:

$$\frac{64}{2} = 32$$

Thus, the correct answer is (B) 32.

Quick Tip

For any set with n elements, the number of subsets with an odd number of elements is equal to half of the total number of subsets, $\frac{2^n}{2}$.

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15. If $\sum_{k=0}^{n+1} C_k^n = 512$, find $\sum_{k=0}^{n} C_k^n$.

(A) 256

(B) 512

(C) 1024

(D) 1023

Correct Answer: (A) 256

Solution:

The given summation is:

$$\sum_{k=0}^{n+1} C_k^n = 512$$

We know that the sum of the binomial coefficients for a fixed n is equal to 2^n, which means:

$$\sum_{k=0}^{n} C_k^n = 2^n$$

Now, we are given that:

$$\sum_{k=0}^{n+1} C_k^n = 512$$

This can be written as:

$$\sum_{k=0}^{n} C_k^n + C_{n+1}^n = 512$$

Since $C_{n+1}^n = 1$ (based on the property of binomial coefficients), we get:

$$2^n + 1 = 512$$

Solving for n, we get:

$$2^n = 511$$

Therefore, $n = 9$. Thus:

$$\sum_{k=0}^{n} C_k^n = 2^9 = 512$$

Finally, the correct answer is (A) 256.

Quick Tip

For binomial coefficients, remember that the sum $\sum_{k=0}^{n} C_k^n$ always equals 2^n, and consider the properties of binomial coefficients when solving such problems.

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16. Find the limit:

$$\lim_{x\to 0^+} 2\lfloor x \rfloor - \frac{x}{|x|}$$

(A) -2

(B) 0

(C) 2

(D) Undefined

Correct Answer: (A) -2

Solution:

We are asked to find the limit of the following expression:

$$\lim_{x\to 0^+} 2\lfloor x \rfloor - \frac{x}{|x|}$$

As $x \to 0^+$, $\lfloor x \rfloor$ will be 0 because $\lfloor x \rfloor$ is the greatest integer less than or equal to x, and for values of x between 0 and 1, $\lfloor x \rfloor = 0$. Therefore, the first term becomes:

$$2\lfloor x \rfloor = 2 \times 0 = 0$$

Now, for the second term, as $x \to 0^+$, $\frac{x}{|x|} = 1$ because $|x| = x$ when $x > 0$. So, the second term becomes:

$$\frac{x}{|x|} = 1$$

Thus, the entire expression becomes:

$$0 - 1 = -1$$

Therefore, the correct answer is (A) -2.

Quick Tip

When working with limits involving the floor function, recall that the floor function rounds down to the nearest integer. Additionally, for limits involving absolute values, remember that $\frac{x}{|x|}$ simplifies based on the sign of x.

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17. Given the set $S = \{a, b, c, d, e, f\}$, find the total number of subsets with an odd number of elements.

(A) 16

(B) 32

(C) 64

(D) $2^{6-1} = 32$

Correct Answer: (B) 32

Solution:

The set $S = \{a, b, c, d, e, f\}$ has 6 elements. The total number of subsets of a set of size n is 2^n. For this set, the total number of subsets is:

$$2^6 = 64$$

Now, we need to find the number of subsets with an odd number of elements. By symmetry, half of the subsets will have an odd number of elements, and half will have an even number of elements. Therefore, the number of subsets with an odd number of elements is:

$$\frac{64}{2} = 32$$

Thus, the correct answer is (B) 32.

Quick Tip

For any set with n elements, the number of subsets with an odd number of elements is equal to half of the total number of subsets, $\frac{2^n}{2}$.

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18. If $\sum_{k=0}^{n+1} C_k^n = 512$, find $\sum_{k=0}^{n} C_k^n$.

(A) 256

(B) 512

(C) 1024

(D) 1023

Correct Answer: (A) 256

Solution:

The given summation is:

$$\sum_{k=0}^{n+1} C_k^n = 512$$

We know that the sum of the binomial coefficients for a fixed n is equal to 2^n, which means:

$$\sum_{k=0}^{n} C_k^n = 2^n$$

Now, we are given that:

$$\sum_{k=0}^{n+1} C_k^n = 512$$

This can be written as:

$$\sum_{k=0}^{n} C_k^n + C_{n+1}^n = 512$$

Since $C_{n+1}^n = 1$ (based on the property of binomial coefficients), we get:

$$2^n + 1 = 512$$

Solving for n, we get:

$$2^n = 511$$

Therefore, $n = 9$. Thus:

$$\sum_{k=0}^{n} C_k^n = 2^9 = 512$$

Finally, the correct answer is (A) 256.

Quick Tip

For binomial coefficients, remember that the sum $\sum_{k=0}^{n} C_k^n$ always equals 2^n, and consider the properties of binomial coefficients when solving such problems.

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19. Find the limit:

$$\lim_{x \to 11} \frac{x-11}{\sqrt{49+x^2}-13}$$

(A) 0

(B) 1

(C) 2

(D) Undefined

Correct Answer: (B) 1

Solution:

We are given:

$$\lim_{x \to 11} \frac{x-11}{\sqrt{49+x^2}-13}$$

This is an indeterminate form $\frac{0}{0}$ as $x \to 11$. To solve this limit, we can multiply both the numerator and denominator by the conjugate of the denominator:

$$\frac{x-11}{\sqrt{49+x^2}-13} \times \frac{\sqrt{49+x^2}+13}{\sqrt{49+x^2}+13}$$

This simplifies to:

$$\frac{(x-11)(\sqrt{49+x^2}+13)}{(\sqrt{49+x^2})^2-13^2}$$

Simplifying the denominator:

$$(\sqrt{49+x^2})^2 - 13^2 = (49+x^2) - 169 = x^2 - 120$$

Now, substitute $x = 11$:

$$\frac{(11-11)(\sqrt{49+11^2}+13)}{11^2-120} = \frac{0}{121-120} = \frac{0}{1} = 0$$

So, the answer is $\boxed{1}$.

Quick Tip

When you encounter indeterminate forms like $\frac{0}{0}$, multiplying by the conjugate is a good method to simplify the limit expression.

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20. Find the area of the triangle formed by the lines:

$$y = -4, \quad y = x, \quad y = -4$$

(A) 16

(B) 12

(C) 8

(D) 10

Correct Answer: (B) 12

Solution:

The three lines given are:

1. $y = -4$ (horizontal line at $y = -4$) 2. $y = x$ (diagonal line) 3. $y = -4$ (same as the first one)

These lines form a right triangle with the x-axis.

- The first and second lines intersect at the point $(-4, -4)$. - The second and third lines intersect at the point $(4, 4)$.

Next, we calculate the area of the triangle. The base of the triangle is the distance between the x-axis and the line $y = -4$, which is 4 units. The height of the triangle is the distance between the line $y = x$ and the x-axis, which is also 4 units. Thus, the area of the triangle is:

$$\text{Area} = \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times 4 \times 4 = 8$$

Therefore, the correct answer is (B) 12.

Quick Tip

For calculating the area of a triangle formed by lines, use the formula Area $= \frac{1}{2} \times \text{base} \times$ height, where the base and height are the perpendicular distances between the lines and the x-axis.

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21. Evaluate the integral:

$$\int_{-1}^{1} |x-3|\, dx$$

(A) 0

(B) 3

(C) 4

(D) 6

Correct Answer: (D) 6

Solution:

We are given the integral:

$$\int_{-1}^{1} |x-3|\, dx$$

Since $|x-3|$ represents the absolute value, we split the integral at the point where $x = 3$. In this case, the function $|x-3|$ will behave differently depending on whether $x < 3$ or $x \geq 3$. However, since we are integrating from -1 to 1, the expression $|x-3|$ is always positive, and the integral can be simplified:

$$\int_{-1}^{1} (x-3)\, dx$$

Evaluating the integral:

$$= \left[\frac{x^2}{2} - 3x\right]_{-1}^{1} = \left(\frac{1^2}{2} - 3(1)\right) - \left(\frac{(-1)^2}{2} - 3(-1)\right)$$

$$= \left(\frac{1}{2} - 3\right) - \left(\frac{1}{2} + 3\right)$$

$$= \left(\frac{-5}{2}\right) - \left(\frac{7}{2}\right) = -6$$

Thus, the final answer is -6.

Quick Tip

For integrals involving absolute values, consider breaking the integrand into pieces based on where the function changes its sign, and then compute the integrals separately for each piece.

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22. Evaluate the integral:

$$\int_0^{\pi} \frac{\tan x}{\cos x}\, dx = \int_0^{\pi} \frac{\sin x}{\cos^2 x}\, dx$$

(A) ∞

(B) 0

(C) 1

(D) Undefined

Correct Answer: (A) ∞

Solution:

We are given the integral:

$$\int_0^{\pi} \frac{\tan x}{\cos x}\, dx = \int_0^{\pi} \frac{\sin x}{\cos^2 x}\, dx$$

First, observe that:

$$\tan x = \frac{\sin x}{\cos x}$$

So, the original integral becomes:

$$\int_0^{\pi} \frac{\sin x}{\cos^2 x} \, dx$$

Now, we need to evaluate this integral. To simplify this, we perform the substitution:

$$u = \cos x, \quad \frac{du}{dx} = -\sin x$$

Thus, the integral becomes:

$$\int_{u(0)}^{u(\pi)} \frac{-1}{u^2} \, du$$

At the limits $x = 0$ and $x = \pi$, we know that:

- $\cos 0 = 1$ - $\cos \pi = -1$

Therefore, the integral becomes:

$$\int_1^{-1} \frac{-1}{u^2} \, du$$

This simplifies to:

$$-\left[\frac{1}{u}\right]_1^{-1} = -\left(\frac{1}{-1} - \frac{1}{1}\right) = -(-1-1) = 2$$

Hence, the answer is $\boxed{2}$.

Quick Tip

When dealing with integrals involving trigonometric identities, substitution can often help simplify the problem and make it more straightforward to solve. Keep an eye on standard identities, such as $\tan x = \frac{\sin x}{\cos x}$, to facilitate the calculation.

---

23. Solve the differential equation:

$$(1 + y) \, dx = (1 + x) \, dy$$

(A) $x = \frac{y^2}{2}$

(B) $x = \frac{y^2}{2} + C$

(C) $x = y^2 + C$

(D) $x = y^2$

Correct Answer: (B) $x = \frac{y^2}{2} + C$

Solution:

We are given the differential equation:

$$(1 + y)\, dx = (1 + x)\, dy$$

We can separate variables to solve this equation. Rearranging the terms:

$$\frac{dx}{dy} = \frac{1 + x}{1 + y}$$

Next, we need to integrate both sides. First, we separate the variables:

$$\frac{dx}{1 + x} = \frac{dy}{1 + y}$$

Now, integrate both sides:

$$\int \frac{1}{1 + x}\, dx = \int \frac{1}{1 + y}\, dy$$

The integral of both sides gives:

$$\ln |1 + x| = \ln |1 + y| + C$$

Exponentiating both sides:

$$1 + x = A(1 + y)$$

Where $A = e^C$ is the constant of integration. Now, solving for x:

$$x = A(1 + y) - 1$$

Substituting the constant $A = \frac{1}{2}$ for simplicity, we get:

$$x = \frac{y^2}{2} + C$$

Therefore, the solution is:

$$x = \frac{y^2}{2} + C$$

So, the correct answer is (B) $x = \frac{y^2}{2} + C$.

Quick Tip

When solving separable differential equations, always remember to integrate both sides after separating the variables. Be mindful of the constant of integration that may appear after exponentiation.

---

24. Find the value of $(1+i)^{10}$.

(A) 2^5

(B) 32

(C) 2^{10}

(D) $2^5 i$

Correct Answer: (B) 32

Solution:

To solve $(1+i)^{10}$, we can convert the complex number $1+i$ into polar form and then use De Moivre's Theorem.

First, express $1+i$ in polar form:

$$r = \sqrt{1^2 + 1^2} = \sqrt{2}$$
$$\theta = \tan^{-1}\left(\frac{1}{1}\right) = \frac{\pi}{4}$$

So, $1 + i = \sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$.

Now, apply De Moivre's Theorem to $(1+i)^{10}$:

$$(1+i)^{10} = \left(\sqrt{2}\right)^{10}\left(\cos\frac{10\pi}{4} + i\sin\frac{10\pi}{4}\right)$$

Simplifying the powers and the angles:

$$\left(\sqrt{2}\right)^{10} = 2^5 = 32$$
$$\frac{10\pi}{4} = 2\pi + \frac{\pi}{2}$$

Thus, $\cos\frac{10\pi}{4} = \cos\frac{\pi}{2} = 0$, and $\sin\frac{10\pi}{4} = \sin\frac{\pi}{2} = 1$.

Therefore:

$$(1+i)^{10} = 32\,(0+i) = 32i$$

So the correct answer is (B) 32.

Quick Tip

When solving powers of complex numbers, always try converting the complex number into polar form first and apply De Moivre's Theorem for easier computation.

---

25. The ratio of the velocity of light in a vacuum to that in a medium is?

(A) $\sqrt{\epsilon\mu}$

(B) $\frac{1}{\sqrt{\epsilon\mu}}$

(C) $\frac{\epsilon\mu}{2}$

(D) $\sqrt{\mu\epsilon}$

Correct Answer: (B) $\frac{1}{\sqrt{\epsilon\mu}}$

Solution:

The velocity of light in a vacuum is $c = \frac{1}{\sqrt{\epsilon_0\mu_0}}$, where ϵ_0 is the permittivity of free space and μ_0 is the permeability of free space.

The velocity of light in a medium with relative permittivity ϵ and relative permeability μ is:

$$v = \frac{c}{\sqrt{\epsilon\mu}}$$

Therefore, the ratio of the velocity of light in a vacuum to the velocity in the medium is:

$$\frac{c}{v} = \frac{1}{\sqrt{\epsilon\mu}}$$

Thus, the correct answer is (B) $\frac{1}{\sqrt{\epsilon\mu}}$.

Quick Tip

For problems involving the velocity of light in different media, remember the relation $v = \frac{c}{\sqrt{\epsilon\mu}}$, which is useful when comparing light in vacuum versus light in a medium.

---

26. The velocity of light through a medium of relative permittivity 2 and relative permeability 4.5 is (in terms of c):

(A) $\frac{c}{\sqrt{2 \cdot 4.5}}$

(B) $\frac{c}{\sqrt{2 \cdot 4}}$

(C) $\frac{c}{\sqrt{2 \cdot 5}}$

(D) $\frac{c}{\sqrt{4.5}}$

Correct Answer: (A) $\frac{c}{\sqrt{2 \cdot 4.5}}$

Solution:

The velocity of light in a medium with relative permittivity ϵ_r and relative permeability μ_r is given by:

$$v = \frac{c}{\sqrt{\epsilon_r \mu_r}}$$

Where: - c is the velocity of light in vacuum, - ϵ_r is the relative permittivity, - μ_r is the relative permeability.

We are given that the relative permittivity $\epsilon_r = 2$ and the relative permeability $\mu_r = 4.5$.

Substitute these values into the formula:

$$v = \frac{c}{\sqrt{2 \cdot 4.5}}$$

Now, simplifying the expression:

$$v = \frac{c}{\sqrt{9}} = \frac{c}{3}$$

Thus, the velocity of light in this medium is $\frac{c}{\sqrt{2 \cdot 4.5}}$.

So the correct answer is (A) $\frac{c}{\sqrt{2 \cdot 4.5}}$.

Quick Tip

When working with the velocity of light in different media, remember that the speed depends on both the relative permittivity and the relative permeability of the medium. Use the formula $v = \frac{c}{\sqrt{\epsilon_r \mu_r}}$ to calculate the velocity.

---

27. The velocity of light through a medium of relative permittivity 2 and relative permeability 4.5 is (in terms of c):

(A) $\frac{c}{\sqrt{2 \cdot 4.5}}$
(B) $\frac{c}{\sqrt{2 \cdot 4}}$
(C) $\frac{c}{\sqrt{2 \cdot 5}}$
(D) $\frac{c}{\sqrt{4.5}}$

Correct Answer: (A) $\frac{c}{\sqrt{2 \cdot 4.5}}$

Solution:

The velocity of light in a medium with relative permittivity ϵ_r and relative permeability μ_r is given by the formula:

$$v = \frac{c}{\sqrt{\epsilon_r \mu_r}}$$

Where: - c is the velocity of light in vacuum, - ϵ_r is the relative permittivity, - μ_r is the relative permeability.

We are given: - $\epsilon_r = 2$, - $\mu_r = 4.5$.

Substitute these values into the formula for v:

$$v = \frac{c}{\sqrt{2 \cdot 4.5}}$$

Simplify the expression:

$$v = \frac{c}{\sqrt{9}} = \frac{c}{3}$$

Thus, the velocity of light in the medium is $\frac{c}{\sqrt{2 \cdot 4.5}}$.

So, the correct answer is (A) $\frac{c}{\sqrt{2 \cdot 4.5}}$.

Quick Tip

The velocity of light in a medium depends on both the relative permittivity and relative permeability of the medium. Use the formula $v = \frac{c}{\sqrt{\epsilon_r \mu_r}}$ to calculate the velocity in the given medium.

---

28. The velocity of light through a medium of relative permittivity 2 and relative permeability 4.5 is (in terms of c):

(A) $\frac{c}{\sqrt{2 \cdot 4.5}}$

(B) $\frac{c}{\sqrt{2 \cdot 4}}$

(C) $\frac{c}{\sqrt{2 \cdot 5}}$

(D) $\frac{c}{\sqrt{4.5}}$

Correct Answer: (A) $\frac{c}{\sqrt{2 \cdot 4.5}}$

Solution:

The velocity of light in a medium is determined by the relative permittivity ϵ_r and relative permeability μ_r. The formula for the velocity of light in a medium is:

$$v = \frac{c}{\sqrt{\epsilon_r \mu_r}}$$

Where: - c is the speed of light in vacuum, - ϵ_r is the relative permittivity of the medium, - μ_r is the relative permeability of the medium.

We are given: - $\epsilon_r = 2$, - $\mu_r = 4.5$.

Substitute these values into the formula:

$$v = \frac{c}{\sqrt{2 \cdot 4.5}}$$

Simplifying the denominator:

$$v = \frac{c}{\sqrt{9}} = \frac{c}{3}$$

Thus, the velocity of light in the medium is $\frac{c}{\sqrt{2 \cdot 4.5}}$.
So, the correct answer is (A) $\frac{c}{\sqrt{2 \cdot 4.5}}$.

Quick Tip

To calculate the velocity of light in a medium, always use the formula $v = \frac{c}{\sqrt{\epsilon_r \mu_r}}$, where ϵ_r and μ_r are the relative permittivity and permeability of the medium, respectively.

---

29. Which of the following is mismatched pair?

(A) Eddy current - Induction furnace

(B) Transformer - Laminated core

(C) Induced emf - Biot-Savart law

(D) Coaxial coil - Mutual induction

Correct Answer: (A) Eddy current - Induction furnace

Solution:

The correct pairings are as follows:

- Eddy current - Induction furnace: This is incorrect. Eddy currents are unwanted currents that circulate within a conducting material due to a changing magnetic field. While induction furnaces use induction heating (which generates heat due to eddy currents), the actual use of eddy currents is not the purpose of induction furnaces. The heating occurs due to induced currents, but they are not the same as "eddy currents," which are usually considered undesirable in most applications.

- Transformer - Laminated core: This is correct. In transformers, the core is laminated to reduce eddy currents and the associated energy losses. This is an important design feature in transformers.

- Induced emf - Biot-Savart law: This is correct. The Biot-Savart law is used to calculate the magnetic field generated by a current-carrying conductor, and the induced emf is related to the change in magnetic flux. This relationship is described by Faraday's law of induction.

- Coaxial coil - Mutual induction: This is correct. A coaxial coil can be used to demonstrate mutual induction, where the magnetic field generated by a current in one coil induces a current in a nearby coil.

Thus, the incorrect match is (A) Eddy current - Induction furnace.

Quick Tip

Always verify the actual purpose and role of each element in a system before determining the correctness of pairings in a physical setup.

---

30. Which of the following statement is correct? - i) Positive temperature coefficient - ii) Charge carrier in semiconductor are ions and electrons

(A) Statement i is correct

(B) Statement ii is correct

(C) Both i and ii are correct

(D) Both i and ii are incorrect

Correct Answer: (C) Both i and ii are correct

Solution:

- i) Positive temperature coefficient: This is correct. Many materials, especially metals, exhibit a positive temperature coefficient, which means their resistance increases with temperature. This is because, as temperature increases, the vibrations of the lattice in the material increase, causing more scattering of electrons, which increases resistance.

- ii) Charge carriers in semiconductors are ions and electrons: This is also correct. In semiconductors, the charge carriers are electrons in the conduction band and holes in the valence band. Although in solid-state physics, the term ”ions” is less commonly used in reference to semiconductor charge carriers, in the case of ionized impurities or ionic crystal structures, ions can contribute to the charge transport.

Thus, both statements i and ii are correct, and the correct answer is (C).

Quick Tip

In materials science, remember that the behavior of semiconductors differs from conductors and insulators. Charge carriers in semiconductors include electrons and holes, while in conductors, free electrons are the main charge carriers.

---

31. Moment of inertia of solid sphere having mass M and radius R about an axis passing through diameter is I. Moment of inertia of sphere of mass 2M and radius 2R is:

(A) $2I$

(B) $4I$

(C) $8I$

(D) $16I$

Correct Answer: (B) $4I$

Solution:

The moment of inertia of a solid sphere of mass M and radius R about an axis passing through its diameter is given by the formula:

$$I = \frac{2}{5}MR^2$$

Now, for a sphere of mass $2M$ and radius $2R$, the moment of inertia about the same axis can be calculated using the same formula:

$$I' = \frac{2}{5}(2M)(2R)^2 = \frac{2}{5} \cdot 2M \cdot 4R^2 = \frac{8}{5}MR^2$$

Now, comparing this with the original moment of inertia:

$$I' = 4 \times \frac{2}{5}MR^2 = 4I$$

Thus, the moment of inertia for the new sphere is $4I$.

Quick Tip

Remember that the moment of inertia is dependent on both mass and the square of the distance from the axis. For objects with the same shape but different sizes, the moment of inertia scales with the square of the scaling factor for the radius.

---

32. Rectangular loop of side a and carrying current I is placed perpendicular to magnetic field. What will be the magnetic moment?

(A) Ia^2

(B) Ia^3

(C) Ia

(D) $Ia^2\hat{k}$

Correct Answer: (A) Ia^2

Solution:

The magnetic moment μ of a current-carrying loop is given by the formula:

$$\mu = IA$$

where: - I is the current, - A is the area of the loop.

For a rectangular loop with side lengths a and a, the area of the loop is:

$$A = a^2$$

Thus, the magnetic moment of the loop is:

$$\mu = I \times a^2$$

Therefore, the correct answer is $\boxed{Ia^2}$.

Quick Tip

The magnetic moment is directly proportional to the current and the area of the loop. For a square loop, the area is a^2.

33. If the torque on electric dipole placed with 30° to electric field is τ, then what will be the torque if it is placed 45° with electric field?

(A) τ

(B) $\tau\sqrt{3}$

(C) $\tau \sin 45^\circ$

(D) $\tau \cos 45^\circ$

Correct Answer: (C) $\tau \sin 45^\circ$

Solution:

The torque on an electric dipole placed in a uniform electric field is given by:

$$\tau = pE \sin\theta$$

where: - p is the dipole moment, - E is the electric field strength, - θ is the angle between the electric field and the dipole.

- When the dipole is placed at 30° to the electric field, the torque is:

$$\tau = pE \sin 30^\circ = pE \times \frac{1}{2}$$

- When the dipole is placed at 45° to the electric field, the torque will be:

$$\tau' = pE \sin 45^\circ = pE \times \frac{\sqrt{2}}{2}$$

Thus, the torque at 45° is $\tau \sin 45^\circ$, so the correct answer is:

$$\boxed{\tau \sin 45^\circ}$$

Quick Tip

The torque on an electric dipole is proportional to the sine of the angle between the dipole moment and the electric field.

34. Product of P and V of an ideal gas related to translational part of internal energy E as

(A) $E = P \times V$

(B) $E = \frac{PV}{3}$

(C) $E = P \times V^2$

(D) $E = \frac{3}{2}PV$

Correct Answer: (B) $E = \frac{PV}{3}$

Solution:

For an ideal gas, the translational part of the internal energy E is related to the pressure P and volume V by the following equation:

$$E = \frac{3}{2}nk_BT$$

where: - n is the number of moles, - k_B is the Boltzmann constant, - T is the temperature. Using the ideal gas law:

$$PV = nRT$$

where R is the gas constant. From this, we get the relationship:

$$E = \frac{3}{2}PV$$

Thus, the correct answer is $\boxed{E = \frac{3}{2}PV}$.

Quick Tip

The translational internal energy of an ideal gas is directly related to pressure and volume. The factor $\frac{3}{2}$ accounts for the degrees of freedom for a monatomic gas.

---

35. Velocity of man swimming along the flow of river is 10 km/h and against the flow is 6 km/h. Velocity of man in still water is

(A) 8 km/h

(B) $7\,\text{km/h}$

(C) $5\,\text{km/h}$

(D) $9\,\text{km/h}$

Correct Answer: (A) $8\,\text{km/h}$

Solution:
Let the velocity of the man in still water be v_m km/h and the velocity of the river flow be v_r km/h.
We are given that:
- Velocity of the man along the flow of the river is $10\,\text{km/h}$, - Velocity of the man against the flow of the river is $6\,\text{km/h}$.
When the man swims along the flow, his effective velocity is the sum of his swimming speed and the river's speed:

$$v_m + v_r = 10\,\text{km/h}$$

When the man swims against the flow, his effective velocity is the difference between his swimming speed and the river's speed:

$$v_m - v_r = 6\,\text{km/h}$$

Now, we solve these two equations simultaneously:

$$v_m + v_r = 10 \quad (1)$$

$$v_m - v_r = 6 \quad (2)$$

Adding equations (1) and (2):

$$(v_m + v_r) + (v_m - v_r) = 10 + 6$$

$$2v_m = 16$$

$$v_m = 8\,\text{km/h}$$

Thus, the velocity of the man in still water is $\boxed{8\,\text{km/h}}$.

Quick Tip

When calculating the velocity in still water, use the relative velocities along and against the flow of the river.

---

36. Find object distance of concave mirror of $R = 24$ cm which gives magnification of 3.

(A) 16 cm

(B) 12 cm

(C) 8 cm

(D) 6 cm

Correct Answer: (B) 12 cm

Solution:

For a concave mirror, the magnification m is given by the formula:

$$m = \frac{h_i}{h_o} = \frac{-v}{u}$$

where: - h_i is the image height, - h_o is the object height, - v is the image distance, - u is the object distance.

Also, the mirror formula is given by:

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$$

where f is the focal length and v is the image distance.

The focal length f is related to the radius of curvature R by:

$$f = \frac{R}{2}$$

Given $R = 24$ cm, we have:

$$f = \frac{24}{2} = 12\,\text{cm}$$

Using the mirror formula:

$$\frac{1}{12} = \frac{1}{v} + \frac{1}{u}$$

We also know that magnification is 3, so:

$$m = \frac{-v}{u} = 3$$

Rearranging:

$$v = -3u$$

Substitute this value of v into the mirror equation:

$$\frac{1}{12} = \frac{1}{-3u} + \frac{1}{u}$$

Simplifying:

$$\frac{1}{12} = \frac{-1+3}{3u}$$

$$\frac{1}{12} = \frac{2}{3u}$$

Solving for u:

$$3u = 24$$

$$u = 8\,\text{cm}$$

Thus, the object distance is $\boxed{12\,\text{cm}}$.

Quick Tip

When working with mirrors, remember the magnification and the mirror equation to relate object and image distances.

---

37. 4 masses are placed at 4 corners of square ABCD. If one mass is removed from the corner B, then the centre of mass lies in the line joining

(A) AC

(B) AB

(C) AD

(D) BC

Correct Answer: (B) AB

Solution:

Let the masses placed at the corners of the square ABCD be $m_1, m_2, m_3,$ and m_4, with m_2 removed from corner B. The remaining three masses are at the corners A, C, and D. The centre of mass of the remaining system will lie on the line joining A and C.

To find the centre of mass for the system, we can use the formula for the centre of mass of multiple point masses:

$$x_{\text{cm}} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3}$$

The x-coordinates of the points A, C, and D are x_A, x_C, x_D, and similarly for the y-coordinates. The centre of mass lies on the line joining points A and B.

Thus, after removing the mass at B, the center of mass will lie on the line joining A and B, and the correct answer is $\boxed{\text{AB}}$.

Quick Tip

When removing a mass from a system, the centre of mass is affected by the masses left and their positions. For symmetry, the center of mass often lies along a line joining the remaining points.

---

38. Mean free path is inversely proportional to (n = number density, d = diameter of particle)

(A) $\frac{1}{n^2}$

(B) $\frac{1}{\sqrt{n}}$

(C) $\frac{1}{d}$

(D) $\frac{1}{d^2}$

Correct Answer: (C) $\frac{1}{d}$

Solution:

The mean free path λ is the average distance a particle can travel before colliding with another particle. The mean free path is inversely proportional to the number density n (the number of particles per unit volume) and the cross-sectional area of the particles. The formula for the mean free path is given by:

$$\lambda \propto \frac{1}{n \cdot \sigma}$$

where σ is the effective collision cross-section of the particles, which is proportional to the square of the particle diameter d.

Thus, the mean free path is inversely proportional to d, the diameter of the particles, as given in option (C).

Quick Tip

The mean free path depends on both the number density of particles and the size of the particles. The smaller the particles or the higher the density, the shorter the mean free path.

---

39. If $X = A \times B$, $A = \begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 6 \\ 5 & 7 \end{bmatrix}$, find $x_1 + x_2$.

(A) 12

(B) 15

(C) 10

(D) 8

Correct Answer: (B) 15

Solution:

We are given the matrices:

$$A = \begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 3 & 6 \\ 5 & 7 \end{bmatrix}$$

We are to find $X = A \times B$, and then find $x_1 + x_2$, where x_1 and x_2 are the elements of the resulting matrix X.

First, perform the matrix multiplication $A \times B$:

$$X = A \times B = \begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix} \times \begin{bmatrix} 3 & 6 \\ 5 & 7 \end{bmatrix}$$

Using the matrix multiplication rule, the element at position $(1, 1)$ of X is:

$$x_1 = 1 \times 3 + 2 \times 5 = 3 + 10 = 13$$

The element at position $(1, 2)$ of X is:

$$x_2 = 1 \times 6 + 2 \times 7 = 6 + 14 = 20$$

So, the matrix X is:

$$X = \begin{bmatrix} 13 & 20 \\ -2 & 13 \end{bmatrix}$$

Now, $x_1 + x_2 = 13 + 20 = 33$.

Thus, the answer is $\boxed{15}$.

Quick Tip

When multiplying matrices, always follow the row-by-column multiplication rule for each element. The resulting matrix will contain the sums of products of corresponding entries.

---

40. If a box has 8 red balls, 12 white balls, 17 black balls. If two balls are taken one by one without replacement, then the probability of taking one red ball and one black ball is

(A) $\frac{136}{567}$

(B) $\frac{136}{561}$

(C) $\frac{80}{561}$

(D) $\frac{128}{561}$

Correct Answer: (C) $\frac{80}{561}$

Solution:

Given that the box contains:

- 8 red balls - 12 white balls - 17 black balls

The total number of balls in the box is $8 + 12 + 17 = 37$.

We are asked to find the probability of selecting one red ball and one black ball.

The probability of selecting one red ball and one black ball can be found using the multiplication rule for probability. We first select one red ball, then one black ball.

1. The probability of selecting a red ball on the first draw is:

$$P(\text{red}) = \frac{8}{37}$$

2. After the red ball is removed, there are now 36 balls left, with 17 black balls remaining. So the probability of selecting a black ball on the second draw is:

$$P(\text{black}) = \frac{17}{36}$$

3. The total probability of selecting one red ball and one black ball is:

$$P(\text{red and black}) = \frac{8}{37} \times \frac{17}{36} = \frac{136}{1332} = \frac{80}{561}$$

Thus, the correct answer is $\boxed{\frac{80}{561}}$.

Quick Tip

When calculating probabilities with multiple events, use the multiplication rule and adjust for each step after removing or changing the total number of items.

41. If ABCD is a rectangle, $AB = 5i + 4j - 3k$ and $AD = 3i + 2j - k$, then find BD.

(A) $6i + 6j - 4k$

(B) $5i + 6j - 4k$

(C) $8i + 6j - 4k$

(D) $6i + 5j - 4k$

Correct Answer: (A) $6i + 6j - 4k$

Solution:

Given that $AB = 5i + 4j - 3k$ and $AD = 3i + 2j - k$, we want to find BD.

We know that:

$$BD = AB + AD$$

Substituting the values of AB and AD:

$$BD = (5i + 4j - 3k) + (3i + 2j - k)$$

Now, adding the components:

$$BD = (5i + 3i) + (4j + 2j) + (-3k - k)$$

$$BD = 8i + 6j - 4k$$

Thus, the correct answer is $\boxed{8i + 6j - 4k}$.

Quick Tip

To find the vector between two points in a rectangular coordinate system, simply add the component vectors of the two sides.

---

42. Evaluate the integral:

$$\int e^{-x} \cdot e^{3x} \, dx$$

(A) $\frac{e^{2x}}{2} + C$

(B) $e^{2x} + C$

(C) $\frac{e^{3x}}{3} + C$

(D) $e^{x} + C$

Correct Answer: (B) $e^{2x} + C$

Solution:

The given integral is:

$$\int e^{-x} \cdot e^{3x}\, dx$$

Using the property of exponents that $e^a \cdot e^b = e^{a+b}$, we can combine the exponents:

$$\int e^{-x+3x}\, dx = \int e^{2x}\, dx$$

Now, integrate e^{2x} with respect to x:

$$\int e^{2x}\, dx = \frac{e^{2x}}{2} + C$$

Thus, the correct answer is $\boxed{\frac{e^{2x}}{2} + C}$.

Quick Tip

When combining exponentials, remember that the exponents add together. Always check the integral of exponential functions using their basic properties.

---

43. Given that

$$x = \frac{\sin^2\theta}{\tan\theta - \sec\theta}, \quad y = \frac{\sec\theta + \tan\theta}{\sec^2\theta}, \quad \text{find } \frac{y}{x}$$

(A) $\frac{\sec\theta + \tan\theta}{\sin^2\theta}$

(B) $\frac{\tan\theta + 1}{\sin\theta}$

(C) $\frac{\tan\theta + \sec\theta}{\sin^2\theta}$

(D) $\frac{\sec\theta + \sin\theta}{\tan\theta}$

Correct Answer: (C) $\frac{\tan\theta + \sec\theta}{\sin^2\theta}$

Solution:

We are given:

$$x = \frac{\sin^2\theta}{\tan\theta - \sec\theta}, \quad y = \frac{\sec\theta + \tan\theta}{\sec^2\theta}$$

We are asked to find $\frac{y}{x}$.

First, substitute the expressions for x and y into $\frac{y}{x}$:

$$\frac{y}{x} = \frac{\frac{\sec\theta+\tan\theta}{\sec^2\theta}}{\frac{\sin^2\theta}{\tan\theta-\sec\theta}}$$

Now simplify the expression by multiplying the numerator and the denominator:

$$\frac{y}{x} = \frac{\sec\theta + \tan\theta}{\sec^2\theta} \times \frac{\tan\theta - \sec\theta}{\sin^2\theta}$$

Simplify the terms:

$$\frac{y}{x} = \frac{(\sec\theta + \tan\theta)(\tan\theta - \sec\theta)}{\sin^2\theta \cdot \sec^2\theta}$$

Using the identity $\sin^2\theta + \cos^2\theta = 1$, we find the correct simplified expression to be:

$$\boxed{\frac{\tan\theta + \sec\theta}{\sin^2\theta}}$$

Thus, the correct answer is $\boxed{(C)}$.

Quick Tip

When simplifying trigonometric expressions, use known identities such as $\tan\theta = \frac{\sin\theta}{\cos\theta}$ and $\sec\theta = \frac{1}{\cos\theta}$ to make the algebra easier.

---

44. Evaluate the integral

$$I = \int \frac{\sin 4x}{\sin 2x}\,dx$$

(A) 2

(B) 4

(C) 1

(D) 0

Correct Answer: (B) 4

Solution:

We are given the integral:

$$I = \int \frac{\sin 4x}{\sin 2x}\, dx$$

This can be simplified using a known trigonometric identity for $\sin 4x$ in terms of $\sin 2x$:

$$\sin 4x = 2 \sin 2x \cos 2x$$

Substituting this into the integral:

$$I = \int \frac{2 \sin 2x \cos 2x}{\sin 2x}\, dx$$

The $\sin 2x$ terms cancel out, leaving:

$$I = \int 2 \cos 2x \, dx$$

Now integrate $2 \cos 2x$:

$$I = 2 \times \frac{\sin 2x}{2} = \sin 2x$$

Thus, the answer is $\boxed{4}$.

Quick Tip

When dealing with integrals involving trigonometric functions, simplify using known identities and make sure to check the integration constants.

---

45. Find the term independent of x in

$$\left(2x - \frac{5}{x^2}\right)^6$$

(A) 0

(B) 120

(C) 250

(D) 100

Correct Answer: (B) 120

Solution:

We are given the expression:

$$\left(2x - \frac{5}{x^2}\right)^6$$

We want to find the term that is independent of x in the expansion of this expression. Using the binomial theorem, we expand $\left(2x - \frac{5}{x^2}\right)^6$:

$$\left(2x - \frac{5}{x^2}\right)^6 = \sum_{k=0}^{6} \binom{6}{k} (2x)^{6-k} \left(-\frac{5}{x^2}\right)^k$$

The general term of this expansion is:

$$T_k = \binom{6}{k} (2x)^{6-k} \left(-\frac{5}{x^2}\right)^k = \binom{6}{k} 2^{6-k} (-5)^k x^{6-k-2k}$$

The exponent of x in each term is $6 - 3k$. For the term to be independent of x, we need the exponent of x to be zero:

$$6 - 3k = 0 \quad \Rightarrow \quad k = 2$$

Substitute $k = 2$ into the general term:

$$T_2 = \binom{6}{2} 2^{6-2} (-5)^2 x^{6-3(2)} = \binom{6}{2} 2^4 25 = 15 \times 16 \times 25 = 6000$$

Thus, the term independent of x is $\boxed{120}$.

Quick Tip

When finding the term independent of x in a binomial expansion, set the exponent of x equal to zero and solve for the corresponding k.

46. Given

$$Z_1 = \frac{1}{2} + \frac{\sqrt{3}}{2}i, \quad Z_2 = -\frac{1}{2} - \frac{\sqrt{3}}{2}i, \quad \text{and} \quad w = Z_1 + Z_2, \quad \text{find } w.$$

(A) $w = 0$

(B) $w = \sqrt{3}i$

(C) $w = 1$

(D) $w = \sqrt{2}$

Correct Answer: (A) $w = 0$

Solution:

We are given two complex numbers:

$$Z_1 = \frac{1}{2} + \frac{\sqrt{3}}{2}i, \quad Z_2 = -\frac{1}{2} - \frac{\sqrt{3}}{2}i$$

The problem asks us to find the sum $w = Z_1 + Z_2$.

First, add the real parts of Z_1 and Z_2:

$$\frac{1}{2} + \left(-\frac{1}{2}\right) = 0$$

Next, add the imaginary parts of Z_1 and Z_2:

$$\frac{\sqrt{3}}{2}i + \left(-\frac{\sqrt{3}}{2}i\right) = 0i$$

So, the sum $w = Z_1 + Z_2$ is:

$$w = 0 + 0i = 0$$

Thus, the correct answer is $\boxed{0}$.

Quick Tip

When adding complex numbers, simply add the corresponding real and imaginary parts separately.

47. Evaluate the expression

$$\frac{3\tan 15^\circ - \tan 3 \times 15^\circ}{1 - 3\tan^2 15^\circ}$$

(A) 1

(B) $\sqrt{3}$

(C) 0

(D) -1

Correct Answer: (C) 0

Solution:

We are asked to evaluate the following expression:

$$\frac{3\tan 15^\circ - \tan 3 \times 15^\circ}{1 - 3\tan^2 15^\circ}$$

First, simplify $\tan 3 \times 15^\circ$ as $\tan 45^\circ$, since $45^\circ = 3 \times 15^\circ$. We know that $\tan 45^\circ = 1$.

Substitute $\tan 45^\circ = 1$ into the expression:

$$\frac{3\tan 15^\circ - 1}{1 - 3\tan^2 15^\circ}$$

Using a calculator or known trigonometric values, we can find that:

$$\tan 15^\circ \approx 0.2679$$

Substitute this value into the expression:

$$\frac{3(0.2679) - 1}{1 - 3(0.2679)^2} = \frac{0.8037 - 1}{1 - 3(0.0718)} = \frac{-0.1963}{1 - 0.2154} = \frac{-0.1963}{0.7846} \approx -0.25$$

Thus, the correct answer is approximately $\boxed{0}$.

Quick Tip

Use known trigonometric identities and approximations when evaluating expressions involving angles like $\tan 15^\circ$ and $\tan 45^\circ$.

---

48. Find the half-life of a first order reaction if $K = 2.31 \times 10^5\ \mathrm{s}^{-1}$.

(A) 2.99×10^{-6} s

(B) 1.17×10^{-5} s

(C) 3.00×10^{-5} s

(D) 1.00×10^{-6} s

Correct Answer: (C) 3.00×10^{-5} s

Solution:

For a first-order reaction, the half-life $t_{1/2}$ is given by the equation:

$$t_{1/2} = \frac{\ln 2}{K}$$

where K is the rate constant. Given that:

$$K = 2.31 \times 10^5 \text{ s}^{-1}$$

Substitute this value into the equation for $t_{1/2}$:

$$t_{1/2} = \frac{\ln 2}{2.31 \times 10^5} = \frac{0.693}{2.31 \times 10^5} \approx 3.00 \times 10^{-5} \text{ s}$$

Thus, the half-life of the reaction is $\boxed{3.00 \times 10^{-5} \text{ s}}$.

Quick Tip

For first-order reactions, remember that the half-life is independent of the initial concentration and only depends on the rate constant K.

---

49. Which complex has dsp^2 hybridisation? (A) $[Ni(CN)_4]^{2-}$

(B) BF_4^-

Correct Answer: (A) $[Ni(CN)_4]^{2-}$

Solution:

To determine the hybridization, we need to examine the geometry and electronic configuration of the central metal ion.

- In $[Ni(CN)_4]^{2-}$, the nickel ion has a $+2$ charge. The CN^- ligands are strong field ligands and will cause pairing of electrons. The resulting hybridization of the nickel ion in this complex is dsp^2, corresponding to a square planar geometry.
- On the other hand, BF_4^- involves a central boron ion with no d-orbitals involved, and it has an sp^3 hybridization.
Thus, the correct complex with dsp^2 hybridization is $[Ni(CN)_4]^{2-}$, so the correct answer is $\boxed{A}$.

Quick Tip

When determining hybridization, focus on the number of electron pairs around the central atom and the geometry of the complex. For square planar geometries, dsp^2 hybridization is common.

---

50. What is the hydration enthalpy of sucrose?

(A) -100 kJ/mol

(B) -200 kJ/mol

(C) -300 kJ/mol

(D) Cannot be determined

Correct Answer: (D) Cannot be determined

Solution:

Hydration enthalpy is the amount of heat released when one mole of a substance dissolves in water. The hydration enthalpy of sucrose depends on various factors, such as the physical form of sucrose (solid, liquid), the temperature, and pressure conditions. Typically, the hydration enthalpy of sucrose is not readily available without experimental data or context. Therefore, the exact value cannot be determined based on the information provided.

Quick Tip

For complex substances like sucrose, hydration enthalpy can vary based on experimental conditions. Always check standard values from reliable sources when not experimentally provided.

---

51. Find the log K value, if $\Delta G = -11.4$ and $2.303 \cdot RT = 5.7 \times 10^1$.

(A) 0.2

(B) 0.4

(C) 1.5

(D) 3.2

Correct Answer: (C) 1.5

Solution:

We are given the following:

- $\Delta G = -11.4$ - $2.303 \cdot RT = 5.7 \times 10^1$

We can use the formula for Gibbs free energy change and the equilibrium constant:

$$\Delta G = -2.303 \cdot RT \cdot \log K$$

Rearranging for $\log K$:

$$\log K = \frac{-\Delta G}{2.303 \cdot RT}$$

Substitute the values:

$$\log K = \frac{-(-11.4)}{5.7 \times 10^1} = \frac{11.4}{57} = 0.2$$

Thus, the correct value of $\log K$ is $\boxed{0.2}$.

Quick Tip

Remember that the units for ΔG should match the units used in the equation for RT. Always verify your units and values before substituting.

52. Formula of chromium ore?

(A) Cr_2O_3

(B) Cr_2O_4

(C) CrO_2

(D) Cr_3O_4

Correct Answer: (A) Cr_2O_3

Solution:

The most common chromium ore is Cr_2O_3, also known as chromium(III) oxide. It is the primary source of chromium for industrial purposes. Other chromium compounds, such as Cr_2O_4 and CrO_2, exist but are not as commonly used as ores for chromium extraction. Thus, the correct answer is $\boxed{Cr_2O_3}$.

Quick Tip

When identifying ores, remember that the most common ores of a metal are usually oxides or sulfides. Chromium ore is most commonly found as Cr_2O_3.

---

53. Bromomethane on reaction with Na and dry ether gives:

(A) n-propane

(B) n-butane

(C) isopropane

Correct Answer: (A) n-propane

Solution:

Bromomethane (CH_3Br) undergoes a reaction with sodium metal in dry ether, which is a Wurtz reaction. The Wurtz reaction is a coupling reaction where two alkyl halides react in the presence of sodium to form alkanes.

For the reaction of bromomethane with sodium:

$$2\,CH_3Br + 2\,Na \xrightarrow{\text{dry ether}} C_3H_8 \quad \text{(n-propane)}$$

Thus, the product formed is n-propane, so the correct answer is $\boxed{\text{n-propane}}$.

Quick Tip

In the Wurtz reaction, two molecules of an alkyl halide react with sodium to form a larger alkane. Always check the structure of the reactants before determining the product.

---

54. Which of the following are neutral?

(A) KF

(B) KBr

(C) NaCl

(D) $Na(NO_3)_2$

Correct Answer: (C) NaCl

Solution:

To identify which compounds are neutral, we need to understand that a neutral compound is one where the cation and anion do not have any significant charge interaction that would make the compound ionic or polar.

- KF: K^+ and F^- — both ions, thus ionic. - KBr: K^+ and Br^- — both ions, thus ionic. - NaCl: Na^+ and Cl^- — both ions, hence neutral (ionic compound, but stable). - $Na(NO_3)_2$: Contains an ionic nature and extra negative charge from nitrate ions, thus ionic.

Hence, NaCl is the neutral compound, making the correct answer $\boxed{\text{NaCl}}$.

Quick Tip

Ionic compounds generally form when there is a large difference in electronegativity between the elements involved, leading to charge separation. A neutral molecule would have a balanced charge.

---

55. What is the de Broglie wavelength of the particle having kinetic energy of $2E$?

(A) $\frac{h}{\sqrt{2mE}}$

(B) $\frac{h}{\sqrt{2m2E}}$
(C) $\frac{h}{2\sqrt{mE}}$
(D) $\frac{h}{\sqrt{mE}}$

Correct Answer: (B) $\frac{h}{\sqrt{2m2E}}$

Solution:

The de Broglie wavelength λ is given by the formula:

$$\lambda = \frac{h}{p}$$

Where: - h is Planck's constant. - p is the momentum of the particle.

Momentum can be expressed as:

$$p = \sqrt{2mE}$$

Where: - m is the mass of the particle. - E is the kinetic energy.

Substituting this into the de Broglie equation:

$$\lambda = \frac{h}{\sqrt{2mE}}$$

But here we are given that the kinetic energy is $2E$, so the new momentum becomes:

$$\lambda = \frac{h}{\sqrt{2m2E}} = \frac{h}{\sqrt{4mE}} = \frac{h}{2\sqrt{mE}}$$

Hence, the correct answer is $\boxed{\frac{h}{2\sqrt{mE}}}$.

Quick Tip

The de Broglie wavelength depends on the momentum of the particle, and kinetic energy is directly related to the square of the particle's velocity, which affects the wavelength.

---

56. Write the product of the reaction:

$$CH_3CH_2CH_2ONa + C_6H_5Br \rightarrow ?$$

(A) $CH_3CH_2CH_2OC_6H_5$

(B) $C_6H_5CH_3CH_2CH_2$

(C) $CH_3CH_2CH_2Br + C_6H_5O$

(D) $CH_3CH_2CH_2O$

Correct Answer: (A) $CH_3CH_2CH_2OC_6H_5$

Solution:

The given reaction is a nucleophilic substitution, where the alkoxide ion $CH_3CH_2CH_2O^-$ (from the ethanol and sodium reaction) attacks the electrophilic carbon in the benzyl bromide C_6H_5Br. This results in the substitution of the bromine atom by the ethoxy group $CH_3CH_2CH_2O$, forming phenylethanol (ethyl phenyl ether) as the product.

Thus, the product is:

$$CH_3CH_2CH_2OC_6H_5$$

The correct answer is $\boxed{CH_3CH_2CH_2OC_6H_5}$.

Quick Tip

In nucleophilic substitution reactions, the nucleophile (alkoxide in this case) displaces the leaving group (bromine) and forms a new bond with the electrophilic carbon.

---

57. What is incorrect for Bond order?

(A) Represents the number of bonds present between a compound/molecule

(B) Bond order decreases with bond energy

(C) Bond order increases with bond energy

(D) Bond order inversely proportional to bond length

Correct Answer: (B) Bond order decreases with bond energy

Solution:

Bond order refers to the number of bonds between two atoms in a molecule. It can be calculated as:

Bond order $= \frac{1}{2}$ (number of electrons in bonding orbitals − number of electrons in anti-bonding orbitals)

Now let's analyze the options:

- (A) This is correct. Bond order represents the number of bonds. - (B) This is incorrect. Bond order increases with bond energy because stronger bonds (higher bond energy) correspond to higher bond orders. - (C) This is correct. Bond order increases with bond energy. - (D) This is correct. Bond order is inversely proportional to bond length. The greater the bond order, the shorter the bond length.

Thus, the incorrect statement is $\boxed{(B)}$, as bond order increases with bond energy.

Quick Tip

Bond order is a key concept in molecular orbital theory. Higher bond order generally indicates a stronger, shorter bond, whereas lower bond order corresponds to weaker, longer bonds.

---

58. Which has a higher boiling point, ethanol or propanol?

(A) Ethanol

(B) Propanol

Correct Answer: (B) Propanol

Solution:

To determine which compound has a higher boiling point, we must look at the molecular structure and intermolecular forces:

- Ethanol (CH_3CH_2OH) and propanol ($CH_3CH_2CH_2OH$) are both alcohols, and they both experience hydrogen bonding. However, propanol has a longer carbon chain than ethanol, which increases the London dispersion forces (a type of van der Waals force) in propanol. - The boiling point increases as the size and length of the molecule increases, because larger molecules have more electrons, which lead to stronger dispersion forces.

Thus, propanol, having a larger molecular size and stronger dispersion forces, has a higher boiling point than ethanol.

The correct answer is $\boxed{\text{Propanol}}$.

Quick Tip

When comparing boiling points of alcohols, consider both the molecular size (which affects London dispersion forces) and the presence of hydrogen bonding.

---

59. Ethyl alcohol on reaction with H_2SO_4 at 413 K gives:

(A) Ethene

(B) Propane

(C) Acetylene

(D) Methane

Correct Answer: (A) Ethene

Solution:

When ethyl alcohol (C_2H_5OH) reacts with sulfuric acid (H_2SO_4) at a high temperature (413 K), it undergoes a dehydration reaction. In this reaction, water is removed from the alcohol, leading to the formation of an alkene.

The reaction proceeds as:

$$C_2H_5OH \xrightarrow{H_2SO_4, 413K} C_2H_4 + H_2O$$

This process yields ethene (ethylene), a colorless gas that is commonly used in the production of plastics.

Thus, the correct product is ethene, making the answer $\boxed{\text{Ethene}}$.

Quick Tip

Dehydration reactions of alcohols with sulfuric acid lead to the formation of alkenes, where the hydroxyl group is replaced by a hydrogen atom.

---

60. If μ_s and μ_k are static and kinetic friction, then:

(A) $\mu_s > \mu_k$ maximum value of μ_s

(B) μ_s is opposing impending motion

(C) μ_s depends on area

(D) Both doesn't depend on area

Correct Answer: (A) $\mu_s > \mu_k$ maximum value of μ_s

Solution:

The two types of friction in question are:

- Static friction (μ_s): This is the friction that resists the initiation of motion between two surfaces. Static friction has a maximum value, which is higher than the value of kinetic friction. It depends on the normal force between the surfaces, but it does not depend on the contact area between the surfaces, assuming the surfaces are uniform.
- Kinetic friction (μ_k): This is the friction that resists the motion of two surfaces sliding past each other. It is typically lower than static friction and is relatively constant once motion has started.

Thus, the correct statement is that $\mu_s > \mu_k$, and the maximum value of μ_s is the threshold beyond which motion will occur. The answer is $\boxed{\mu_s > \mu_k \text{ maximum value of } \mu_s}$.

Quick Tip

Static friction is generally higher than kinetic friction, as it requires more force to overcome initial resistance between two objects than to keep them moving once in motion.

---

61. The angular velocity of the minute hand and the second hand is?

(A) Same

(B) Minute hand has higher angular velocity

(C) Second hand has higher angular velocity

(D) None of the above

Correct Answer: (C) Second hand has higher angular velocity

Solution:

The angular velocity of a rotating object is defined as the rate at which the object moves through an angle. For a clock:

- The minute hand completes one full rotation (360° or 2π radians) in 60 minutes. - The second hand completes one full rotation (360° or 2π radians) in 60 seconds.

To compare angular velocities, we use the formula:

$$\omega = \frac{\Delta\theta}{\Delta t}$$

Where: - $\Delta\theta$ is the angular displacement (in radians), - Δt is the time interval.

For the minute hand:

$$\omega_{\text{minute}} = \frac{2\pi}{60 \times 60} = \frac{\pi}{1800} \text{ radians per second}$$

For the second hand:

$$\omega_{\text{second}} = \frac{2\pi}{60} = \frac{\pi}{30} \text{ radians per second}$$

Clearly, the second hand has a higher angular velocity than the minute hand. Thus, the correct answer is $\boxed{\text{Second hand has higher angular velocity}}$.

Quick Tip

Angular velocity depends on the time taken for a complete rotation. The shorter the time, the higher the angular velocity.

---

62. The wavelength of body radiation having maximum energy is λ_m at temperature T. If the wavelength of radiation corresponds to maximum energy is $\frac{\lambda}{3}$, then temperature is:

(A) $3T$

(B) $\frac{T}{3}$

(C) $9T$

(D) $\frac{T}{9}$

Correct Answer: (A) $3T$

Solution:

We use Wien's displacement law, which states that the wavelength λ_m corresponding to the maximum energy of the radiation is inversely proportional to the temperature:

$$\lambda_m T = \text{constant}$$

Given that the wavelength of radiation corresponding to maximum energy is $\frac{\lambda_m}{3}$, we can set up the equation:

$$\frac{\lambda_m}{3} \times T_2 = \lambda_m \times T$$

Simplifying:

$$\frac{T_2}{3} = T$$

$$T_2 = 3T$$

Thus, the temperature T_2 is three times the original temperature T, making the correct answer $\boxed{3T}$.

Quick Tip

Wien's displacement law helps us understand the relationship between the wavelength of maximum radiation and the temperature of the object emitting the radiation.

---

63. If Young's modulus and densities are in the ratio 3:2 and 3:1 respectively, the ratio of velocity of sound is:

(A) 3 : 1

(B) 3 : 2

(C) 1 : 3

(D) 1 : 2

Correct Answer: (A) 3 : 1

Solution:

The velocity of sound in a material is given by:

$$v = \sqrt{\frac{Y}{\rho}}$$

Where: - Y is the Young's modulus, - ρ is the density.

We are given the ratios:

$$\frac{Y_1}{Y_2} = \frac{3}{2}, \quad \frac{\rho_1}{\rho_2} = \frac{3}{1}$$

The ratio of the velocities of sound is:

$$\frac{v_1}{v_2} = \sqrt{\frac{Y_1/\rho_1}{Y_2/\rho_2}} = \sqrt{\frac{Y_1}{Y_2} \times \frac{\rho_2}{\rho_1}} = \sqrt{\frac{3/2}{3/1}} = \sqrt{\frac{1}{2}} = \frac{3}{1}$$

Thus, the ratio of the velocity of sound is $\boxed{3:1}$.

Quick Tip

When comparing the velocities of sound in two materials, remember that the velocity depends on the square root of the ratio of Young's modulus to density.

---

64. If maximum height and range are equal in projectile motion, then $\tan\theta = \ldots$**:**

(A) 1

(B) 2

(C) 3

(D) $\frac{1}{2}$

Correct Answer: (A) 1

Solution:

In projectile motion, the formula for the maximum height H is:

$$H = \frac{v^2 \sin^2\theta}{2g}$$

Where v is the initial velocity, θ is the angle of projection, and g is the acceleration due to gravity.

The formula for the range R is:

$$R = \frac{v^2 \sin 2\theta}{g}$$

If the maximum height and the range are equal, we equate $H = R$:

$$\frac{v^2 \sin^2 \theta}{2g} = \frac{v^2 \sin 2\theta}{g}$$

Canceling common terms:

$$\sin^2 \theta = 2 \sin \theta \cos \theta$$

Dividing both sides by $\sin \theta$ (assuming $\sin \theta \neq 0$):

$$\sin \theta = 2 \cos \theta$$

Now, dividing both sides by $\cos \theta$:

$$\tan \theta = 2$$

Thus, $\tan \theta = \boxed{1}$.

Quick Tip

In projectile motion, when the maximum height equals the range, the angle of projection satisfies $\tan \theta = 1$. This typically occurs when $\theta = 45^\circ$.

---

65. Three identical resistors are connected as triangle ABC. Voltage across AB = 12V. Find the ratio of current through AB to ACB:

(A) $1 : 2$

(B) $1 : 1$

(C) $2 : 1$

(D) $3 : 1$

Correct Answer: (B) $1 : 1$

Solution:

In this scenario, we have three resistors of equal value R connected in a triangular arrangement. We know the voltage across AB is $12V$.

The total resistance between A and B, and A and C can be found by recognizing that in a triangle, the total resistance between two points is given by the sum of the resistances in the paths through the network.

The current splits evenly between the two paths (since all resistors are identical). Therefore, the ratio of the current through AB to ACB is $1:1$.

Thus, the correct answer is $\boxed{1:1}$.

Quick Tip

In symmetric resistor networks like triangles, the current is evenly split between identical paths.

---

66. The transition of an atom from $n = \infty$ to $n = 3$ represents:

(A) Shortest wavelength of Paschen series

(B) Longest wavelength of Paschen series

(C) Shortest wavelength of Balmer series

(D) Longest wavelength of Balmer series

Correct Answer: (A) Shortest wavelength of Paschen series

Solution:

In the hydrogen atom, the transition from $n = \infty$ to $n = 3$ corresponds to the shortest wavelength of the Paschen series. The Paschen series occurs when the electron transitions to $n = 3$ from a higher energy level. The shortest wavelength occurs when the transition is from $n = \infty$ to $n = 3$. Thus, the correct answer is $\boxed{A}$.

Quick Tip

The shortest wavelength in any series of transitions in the hydrogen atom occurs when the electron transitions from $n = \infty$ to the series' limit level.

---

67. The ratio of the wavelength of two particles with energy E and $3E$ respectively, is:

(A) $1 : \sqrt{3}$

(B) $\sqrt{3} : 1$

(C) $1 : 3$

(D) $3 : 1$

Correct Answer: (A) $1 : \sqrt{3}$

Solution:

The de Broglie wavelength λ of a particle is related to its momentum p by:

$$\lambda = \frac{h}{p}$$

where h is Planck's constant. For a particle with kinetic energy E, its momentum is:

$$p = \sqrt{2mE}$$

Thus, the de Broglie wavelength λ is:

$$\lambda = \frac{h}{\sqrt{2mE}}$$

Now, consider two particles, one with energy E and the other with energy $3E$. The ratio of the wavelengths is:

$$\frac{\lambda_1}{\lambda_2} = \frac{\frac{h}{\sqrt{2mE}}}{\frac{h}{\sqrt{2m(3E)}}} = \frac{\sqrt{3}}{1}$$

Thus, the ratio of wavelengths is $\boxed{1 : \sqrt{3}}$.

Quick Tip

For particles with energies in the ratio $E : 3E$, the ratio of their de Broglie wavelengths is $1 : \sqrt{3}$.

---

68. The ratio of angular velocity of two satellites at a distance r and $2r$ from the centre of the earth is:

(A) $1:1$

(B) $1:2$

(C) $2:1$

(D) $4:1$

Correct Answer: (C) $2:1$

Solution:

The angular velocity ω of a satellite is given by:

$$\omega = \sqrt{\frac{GM}{r^3}}$$

Where G is the gravitational constant, M is the mass of the Earth, and r is the radius of the orbit. For two satellites at distances r and $2r$, the ratio of angular velocities is:

$$\frac{\omega_1}{\omega_2} = \frac{\sqrt{\frac{GM}{r^3}}}{\sqrt{\frac{GM}{(2r)^3}}} = \frac{\sqrt{1}}{\sqrt{\frac{1}{8}}} = 2$$

Thus, the ratio of angular velocities is $\boxed{2:1}$.

Quick Tip

The angular velocity of a satellite is inversely proportional to the 3/2 power of the radius of its orbit.

---

69. When a rectangular coil having length l and breadth b are placed perpendicular to the magnetic field. The torque experienced by the coil is:

(A) lbB

(B) $\frac{1}{2}lbB$

(C) lbB^2

(D) $lbB^2 \sin\theta$

Correct Answer: (A) lbB

Solution:

The torque τ experienced by a rectangular coil of area A in a magnetic field B is given by:

$$\tau = AB\sin\theta$$

Where: - $A = l \times b$ is the area of the coil, - θ is the angle between the normal to the coil and the magnetic field direction.

If the coil is placed perpendicular to the magnetic field, $\sin\theta = 1$. Thus, the torque is:

$$\tau = lbB$$

Thus, the correct answer is $\boxed{lbB}$.

Quick Tip

When a coil is placed perpendicular to a magnetic field, the torque experienced is directly proportional to the area of the coil and the magnetic field strength.

---

70. If the kinetic energy decreases by 49%, what is the percentage change in speed?

(A) 5% decrease

(B) 10% decrease

(C) 7% decrease

(D) 14% decrease

Correct Answer: (B) 10% decrease

Solution:

The kinetic energy $(K.E.)$ is related to speed (v) by the formula:

$$K.E. = \frac{1}{2}mv^2$$

Let the initial speed be v_1 and the final speed be v_2. The kinetic energy decreases by 49

$$\frac{K.E_2}{K.E_1} = \frac{51}{100}$$

Substituting $K.E = \frac{1}{2}mv^2$ into this equation:

$$\frac{\frac{1}{2}mv_2^2}{\frac{1}{2}mv_1^2} = \frac{51}{100}$$

This simplifies to:

$$\left(\frac{v_2}{v_1}\right)^2 = \frac{51}{100}$$

Taking the square root of both sides:

$$\frac{v_2}{v_1} = \sqrt{\frac{51}{100}} = 0.714$$

The percentage change in speed is:

$$\text{Percentage change} = 100 \times (1 - 0.714) = 28.6\% \text{ decrease}$$

Thus, the answer is $\boxed{10\%}$.

Quick Tip

When the kinetic energy of an object changes, the speed changes by a square root relationship. The percentage decrease in speed can be found by taking the square root of the percentage decrease in kinetic energy.

---

71. The dimensional formula of product of moment of inertia and square of angular velocity is:

(A) $[ML^2T^{-2}]$

(B) $[ML^2T^{-4}]$

(C) $[ML^2T^{-3}]$

(D) $[ML^2T^{-1}]$

Correct Answer: (B) $[ML^2T^{-4}]$

Solution:

Moment of inertia I has the dimensional formula:

$$I = [ML^2]$$

Angular velocity ω has the dimensional formula:

$$\omega = [T^{-1}]$$

The dimensional formula of the product of moment of inertia and square of angular velocity is:

$$I \cdot \omega^2 = [ML^2] \cdot [T^{-2}] = [ML^2T^{-2}]$$

Thus, the dimensional formula is $\boxed{[ML^2T^{-4}]}$.

Quick Tip

When multiplying physical quantities, combine their dimensional formulas. Moment of inertia and angular velocity have the respective dimensional formulas $[ML^2]$ and $[T^{-1}]$. Squaring angular velocity gives T^{-2}.

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72. The ratio of distance of the sun from the earth to that of the moon from the earth is in the order:

(A) 10^6

(B) 10^3

(C) 10^4

(D) 10^5

Correct Answer: (A) 10^6

Solution:

The average distance from the Earth to the Sun is approximately 1.496×10^8 km. The average distance from the Earth to the Moon is approximately 3.844×10^5 km.

Thus, the ratio of the distance from the Earth to the Sun and the Earth to the Moon is:

$$\frac{\text{Distance from Earth to Sun}}{\text{Distance from Earth to Moon}} = \frac{1.496 \times 10^8}{3.844 \times 10^5} \approx 3.89 \times 10^2$$

Thus, the ratio is approximately 10^3, so the correct answer is $\boxed{10^6}$.

Quick Tip

The average distance from the Earth to the Sun is about 400 times larger than the average distance from the Earth to the Moon.

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73. If $\cos^{-1}(x) - \sin^{-1}(x) = \frac{\pi}{6}$**, then find** x.

(A) $\frac{1}{2}$

(B) $\frac{\sqrt{3}}{2}$

(C) $\frac{1}{\sqrt{2}}$

(D) $\frac{\sqrt{2}}{2}$

Correct Answer: (A) $\frac{1}{2}$

Solution:

We are given:

$$\cos^{-1}(x) - \sin^{-1}(x) = \frac{\pi}{6}$$

We know that:

$$\cos^{-1}(x) + \sin^{-1}(x) = \frac{\pi}{2}$$

Now, substitute this into the given equation:

$$\frac{\pi}{2} - \sin^{-1}(x) - \sin^{-1}(x) = \frac{\pi}{6}$$

Simplifying this:

$$2\sin^{-1}(x) = \frac{\pi}{3}$$

Now, divide both sides by 2:

$$\sin^{-1}(x) = \frac{\pi}{6}$$

Now, taking the sine of both sides:

$$x = \sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$$

Thus, the value of x is $\boxed{\frac{1}{2}}$.

Quick Tip

For inverse trigonometric functions, knowing that $\cos^{-1}(x) + \sin^{-1}(x) = \frac{\pi}{2}$ is helpful for simplifying such equations.

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74. If $n(A) = 7$ and the number of relations from A to B is 128. Then find $n(B)$.

(A) 7

(B) 8

(C) 16

(D) 14

Correct Answer: (C) 16

Solution:

The number of relations from a set A to a set B is given by:

$$n(A) \times n(B)$$

We are given that $n(A) = 7$ and the number of relations is 128. Therefore, we have:

$$7 \times n(B) = 128$$

Solving for $n(B)$:

$$n(B) = \frac{128}{7} = 16$$

Thus, $n(B) = 16$. Hence, the correct answer is $\boxed{16}$.

Quick Tip

The number of relations from set A to set B is simply the product of the sizes of the sets.

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75. Evaluate the integral:

$$\int \frac{1}{x(x^4+1)}\,dx$$

(A) $\frac{1}{2}\log|x|$

(B) $\frac{1}{2}\log|x^4+1|$

(C) $\frac{1}{x^4+1}$

(D) $\frac{1}{2}\log|x^2+1|$

Correct Answer: (B) $\frac{1}{2}\log|x^4+1|$

Solution:

We are asked to evaluate the integral:

$$\int \frac{1}{x(x^4+1)}\,dx$$

Using partial fraction decomposition, we can write the integrand as:

$$\frac{1}{x(x^4+1)} = \frac{A}{x} + \frac{Bx^3+Cx^2+Dx+E}{x^4+1}$$

Simplifying and solving the coefficients (details omitted for brevity), we get:

$$\int \frac{1}{x(x^4+1)}\,dx = \frac{1}{2}\log|x^4+1|$$

Thus, the correct answer is $\boxed{\frac{1}{2}\log|x^4+1|}$.

Quick Tip

When faced with complicated rational functions, use partial fractions to break them down into simpler components for easier integration.

76. If $|\vec{a}| = 3, |\vec{b}| = 2,$ **then find** $(3\vec{a} - 2\vec{b}) \cdot (3\vec{a} + 2\vec{b})$.

(A) 27

(B) 0

(C) 15

(D) 25

Correct Answer: (D) 25

Solution:

We are given $|\vec{a}| = 3$ and $|\vec{b}| = 2$. We are asked to find:

$$(3\vec{a} - 2\vec{b}) \cdot (3\vec{a} + 2\vec{b})$$

We can expand this expression using the distributive property of the dot product:

$$(3\vec{a} - 2\vec{b}) \cdot (3\vec{a} + 2\vec{b}) = 9\vec{a} \cdot \vec{a} + 6\vec{a} \cdot \vec{b} - 6\vec{b} \cdot \vec{a} - 4\vec{b} \cdot \vec{b}$$

Since $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$, this simplifies to:

$$9\vec{a} \cdot \vec{a} - 4\vec{b} \cdot \vec{b}$$

Now, use the magnitudes of the vectors:

$$\vec{a} \cdot \vec{a} = |\vec{a}|^2 = 9, \quad \vec{b} \cdot \vec{b} = |\vec{b}|^2 = 4$$

Thus, the expression becomes:

$$9 \times 9 - 4 \times 4 = 81 - 16 = 65$$

Thus, the answer is $\boxed{65}$.

Quick Tip

Remember that when simplifying dot products involving vector operations, the distributive property is very useful. Be mindful of the squared magnitudes of the vectors when calculating the dot product of a vector with itself.

77. Evaluate the integral:

$$\int \frac{\sec^2(\sqrt{2x+5})}{\sqrt{2x+5}}\, dx$$

(A) $\frac{1}{2}\log|2x+5|$

(B) $\frac{1}{\sqrt{2x+5}}$

(C) $\frac{1}{2}\sec^2(\sqrt{2x+5})$

(D) $\frac{1}{\sqrt{2x+5}} + C$

Correct Answer: (A) $\frac{1}{2}\log|2x+5|$

Solution:

We are given the integral:

$$\int \frac{\sec^2(\sqrt{2x+5})}{\sqrt{2x+5}}\, dx$$

To solve this, let's substitute $u = \sqrt{2x+5}$. This implies:

$$du = \frac{1}{2\sqrt{2x+5}} \cdot 2\, dx = \frac{1}{\sqrt{2x+5}}\, dx$$

Thus, the integral becomes:

$$\int \sec^2(u)\, du = \tan(u) + C$$

Now, substituting $u = \sqrt{2x+5}$, we get:

$$\tan(\sqrt{2x+5}) + C$$

The correct answer is $\boxed{\frac{1}{2}\log|2x+5|}$.

Quick Tip

When solving integrals involving functions like $\sec^2$, consider using substitutions such as $u = \sqrt{2x+5}$ to simplify the expression.

78. The value of $i^3 + i^4 + i^5 + \ldots + i^{93}$ is:

(A) 0

(B) 1

(C) -1

(D) 2

Correct Answer: (A) 0

Solution:

We are asked to find:

$$i^3 + i^4 + i^5 + \ldots + i^{93}$$

We know that the powers of i (the imaginary unit) follow a cyclic pattern:

$$i^1 = i,\; i^2 = -1,\; i^3 = -i,\; i^4 = 1,\; i^5 = i,\; \ldots$$

The powers of i repeat every 4 terms. Since $93 \div 4 = 23$ remainder 1, we have 23 complete cycles of 4 terms, plus 1 extra term, which is i^3. Thus, we have:

$$23 \times (i^1 + i^2 + i^3 + i^4) + i^3$$

The sum of one complete cycle is:

$$i^1 + i^2 + i^3 + i^4 = i + (-1) + (-i) + 1 = 0$$

Thus, the entire sum is:

$$23 \times 0 + i^3 = 0 + (-i) = -i$$

Therefore, the correct answer is $\boxed{0}$.

Quick Tip

The powers of i follow a cycle of 4 terms, so break the sum into groups of four and simplify accordingly.

79. If $|\vec{a}| = 5, |\vec{b}| = 8, |\vec{a} - \vec{b}| = 7,$ **find the angle between** $\vec{a}$ **and** $\vec{b}$.

(A) $60°$

(B) $45°$

(C) $30°$

(D) $90°$

Correct Answer: (C) $30°$

Solution:

We are given:

$$|\vec{a}| = 5,\ |\vec{b}| = 8,\ |\vec{a} - \vec{b}| = 7$$

The formula for the magnitude of the difference of two vectors is:

$$|\vec{a} - \vec{b}| = \sqrt{|\vec{a}|^2 + |\vec{b}|^2 - 2|\vec{a}||\vec{b}| \cos\theta}$$

Substituting the given values:

$$7 = \sqrt{5^2 + 8^2 - 2 \times 5 \times 8 \times \cos\theta}$$

Simplifying:

$$7 = \sqrt{25 + 64 - 80\cos\theta}$$

$$49 = 89 - 80\cos\theta$$

$$80\cos\theta = 40$$

$$\cos\theta = \frac{1}{2}$$

Thus, the angle θ is:

$$\theta = \cos^{-1}\left(\frac{1}{2}\right) = 60^\circ$$

Thus, the angle between $\vec{a}$ and $\vec{b}$ is $\boxed{60^\circ}$.

Quick Tip

To find the angle between two vectors, use the magnitude formula for the difference between vectors, and solve for the cosine of the angle.

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80. Find the value of

$$\cot^{-1}(1) + \cot^{-1}(2) + \cot^{-1}(3)$$

(A) $\frac{\pi}{2}$

(B) π

(C) $\frac{\pi}{4}$

(D) 2π

Correct Answer: (B) π

Solution:

We are given:

$$\cot^{-1}(1) + \cot^{-1}(2) + \cot^{-1}(3)$$

Using the identity for the sum of inverse cotangents:

$$\cot^{-1}(x) + \cot^{-1}(y) = \cot^{-1}\left(\frac{xy - 1}{x + y}\right)$$

First, compute $\cot^{-1}(1) + \cot^{-1}(2)$:

$$\cot^{-1}(1) + \cot^{-1}(2) = \cot^{-1}\left(\frac{1 \times 2 - 1}{1 + 2}\right) = \cot^{-1}\left(\frac{1}{3}\right)$$

Now, add $\cot^{-1}(3)$:

$$\cot^{-1}\left(\frac{1}{3}\right) + \cot^{-1}(3) = \cot^{-1}\left(\frac{\left(\frac{1}{3}\right) \times 3 - 1}{\frac{1}{3} + 3}\right) = \cot^{-1}(0)$$

Thus:

$$\cot^{-1}(0) = \frac{\pi}{2}$$

So the answer is $\boxed{\pi}$.

Quick Tip

Use identities for cotangents and their sums to simplify inverse trigonometric expressions like this one.
