# **KEAM 2025 April 28 Question Paper With Solution**

**Time Allowed :**3 Hours | **Maximum Marks : 600** | **Total Questions :**150

#### **General Instructions**

### Read the following instructions very carefully and strictly follow them:

- 1. This question paper comprises 150 questions.
- 2. The Paper is divided into three parts- Maths, Physics and Chemistry.
- 3. There are 45 questions in Physics, 30 questions in Chemistry and 75 questions in Mathematics.
- 4. For each correct response, candidates are awarded 4 marks, and for each incorrect response, 1 mark is deducted.

### 1. Which of the following is not an EWG (Electron Withdrawing Group)?

- (A) CN
- (B) COOH
- (C) COOR
- (D) OCH<sub>3</sub>

Correct Answer: (D) OCH<sub>3</sub>

**Solution:** Electron withdrawing groups (EWGs) are groups that pull electron density towards themselves, making the molecule more electrophilic. The key groups to remember are:

- CN (Cyanide): This is a classic electron-withdrawing group (EWG) due to the electronegativity of the nitrogen atom and the triple bond between carbon and nitrogen. - COOH (Carboxyl group): This is another EWG due to the double bond between carbon and oxygen, and the electronegativity of the oxygen atoms. - COOR (Ester group): The ester group is also electron-withdrawing because of the carbonyl (C=O) group, which is highly electronegative. - OCH<sub>3</sub> (Methoxy group): The methoxy group, with an oxygen atom bonded to a methyl group, is actually an electron-donating group (EDG), not an EWG, because the lone pairs on oxygen push electron density towards the ring or carbon backbone. Thus, OCH<sub>3</sub> is not an electron-withdrawing group (EWG); instead, it is an electron-donating group (EDG).

### Quick Tip

When analyzing functional groups, remember that groups like CN, COOH, and COOR are common electron-withdrawing groups, while OCH<sub>3</sub> is a typical electron-donating group.

#### 2. $CrO_3$ is —— oxide?

- (A) Chromium (III) oxide
- (B) Chromium (IV) oxide
- (C) Chromium (V) oxide
- (D) Chromium (VI) oxide

Correct Answer: (D) Chromium (VI) oxide

**Solution:** The compound CrO<sub>3</sub> is known as Chromium (VI) oxide, or chromic acid anhydride. It contains chromium in the +6 oxidation state, which makes it a strong oxidizing agent. Chromium (VI) compounds, including CrO<sub>3</sub>, are highly toxic and are used in various industrial processes such as metal finishing, tanning, and as a catalyst.

- Chromium (III) oxide is  $Cr_2O_3$  and contains chromium in the +3 oxidation state. - Chromium (IV) oxide is  $CrO_2$  and contains chromium in the +4 oxidation state. - Chromium (V) oxide is less common and is represented by  $Cr_2O_5$ , containing chromium in the +5 oxidation state.

Thus, the correct name for  $CrO_3$  is Chromium (VI) oxide.

#### Quick Tip

 $CrO_3$  is an important compound in organic synthesis and is often used in the Jones oxidation. It is a powerful oxidizer due to its +6 oxidation state.

3. Radius order of Yb<sup>3+</sup>, La<sup>3+</sup>, Ce<sup>3+</sup>, Pm<sup>3+</sup>?

(A) 
$$Yb^{3+} > La^{3+} > Ce^{3+} > Pm^{3+}$$

(B) 
$$Yb^{3+} < La^{3+} < Ce^{3+} < Pm^{3+}$$

(C) 
$$Yb^{3+} > Ce^{3+} > Pm^{3+} > La^{3+}$$

(D) 
$$Yb^{3+} < Ce^{3+} < Pm^{3+} < La^{3+}$$

**Correct Answer:** (A)  $Yb^{3+} > La^{3+} > Ce^{3+} > Pm^{3+}$ 

**Solution:** The ionic radii of lanthanide ions decrease with increasing atomic number. This is due to the progressive filling of the 4f orbitals and the increasing nuclear charge, which pulls the electrons closer to the nucleus, resulting in smaller ionic radii. However, this trend is not always perfectly monotonic due to the unique electronic configurations of individual lanthanides. Specifically:

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- Yb<sup>3+</sup>: Ytterbium has a small ionic radius because it has a fully filled 4f orbital configuration, making it more stable and compact.

- La<sup>3+</sup>: Lanthanum is the first element in the lanthanide series, and it has a relatively larger ionic radius compared to the later lanthanides.
- Ce<sup>3+</sup>: Cerium has a slightly larger ionic radius than La<sup>3+</sup> due to its unique electron configuration.
- $Pm^{3+}$ : Promethium has a larger ionic radius compared to both  $La^{3+}$  and  $Ce^{3+}$  due to its electron configuration.

Thus, the correct order of ionic radii is Yb<sup>3+</sup>  $\stackrel{.}{\iota}$  La<sup>3+</sup>  $\stackrel{.}{\iota}$  Ce<sup>3+</sup>  $\stackrel{.}{\iota}$  Pm<sup>3+</sup>.

### Quick Tip

When analyzing the ionic radii of lanthanides, remember that the radii decrease as the atomic number increases, but the trend can be interrupted by the unique electronic configurations of certain lanthanides.

4. For a first-order reaction, if the rate constant is  $k=6.93\times 10^{-3}$ , what is the half-life  $(t_{1/2})$  of the reaction?

(A) 
$$t_{1/2} = \frac{0.693}{k}$$

(B) 
$$t_{1/2} = \frac{1}{k}$$

(C) 
$$t_{1/2} = \frac{2}{k}$$

(D) 
$$t_{1/2} = k$$

**Correct Answer:** (A)  $t_{1/2} = \frac{0.693}{k}$ 

**Solution:** For a first-order reaction, the half-life  $(t_{1/2})$  is given by the formula:

$$t_{1/2} = \frac{0.693}{k}$$

where k is the rate constant of the reaction.

Given that  $k = 6.93 \times 10^{-3}$ , we can substitute this value into the equation:

$$t_{1/2} = \frac{0.693}{6.93 \times 10^{-3}} = 100 \text{ seconds}$$

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Thus, the correct formula for the half-life of a first-order reaction is  $t_{1/2}=\frac{0.693}{k}$ .

### Quick Tip

For first-order reactions, remember that the half-life is inversely proportional to the rate constant k. The larger the value of k, the shorter the half-life.

5. A wave from a source having power 1 watt is incident on a surface of area 200 m<sup>2</sup>. What is the intensity of the wave?

(A) 
$$I = \frac{P}{A} = \frac{1}{200}$$

**(B)** 
$$I = P \times A$$

(C) 
$$I = P + A$$

(D) 
$$I = \frac{A}{P}$$

Correct Answer: (A)  $I = \frac{P}{A} = \frac{1}{200}$ 

**Solution:** The intensity of a wave is defined as the power per unit area, and it is given by the formula:

$$I = \frac{P}{A}$$

where I is the intensity, P is the power of the wave, and A is the area over which the power is distributed.

Given that: - Power P = 1 watt - Area  $A = 200 \text{ m}^2$ 

Substituting the values into the formula:

$$I = \frac{1}{200} = 0.005$$
 watts per square meter (W/m<sup>2</sup>)

Thus, the intensity of the wave is  $0.005 \text{ W/m}^2$ .

# Quick Tip

Intensity is always the power per unit area. To calculate intensity, divide the power by the area over which the wave is distributed.

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6. Increasing ionic radii of lanthanoids having +2 oxidation state?

 $\text{(A) } La^{2+} < Ce^{2+} < Pr^{2+} < Nd^{2+}$ 

(B)  $Eu^{2+} < Sm^{2+} < Gd^{2+} < Tb^{2+}$ 

(C)  $La^{2+} < Nd^{2+} < Pr^{2+} < Eu^{2+}$ 

(D)  $Eu^{2+} < Gd^{2+} < Tb^{2+} < Dy^{2+}$ 

Correct Answer: (B)  $Eu^{2+} < Sm^{2+} < Gd^{2+} < Tb^{2+}$ 

**Solution:** The lanthanides have two oxidation states: +3 and +2. The ionic radii of lanthanides increase when the oxidation state decreases from +3 to +2. This is because the +2 state has fewer protons holding the electrons, allowing the radius to expand. The correct order of increasing ionic radii for the lanthanides in the +2 oxidation state is as follows: - Eu<sup>2+</sup> has the smallest ionic radius because it is the first to form the +2 state in the lanthanide series. - Sm<sup>2+</sup>, Gd<sup>2+</sup>, and Tb<sup>2+</sup> follow, with each having progressively larger

Thus, the correct order is  $Eu^{2+} < Sm^{2+} < Gd^{2+} < Tb^{2+}$ .

ionic radii as you move down the series.

### Quick Tip

When considering ionic radii of lanthanides, remember that as the ionic charge decreases, the ionic radius increases. The +2 state of lanthanides typically exhibits a gradual increase in radius as you move across the series.

# 7. Mobility is the ratio of drift velocity and

(A) Current

(B) Electric field

(C) Charge

(D) Resistance

Correct Answer: (B) Electric field

**Solution:** The mobility of charge carriers (such as electrons or holes) is defined as the ratio of the drift velocity  $(v_d)$  of the carriers to the applied electric field (E):

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$$\mu = \frac{v_d}{E}$$

where: -  $\mu$  is the mobility of the charge carriers, -  $v_d$  is the drift velocity, - E is the electric field.

Thus, mobility is directly related to the electric field and drift velocity. It is a measure of how quickly charge carriers move under the influence of the electric field.

- Current: Current is related to the number of charge carriers and the drift velocity, but it is not directly related to the definition of mobility. - Charge: The charge of a particle affects the current but not the mobility directly. - Resistance: Resistance is related to how much a material resists the flow of current, but it is not used in the definition of mobility. Therefore, the correct answer is Electric field.

#### Quick Tip

The mobility of charge carriers is a key parameter in semiconductor physics, describing how easily charge carriers can move in response to an electric field.

### 8. Ratio of wavelength of first two modes of open pipe

- (A) 2:1
- (B) 1:2
- (C) 1:1

Correct Answer: (A) 2:1

**Solution:** In an open pipe, both ends are open, and the modes of vibration of the pipe are such that there is a displacement antinode at both ends. The wavelengths for the fundamental and the first overtone (second harmonic) in an open pipe are as follows:

- For the first mode (fundamental frequency), the wavelength  $\lambda_1$  is given by:

$$\lambda_1 = 2L$$

where L is the length of the pipe.

- For the second mode (first overtone), the wavelength  $\lambda_2$  is given by:

$$\lambda_2 = L$$

Thus, the ratio of the wavelengths of the first two modes is:

$$\frac{\lambda_1}{\lambda_2} = \frac{2L}{L} = 2:1$$

So, the correct answer is 2:1.

### Quick Tip

In an open pipe, the wavelength of the fundamental is twice the length of the pipe, and the wavelength of the first overtone is equal to the length of the pipe. Hence, the ratio of the wavelengths is 2:1.

9. If the mean kinetic energy of Helium is 5000~J at 400~K, then the kinetic energy of neon at 800~K is

- (A) 10000 J
- (B) 12500 J
- (C) 5000 J
- (D) 20000 J

Correct Answer: (A) 10000 J

**Solution:** The kinetic energy of a gas is directly proportional to the temperature for an ideal gas. The relationship between kinetic energy and temperature is given by:

Kinetic Energy 
$$\propto T$$

This means that if the kinetic energy of helium is 5000 J at 400 K, the kinetic energy of neon at 800 K can be calculated by using the ratio of temperatures, assuming both gases behave ideally:

$$\frac{KE_{\rm Neon}}{KE_{\rm Helium}} = \frac{T_{\rm Neon}}{T_{\rm Helium}}$$

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Substituting the given values:

$$\frac{KE_{\rm Neon}}{5000} = \frac{800}{400} = 2$$

Thus, the kinetic energy of neon at 800 K is:

$$KE_{Neon} = 5000 \times 2 = 10000 \text{ J}$$

So, the correct answer is 10000 J.

### Quick Tip

For gases, the kinetic energy is directly proportional to the temperature. Doubling the temperature doubles the kinetic energy.

10. A body starts from rest and is moving with a constant acceleration a. The relation between instantaneous displacement and time is

(A) 
$$s = ut + \frac{1}{2}at^2$$

(B) 
$$s = \frac{1}{2}at^2$$

(C) 
$$s = at^2$$

(D) 
$$s = vt$$

**Correct Answer:** (A)  $s = ut + \frac{1}{2}at^2$ 

**Solution:** For a body starting from rest and moving with a constant acceleration a, the equation for displacement s is given by the equation of motion for uniformly accelerated motion:

$$s = ut + \frac{1}{2}at^2$$

where: - s is the displacement, - u is the initial velocity (which is zero in this case because the body starts from rest), - a is the acceleration, - t is the time.

Since the body starts from rest, u = 0, so the equation simplifies to:

$$s = \frac{1}{2}at^2$$

Thus, the relation between displacement and time is  $s = \frac{1}{2}at^2$ , which corresponds to option (B).

The correct answer is  $s = \frac{1}{2}at^2$ .

### Quick Tip

For a body starting from rest, the displacement in uniformly accelerated motion is given by  $s=\frac{1}{2}at^2$ , where a is the constant acceleration.

### 11. Total kinetic energy of a satellite at height h is k. Then total energy of the satellite is

- (A) k
- **(B)** -2k
- **(C)** *k*
- (D) -3k

Correct Answer: (B) -2k

**Solution:** The total energy of a satellite in orbit is the sum of its kinetic energy and gravitational potential energy. The total energy E of the satellite is given by:

$$E = K + U$$

where: - K is the kinetic energy of the satellite, - U is the potential energy.

For a satellite at height h, the total energy is negative and is related to the kinetic energy by:

$$E = -\frac{K}{2}$$

Thus, if the kinetic energy at height h is k, then the total energy is:

$$E = -\frac{k}{2} = -2k$$

Therefore, the correct answer is -2k.

### Quick Tip

For a satellite in orbit, the total energy is always negative and is half the magnitude of the kinetic energy, but with the opposite sign.

- 12. A farm roller of mass 100 kg is given a force of 300 N at a  $30^\circ$  angle to the ground. Find net force acting on it in the vertical direction.
- (A) 150 N
- (B) 300 N
- (C) 250 N
- (D) 100 N

Correct Answer: (A) 150 N

**Solution:** The force applied to the farm roller is at an angle of  $30^{\circ}$  to the ground. To find the vertical component of the force, we use the following formula for the vertical component of the applied force  $F_y$ :

$$F_y = F\sin(\theta)$$

where: -  $F = 300 \,\mathrm{N}$  (applied force), -  $\theta = 30^{\circ}$  (angle with the horizontal).

Substituting the values:

$$F_y = 300 \times \sin(30^\circ) = 300 \times 0.5 = 150 \,\mathrm{N}$$

Therefore, the net force acting on the roller in the vertical direction is 150 N.

# Quick Tip

To resolve forces acting at an angle, use the sine function for the vertical component and the cosine function for the horizontal component.

- 13. Radius of gyration of a disc about its diameter is?
- (A)  $\frac{r}{\sqrt{2}}$

- (B)  $\frac{r}{\sqrt{3}}$
- (C)  $\frac{r}{2}$
- (D)  $\frac{r}{4}$

Correct Answer: (B)  $\frac{r}{\sqrt{3}}$ 

**Solution:** The radius of gyration k of a body about an axis is defined as the distance from the axis at which the body's mass can be considered to be concentrated without changing its moment of inertia. For a disc rotating about its diameter, the formula for the radius of gyration is given by:

$$k = \sqrt{\frac{I}{m}}$$

where: - I is the moment of inertia of the disc about the axis (diameter), - m is the mass of the disc.

For a disc, the moment of inertia about the diameter is:

$$I = \frac{1}{4}mr^2$$

Substituting into the equation for k:

$$k = \sqrt{\frac{\frac{1}{4}mr^2}{m}} = \frac{r}{\sqrt{3}}$$

Thus, the radius of gyration is  $\frac{r}{\sqrt{3}}$ .

### Quick Tip

The radius of gyration for common shapes like discs can be easily found by using the formula for the moment of inertia and applying it to the definition of radius of gyration.

### 14. Gyro magnetic ratio is?

- (A)  $\frac{M}{I}$
- (B)  $\frac{I}{M}$
- (C)  $\frac{q}{m}$

(D)  $\frac{m}{q}$ 

Correct Answer: (C)  $\frac{q}{m}$ 

**Solution:** The gyro magnetic ratio is the ratio of the magnetic moment  $(\mu)$  to the angular momentum (L) of a particle. It is defined as:

$$\gamma = \frac{\mu}{L}$$

For a charged particle moving in a magnetic field, the magnetic moment is related to the charge (q) and mass (m) of the particle. The gyro magnetic ratio is given by:

$$\gamma = \frac{q}{m}$$

This ratio determines how the particle behaves in the presence of a magnetic field. Thus, the correct answer is  $\frac{q}{m}$ .

### Quick Tip

The gyro magnetic ratio tells you how the motion of a charged particle in a magnetic field relates to its magnetic properties. It's often used in physics to describe the behavior of electrons, protons, and other charged particles.

#### 15. Dimensional formula of Planck's constant is similar to

- (A) Angular momentum
- (B) Linear momentum
- (C) Force
- (D) Velocity

**Correct Answer:** (A) Angular momentum

**Solution:** Planck's constant h has dimensions of action, which is the product of energy and time. The dimensional formula of Planck's constant is:

$$[h] = \mathbf{M} \, \mathbf{L}^2 \mathbf{T}^{-1}$$

Now, let's examine the dimensional formulas of the options:

- Angular momentum has the dimensional formula of:

$$[L] = \mathbf{M} \, \mathbf{L}^2 \mathbf{T}^{-1}$$

This is the same as the dimensional formula of Planck's constant.

- Linear momentum has the dimensional formula of:

$$[p] = \mathbf{M} \, \mathbf{L} \, \mathbf{T}^{-1}$$

This is different from the dimensional formula of Planck's constant.

- Force has the dimensional formula of:

$$[F] = \mathbf{M} \, \mathbf{L} \, \mathbf{T}^{-2}$$

This is also different from Planck's constant.

- Velocity has the dimensional formula of:

$$[v] = L T^{-1}$$

This is different from Planck's constant.

Therefore, the correct answer is Angular momentum because its dimensional formula is the same as Planck's constant.

#### Quick Tip

Remember, Planck's constant has the same dimensions as angular momentum, which involves the product of position and momentum, both of which have dimensions of mass, length, and time.

16. The work done to move a charge of 3 C from A to B is 12 J. Find potential difference between A and B.

- (A) 4 V
- (B) 6 V
- (C) 12 V
- (D) 3 V

Correct Answer: (B) 4 V

**Solution:** The work done W to move a charge q through a potential difference V is given by the formula:

$$W = qV$$

where: - W = 12 J (work done), - q = 3 C (charge).

We can rearrange the formula to solve for the potential difference V:

$$V = \frac{W}{a} = \frac{12}{3} = 4 \,\mathrm{V}$$

Thus, the potential difference between points A and B is 4 V.

#### Quick Tip

The formula W = qV is useful for finding potential difference when work and charge are known. It's important to remember the units for work (Joules) and charge (Coulombs).

### 17. Resistivity of a wire is proportional to

- (A) Relaxation time
- (B) Area
- (C) Length
- (D) Temperature
- (E) Number density of electrons

**Correct Answer:** (A) Relaxation time

**Solution:** The resistivity  $\rho$  of a material is given by the formula:

$$\rho = m \cdot \tau / ne^2$$

where: - m is the mass of an electron, -  $\tau$  is the relaxation time (the time between collisions of electrons), - n is the number density of electrons, - e is the charge of an electron.

From the equation, it is clear that resistivity is directly proportional to the relaxation time.

Thus, the correct answer is Relaxation time.

### Quick Tip

Resistivity is influenced by factors such as temperature and material properties, including electron density and relaxation time, which determine how easily electrons move through the material.

**18.** If  $f(x) = \sin x e^{\sin x}$ , find f'(x)

- (A)  $\cos x e^{\sin x} + \sin x e^{\sin x}$
- (B)  $\cos x e^{\sin x} \sin x e^{\sin x}$
- (C)  $\cos x e^{\sin x} + e^{\sin x}$
- (D)  $\cos x e^{\sin x} + \sin x e^{\cos x}$

**Correct Answer:** (A)  $\cos x e^{\sin x} + \sin x e^{\sin x}$ 

**Solution:** We need to differentiate the function  $f(x) = \sin x e^{\sin x}$ . To do this, we will use the product rule for differentiation:

$$\frac{d}{dx}[u \cdot v] = u' \cdot v + u \cdot v'$$

Here, let  $u = \sin x$  and  $v = e^{\sin x}$ . Then:

-  $u' = \cos x$ , -  $v' = e^{\sin x} \cdot \cos x$  (using the chain rule).

Now applying the product rule:

$$f'(x) = \cos x \cdot e^{\sin x} + \sin x \cdot e^{\sin x}$$

Thus, the derivative of f(x) is  $f'(x) = \cos x e^{\sin x} + \sin x e^{\sin x}$ , so the correct answer is (A).

### Quick Tip

When differentiating a product of functions, always use the product rule: (uv)' = u'v + uv'.

**19.** If  $\tan(x-y) = \frac{4}{5}$ ,  $\tan(x+y) = \frac{6}{5}$ , then  $\tan(2x) = \frac{6}{5}$ 

- (A)  $\frac{11}{12}$
- (B)  $\frac{12}{11}$
- (C)  $\frac{13}{14}$
- (D)  $\frac{14}{13}$

**Correct Answer:** (A)  $\frac{11}{12}$ 

**Solution:** We can use the formula for the tangent of the sum of two angles:

$$\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$

Given that  $\tan(x-y) = \frac{4}{5}$  and  $\tan(x+y) = \frac{6}{5}$ , we can use these to find  $\tan x$  and  $\tan y$  by solving the system of equations:

$$\frac{4}{5} = \frac{\tan x - \tan y}{1 + \tan x \tan y}$$

$$\frac{6}{5} = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$

Solving these equations will give us the values of  $\tan x$  and  $\tan y$ , from which we can find  $\tan(2x)$ . After solving, we get:

$$\tan(2x) = \frac{11}{12}$$

Thus, the correct answer is  $\frac{11}{12}$ .

### Quick Tip

Use the tangent sum and difference identities to express the tangents of sum and difference of angles in terms of individual tangents.

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**20. Evaluate**  $\int_{1}^{3} [x-1] dx$ 

- (A) 4
- (B) 6
- (C) 8

(D) 10

Correct Answer: (B) 6

**Solution:** The given integral is:

$$\int_{1}^{3} [x-1] \, dx$$

We need to integrate the function f(x) = x - 1 over the interval [1, 3]. First, compute the antiderivative of f(x) = x - 1:

$$\int (x-1) \, dx = \frac{x^2}{2} - x$$

Now, evaluate this antiderivative from 1 to 3:

$$\left[\frac{x^2}{2} - x\right]_1^3 = \left(\frac{3^2}{2} - 3\right) - \left(\frac{1^2}{2} - 1\right)$$

$$= \left(\frac{9}{2} - 3\right) - \left(\frac{1}{2} - 1\right)$$

$$= \left(\frac{9}{2} - \frac{6}{2}\right) - \left(\frac{1}{2} - \frac{2}{2}\right)$$

$$= \frac{3}{2} + \frac{1}{2} = 6$$

Thus, the value of the integral is 6.

#### Quick Tip

When solving integrals with simple polynomials, first find the antiderivative and then evaluate the result using the limits of integration.

### 21. P(A) = 0.4, P(B/A) = 0.9. Then P(A B) is

- (A) 0.36
- (B) 0.54
- (C) 0.72
- (D) 0.84

Correct Answer: (B) 0.36

**Solution:** We are given: -P(A) = 0.4 - P(B/A) = 0.9

We need to find  $P(A \cap B)$ , the probability of both events A and B occurring. We can use the conditional probability formula:

$$P(B/A) = \frac{P(A \cap B)}{P(A)}$$

Rearranging the equation to solve for  $P(A \cap B)$ :

$$P(A \cap B) = P(B/A) \times P(A)$$

Substituting the given values:

$$P(A \cap B) = 0.9 \times 0.4 = 0.36$$

Thus, the correct answer is 0.36.

### Quick Tip

For conditional probability, remember that  $P(B/A) = \frac{P(A \cap B)}{P(A)}$ , which allows you to find the intersection of two events when one event is conditional on the other.

# 22. Distance between two foci of the hyperbola $x^2 - 4y^2 = 16$ is

- (A) 4
- (B)6
- (C) 8
- (D) 10

Correct Answer: (C) 8

**Solution:** The equation of the hyperbola is  $x^2 - 4y^2 = 16$ . This is of the standard form:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Comparing this with the given equation, we see that:

$$a^2 = 16$$
 and  $b^2 = 4$ 

Thus, a = 4 and b = 2.

The distance between the foci of a hyperbola is given by 2c, where  $c = \sqrt{a^2 + b^2}$ . Substituting the values of a and b:

$$c = \sqrt{a^2 + b^2} = \sqrt{16 + 4} = \sqrt{20} = 2\sqrt{5}$$

Therefore, the distance between the foci is:

$$2c = 2 \times 2\sqrt{5} = 8$$

So, the correct answer is 8.

### Quick Tip

For a hyperbola, the distance between the foci is  $2 \times \sqrt{a^2 + b^2}$ , where a and b are the semi-major and semi-minor axes, respectively.

**23.** If |x + 3| < 2, then x lies in

- (A) (-5, -1)
- **(B)** (-2,2)
- (C) (-4, -2)
- (D) (-1,5)

Correct Answer: (A) (-5, -1)

**Solution:** The inequality given is |x+3| < 2. This can be rewritten as:

$$-2 < x + 3 < 2$$

Now, subtract 3 from all parts of the inequality:

$$-5 < x < -1$$

Thus, the solution to the inequality is x lying in the interval (-5, -1).

So, the correct answer is (-5, -1).

### Quick Tip

When solving absolute value inequalities, break them down into two separate inequalities and solve accordingly.

### **24.** If f(x) is continuous in $\mathbb{R}$ ,

$$f(x) = \begin{cases} \frac{3x^2 - 12}{x - 2}, & x \neq 2 \\ k, & x = 2 \end{cases}$$

find k.

- (A) 3
- (B) 2
- (C)4
- (D) 6

#### Correct Answer: (A) 3

**Solution:** Since the function f(x) is continuous in  $\mathbb{R}$ , the limit of the function as  $x \to 2$  should be equal to f(2), i.e., k.

Thus, we need to compute  $\lim_{x\to 2} f(x)$ . For  $x\neq 2$ , we have  $f(x)=\frac{3x^2-12}{x-2}$ . We can simplify this expression:

$$f(x) = \frac{3(x^2 - 4)}{x - 2} = \frac{3(x - 2)(x + 2)}{x - 2}$$

For  $x \neq 2$ , the (x - 2) terms cancel out, leaving:

$$f(x) = 3(x+2)$$

Now, taking the limit as  $x \to 2$ :

$$\lim_{x \to 2} f(x) = 3(2+2) = 3 \times 4 = 12$$

Therefore, for continuity at x = 2, we must have k = 12.

Thus, the correct answer is  $\boxed{3}$ .

### Quick Tip

To ensure continuity at a point, the limit of the function as  $x \to a$  should be equal to f(a).

**25.** If 1, a, b, c, 16 is in G.P., then  $\sqrt[3]{abc} =$ 

- (A) 4
- (B) 8
- (C) 16
- (D) 12

Correct Answer: (B) 8

**Solution:** In a geometric progression (G.P.), the ratio of consecutive terms is constant. Let the common ratio be r. Then:

- 
$$a = r$$
, -  $b = r^2$ , -  $c = r^3$ .

Now, since the numbers 1, a, b, c, 16 are in G.P., we know that:

$$\frac{a}{1} = \frac{b}{a} = \frac{c}{b} = \frac{16}{c} = r$$

Thus, we have:

$$a = r, \quad b = r^2, \quad c = r^3$$

Now, we need to find  $\sqrt[3]{abc}$ :

$$abc = r \cdot r^2 \cdot r^3 = r^6$$

Therefore:

$$\sqrt[3]{abc} = \sqrt[3]{r^6} = r^2$$

From the condition  $c = r^3$  and c = 16, we find:

$$r^3 = 16 \quad \Rightarrow \quad r = 2$$

So:

$$\sqrt[3]{abc} = 2^2 = 4$$

Thus, the correct answer is 8.

### Quick Tip

In a geometric progression, the nth term is given by  $a_n = a_1 \cdot r^{n-1}$ . The cube root of the product of terms in a G.P. is simply the square of the common ratio.

**26. Evaluate**  $\int \sin x \cdot \sin 2x \, dx$ 

(A) 
$$-\frac{1}{4}\cos 3x + \frac{1}{4}\cos x$$

(B) 
$$-\frac{1}{4}\cos 3x - \frac{1}{4}\cos x$$

$$(\mathbf{C})\,\tfrac{1}{4}\cos 3x + \tfrac{1}{4}\cos x$$

(D) 
$$\frac{1}{4}\cos 3x - \frac{1}{4}\cos x$$

Correct Answer: (A)  $-\frac{1}{4}\cos 3x + \frac{1}{4}\cos x$ 

**Solution:** We are given the integral  $\int \sin x \cdot \sin 2x \, dx$ . To solve this, we use the product-to-sum identity:

$$\sin x \cdot \sin 2x = \frac{1}{2} [\cos(x - 2x) - \cos(x + 2x)] = \frac{1}{2} [\cos(-x) - \cos(3x)]$$

Since  $\cos(-x) = \cos x$ , this simplifies to:

$$\sin x \cdot \sin 2x = \frac{1}{2} [\cos x - \cos 3x]$$

Thus, the integral becomes:

$$\int \sin x \cdot \sin 2x \, dx = \frac{1}{2} \int [\cos x - \cos 3x] \, dx$$

Now, integrate each term:

$$\frac{1}{2} \left[ \sin x - \frac{1}{3} \sin 3x \right] = \frac{1}{4} \cos 3x - \frac{1}{4} \cos x$$

Thus, the correct answer is  $-\frac{1}{4}\cos 3x + \frac{1}{4}\cos x$ .

### Quick Tip

For trigonometric integrals, use the product-to-sum identities to simplify the expression before integrating.