Limit, Continuity, And Differentiability JEE Main PYQ - 2

Total Time: 25 Minute **Total Marks:** 40

Instructions

Instructions

1. Test will auto submit when the Time is up.
2. The Test comprises of multiple choice questions (MCQ) with one or more correct answers.
3. The clock in the top right corner will display the remaining time available for you to complete the examination.

Navigating & Answering a Question

1. The answer will be saved automatically upon clicking on an option amongst the given choices of answer.
2. To deselect your chosen answer, click on the clear response button.
3. The marking scheme will be displayed for each question on the top right corner of the test window.

Limit, Continuity, And Differentiability

1. $\lim_{x \to 0} \frac{x \tan 2x - 2x \tan x}{(1-\cos 2x)^2}$ equals : **(+4, -1)**

a. $\frac{1}{4}$

b. 1

c. $\frac{1}{2}$

d. $-\frac{1}{2}$

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2. Let f be a twice differentiable function defined on R such that $f(0) = 1, f'(0) = 2$ and $f'(x) \neq 0$ for all $x \in R$. If $\begin{vmatrix} f(x) & f'(x) \\ f'(x) & f''(x) \end{vmatrix} = 0$, for all $x \in R$, then the value of $f(1)$ lies in the interval: **(+4, -1)**

a. (9,12)

b. (6,9)

c. (0,3)

d. (3,6)

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3. Let f:[0,1]→R be a function. suppose the function f is twice differentiable,f(0)=0=f(1) and satisfies f"(x)-2f'(x)+f(x)≥ex,x∈[0,1], which of the following is true? **(+4, -1)**

a. 0<f(x)<∞

b. $-\frac{1}{2} < f(x) < \frac{1}{2}$

c. $-\frac{1}{4} < f(x) < 1$

d. $-\infty < f(x) < 0$

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4. The set of all values of a for which $\lim_{x \to a}([x-5]-[2x+2])=0$,where $[\propto]$ denotes the greatest integer less than or equal to $\propto$ is equal to **(+4, -1)**

a. $[-7.5,-6.5)$

b. $(-7.5,-6.5)$

c. $(-7.5,-6.5]$

d. $[-7.5,-6.5]$

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5. $\lim_{n \to \infty} \frac{1}{2^n}\left(\frac{1}{\sqrt{1-\frac{1}{2^n}}}+\frac{1}{\sqrt{1-\frac{2}{2^n}}}+\frac{1}{\sqrt{1-\frac{3}{2^n}}}+\ldots+\frac{1}{\sqrt{1-\frac{2^n-1}{2^n}}}\right)$ is equal to **(+4, -1)**

a. $\frac{1}{2}$

b. 1

c. 2

d. -2

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6. Let $f(x)=\begin{cases} x^2 \sin\left(\frac{1}{x}\right) & ,x \neq 0 \\ 0 & ,x=0 \end{cases}$ Then at $x=0$ **(+4, -1)**

a. f is continuous but not differentiable

b. f is continuous but f' is not continuous

c. f' is continuous but not differentiable

d. f and f' both are continuous

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7. If the function $f(x)=\begin{cases} (1+|\cos x|)^{\frac{\lambda}{|\cos x|}} & ,0<x<\frac{\pi}{2} \\ \mu & , \quad x=\frac{\pi}{2} \\ \frac{\cot 6x}{e^{\cot 4x}} & \frac{\pi}{2}<x<\pi \end{cases}$ is continuous at $x=\frac{\pi}{2}$, then $9\lambda+6\log_e \mu+\mu^6-e^{6\lambda}$ is equal to **(+4, -1)**

a. $2e^4+8$

b. 11

c. 8

d. 10

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8. $\lim_{x\to 1}\left\{log_e\left(e^{x(x-1)}-e^{x(1-x)}\right)-log_e\left(4x\left(x-1\right)\right)\right\}$ is equal to : **(+4, -1)**

a. $-2\,log_e\,2$

b. 1

c. $1-\ log_e\,2$

d. $-\,log_e\,2$

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9. Let $f:R-\{0,1\}\to R$ be a function such that $f(x)+f\left(\frac{1}{1-x}\right)=1+x$ Then $f(2)$ is equal to **(+4, -1)**

a. $\frac{7}{3}$

b. $\frac{9}{2}$

c. $\frac{9}{4}$

d. $\frac{7}{4}$

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10. for some a,b,c ∈ $\mathbb{N}$, let f(x) = ax-3 and g(x)=xb+c, x ∈ $\mathbb{R}$. If $(fog)^{-1}(x)$ = $\left(\frac{x-7}{2}\right)^{\frac{1}{3}}$ then (fog) (ac) + (gof) (b) is equal to __________ . **(+4, -1)**

Answers

1. Answer: c

Explanation:

Given: $\lim_{x\to 0} \frac{x\tan 2x - 2x\tan x}{(1-\cos 2x)^2}$

$\Rightarrow \lim_{x\to 0} \frac{\frac{2x\tan x}{1-\tan^2 x} - 2x\tan x}{\left(1-1+2\sin^2 x\right)^2} \Rightarrow \lim_{x\to 0} \frac{2x\tan x}{1-\tan^2 x}\left(\frac{1-1+\tan^2 x}{4\sin^4 x}\right)$

$\Rightarrow \lim_{x\to 0} \frac{1}{2}\left(\frac{x}{\sin x}\right)\left(\frac{\tan^3 x}{x^3}\right)\left(\frac{x^3}{\sin^3 x}\right) = \frac{1}{2}$

Concepts:

1. Limits And Derivatives:

Mathematically, a limit is explained as a value that a function approaches as the input, and it produces some value. Limits are essential in calculus and mathematical analysis and are used to define derivatives, integrals, and continuity.

$$\lim_{n\to c} f(n) = L$$

Limits Formula:

- $\lim_{x \to 0} sinx = 0$
- $\lim_{x \to 0} cosx = 1$
- $\lim_{x \to 0} \frac{1-cosx}{x} = 0$
- $\lim_{x \to 0} \frac{sin^{-1}x}{x} = 1$
- $\lim_{x \to 0} \frac{tan^{-1}x}{x} = 1$
- $\lim_{x \to 0} \frac{sinx}{x} = 1$
- $\lim_{x \to 0} \frac{tanx}{x} = 1$

Derivatives of a Function:

A derivative is referred to the instantaneous rate of change of a quantity with response to the other. It helps to look into the moment-by-moment nature of an amount. The derivative of a function is shown in the below-given formula.

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Properties of Derivatives:

1. $\frac{d}{dx}[p(x)+q(x)] = \frac{d}{dx}(p(x)) + \frac{d}{dx}(q(x))$
2. $\frac{d}{dx}[p(x)-q(x)] = \frac{d}{dx}(p(x)) - \frac{d}{dx}(q(x))$
3. $\frac{d}{dx}[p(x)\times q(x)] = \frac{d}{dx}[p(x)]\,q(x) + p(x)\,\frac{d}{dx}[q(x)]$
4. $\frac{d}{dx}\left[\frac{p(x)}{q(x)}\right] = \frac{\frac{d}{dx}[p(x)]\,q(x) - p(x)\,\frac{d}{dx}[q(x)]}{(q(x))^2}$

Read More: Limits and Derivatives

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2. **Answer: b**

Explanation:

$f(x)f''(x) - (f'(x))^2 = 0$ $\frac{f''(x)}{f'(x)} = \frac{f'(x)}{f(x)}$ In $(f'(x)) = \ln f(x) + \ln c$ $f'(x) = cf(x)$ $\frac{f'(x)}{f(x)} = c$ Inf$(x) = cx + k_1$ $f(x) = ke^{cx}$ $f(0) = 1 = k$ $f'(0) = c = 2$ $f(x) = e^{2x}$ $f(1) = e^2 \in (6, 9)$

Concepts:

1. Limits And Derivatives:

Mathematically, a limit is explained as a value that a function approaches as the input, and it produces some value. Limits are essential in calculus and mathematical analysis and are used to define derivatives, integrals, and continuity.

$$\lim_{n \to c} f(n) = L$$

Limits Formula:

- $\lim_{x \to 0} sinx = 0$
- $\lim_{x \to 0} cosx = 1$
- $\lim_{x \to 0} \frac{1 - cosx}{x} = 0$
- $\lim_{x \to 0} \frac{sin^{-1}x}{x} = 1$
- $\lim_{x \to 0} \frac{tan^{-1}x}{x} = 1$
- $\lim_{x \to 0} \frac{sinx}{x} = 1$
- $\lim_{x \to 0} \frac{tanx}{x} = 1$

Derivatives of a Function:

A derivative is referred to the instantaneous rate of change of a quantity with response to the other. It helps to look into the moment-by-moment nature of an amount. The derivative of a function is shown in the below-given formula.

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Properties of Derivatives:

$$1.\ \frac{d}{dx}\left[p(x)+q(x)\right]=\frac{d}{dx}\left(p(x)\right)+\frac{d}{dx}\left(q(x)\right)$$

$$2.\ \frac{d}{dx}\left[p(x)-q(x)\right]=\frac{d}{dx}\left(p(x)\right)-\frac{d}{dx}\left(q(x)\right)$$

$$3.\ \frac{d}{dx}\left[p(x)\times q(x)\right]=\frac{d}{dx}\left[p(x)\right]q(x)+p(x)\frac{d}{dx}\left[q(x)\right]$$

$$4.\ \frac{d}{dx}\left[\frac{p(x)}{q(x)}\right]=\frac{\frac{d}{dx}\left[p(x)\right]q(x)-p(x)\frac{d}{dx}\left[q(x)\right]}{\left(q(x)\right)^2}$$

Read More: Limits and Derivatives

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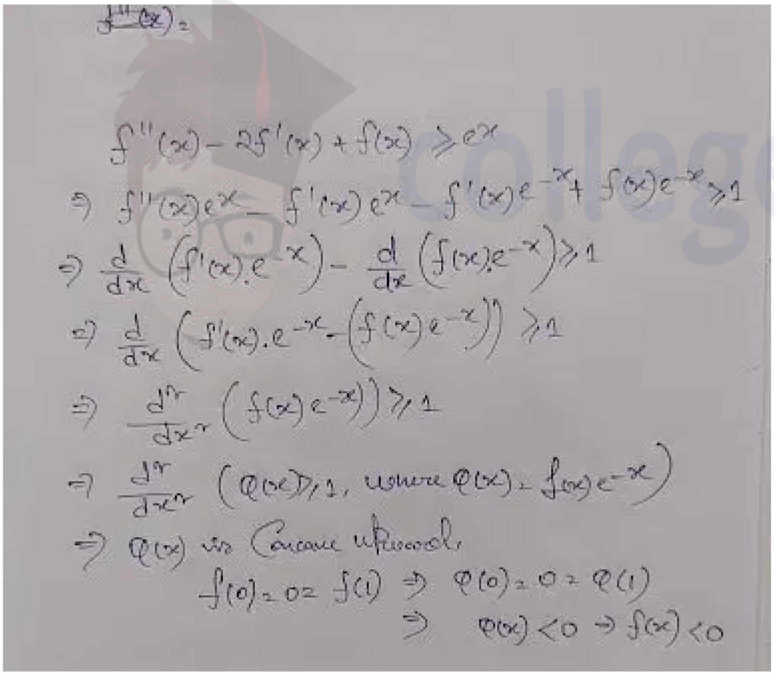
3. Answer: d

Explanation:

The correct answer is option (D) : $-\infty < f(x) < 0$



$f''(x) =$

$f''(x) - 2f'(x) + f(x) \geq e^x$

$\Rightarrow f''(x)e^{-x} - f'(x)e^{-x} - f'(x)e^{-x} + f(x)e^{-x} \geq 1$

$\Rightarrow \frac{d}{dx}\left(f'(x).e^{-x}\right) - \frac{d}{dx}\left(f(x)e^{-x}\right) \geq 1$

$\Rightarrow \frac{d}{dx}\left(f'(x).e^{-x} - \left(f(x)e^{-x}\right)'\right) \geq 1$

$\Rightarrow \frac{d^2}{dx^2}\left(f(x)e^{-x}\right) \geq 1$

$\Rightarrow \frac{d^2}{dx^2}\left(\phi(x)\right) \geq 1$, where $\phi(x) = f(x)e^{-x}$

$\Rightarrow \phi(x)$ is concave upward,

$f(0) = 0 = f(1) \Rightarrow \phi(0) = 0 = \phi(1)$

$\Rightarrow \phi(x) < 0 \Rightarrow f(x) < 0$

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4. Answer: b

Explanation:

The correct answer is (B) : $(-7.5, -6.5)$

$$\lim_{x\to a}([x-5]-[2x+2])=0$$

$$\lim_{x\to a}([x]-5-[2x]-2)=0$$

$\lim_{x \to a}([x] - [2x]) = 7$

$[a] - [2a] = 7$

$a \in I, a = -7$

$a \notin I, a = I + f$

Now, $[a] - [2a] = 7$

$-I - [2f] = 7$

Case-I: $f \in (0, \frac{1}{2})$

$2f \in (0, 1)$

$-I = 7$

$I = -7 \Rightarrow a \in (-7, -6.5)$

Case-II: $f \in (\frac{1}{2}, 1)$

$2f \in (1, 2)$

$-I - 1 = 7$

$I = -8 \Rightarrow a \in (-7.5, -7)$

Hence, $a \in (-7.5, -6.5)$

Concepts:

1. Limits:

A function's limit is a number that a function reaches when its independent variable comes to a certain value. The value (say a) to which the function f(x) approaches casually as the independent variable x approaches casually a given value "A" denoted as f(x) = A.

If $\lim_{x \to a^-}$ f(x) is the expected value of f when x = a, given the values of 'f' near x to the left of 'a'. This value is also called the left-hand limit of 'f' at a.

If $\lim_{x \to a^+}$ f(x) is the expected value of f when x = a, given the values of 'f' near x to the right of 'a'. This value is also called the right-hand limit of f(x) at a.

If the right-hand and left-hand limits concur, then it is referred to as a common value as the limit of f(x) at x = a and denote it by $\lim_{x \to a}$ f(x).

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5. **Answer: c**

Explanation:

$$\lim_{n\to\infty}\frac{1}{2^n}\sum_{r=1}^{2^n}\frac{1}{\sqrt{1-\frac{r}{2^n}}}$$
$$\therefore \frac{1}{2^n}\to dx \Leftarrow \frac{r}{2^n}=x\left(\frac{r}{n'}=x, \frac{1}{x}=dx\right)$$
$$2^n=n'$$
$$\lim_{n'\to\infty}\frac{1}{n'}\sum_{r=1}^{n'-1}\frac{1}{\sqrt{1-\frac{r}{n'}}}=\int_0^1\frac{1}{\sqrt{1-x}}dx$$
$$=-\left.\frac{(1-x)^{1/2}}{1/2}\right]_0^1=-2[0-1]=2$$

Concepts:

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If the right-hand and left-hand limits concur, then it is referred to as a common value as the limit of f(x) at x = a and denote it by $\lim_{x\to a}$ f(x).

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6. Answer: b

Explanation:

Continuity of $f(x): f\left(0^{+}\right)=h^2\cdot\sin\frac{1}{h}=0$
$$f\left(0^{-}\right)=(-h)^2\cdot\sin\left(\frac{-1}{h}\right)=0$$
$$f(0)=0$$
$f(x)$ is continuous
$$f'\left(0^{+}\right)=\lim_{h\to0}\frac{f(0+h)-f(0)}{h}=\frac{h^2\cdot\sin\left(\frac{1}{h}\right)-0}{h}=0$$
$$f'\left(0^{-}\right)=\lim_{h\to0}\frac{f(0-h)-f(0)}{-h}=\frac{h^2\cdot\sin\left(\frac{1}{-h}\right)-0}{-h}=0$$

$f(x)$ is differentiable.

$f'(x) = 2x \cdot \sin\left(\frac{1}{x}\right) + x^2 \cdot \cos\left(\frac{1}{x}\right) \cdot \frac{-1}{x^2}$

$$f'(x) = \begin{cases} 2x \cdot \sin\left(\frac{1}{x}\right) - \cos\left(\frac{1}{x}\right), & x \neq 0 \\ 0, & x = 0 \end{cases}$$

$\Rightarrow f'(x)$ is not continuous (as $\cos\left(\frac{1}{x}\right)$ is highly oscillating at $x = 0$)

Concepts:

1. Continuity:

A function is said to be continuous at a point *x* = *a,* if

$\lim_{x \to a}$

f(x) Exists, and

$\lim_{x \to a}$

f(x) = *f*(*a*)

It implies that if the left hand limit (L.H.L), right hand limit (R.H.L) and the value of the function at *x*=*a* exists and these parameters are equal to each other, then the function *f* is said to be continuous at *x*=*a*.

If the function is undefined or does not exist, then we say that the function is discontinuous.

Conditions for continuity of a function: For any function to be continuous, it must meet the following conditions:

- The function f(x) specified at x = a, is continuous only if f(a) belongs to real number.
- The limit of the function as x approaches a, exists.
- The limit of the function as x approaches a, must be equal to the function value at x = a.

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7. Answer: d

Explanation:

The correct answer is (D) : 10

$\Rightarrow \lim\limits_{x \to \frac{\pi}{2}^{+}} e^{\frac{\cot 6x}{\cot 4x}} = \lim\limits_{x \to \frac{\pi}{2}^{+}} e^{\frac{\sin 4x \times \cos 6x}{\sin 6x \cdot \cos 4x}} = e^{2/3}$

$\Rightarrow \lim\limits_{x \to \frac{\pi}{2}^{-}} (1 + |\cos x|)^{\frac{\lambda}{\cos x}|} = e^{\lambda}$

$\Rightarrow f(\pi/2) = \mu$

For continuous function $\Rightarrow e^{2/3} = e^{\lambda} = \mu$

$\lambda = \frac{2}{3}, \mu = e^{2/3}$

Now, $9\lambda + 6\log_e \mu + \mu^6 - e^{6\lambda} = 10$

Concepts:

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- The limit of the function as x approaches a, must be equal to the function value at x = a.

8. **Answer: d**

Explanation:

The correct option is (D): $-log_e 2$

Concepts:

1. Continuity & Differentiability:

Definition of Differentiability

f(*x*) is said to be differentiable at the point *x* = *a,* if the derivative f ‘(a) be at every point in its domain. It is given by

The maxima lie between the minima and the width of the central maximum is simply the distance between the 1st order minima from the centre of the screen on both sides of the centre.

The position of the minima given by y (measured from the centre of the screen) is:

$$\tan\theta \approx \theta \approx y/D$$

For small θ,

$\sin\theta \approx \theta$

$\Rightarrow \lambda = a \sin\theta \approx a\theta$

$\Rightarrow \theta = y/D = \lambda a$

$\Rightarrow y = \lambda D a$

The width of the central maximum is simply twice this value

⇒ **Width of central maximum** = $2\lambda Da$

⇒ **Angular width of central maximum** = $2\theta = 2\lambda a$

Definition of Continuity

Mathematically, a function is said to be continuous at a point *x* = *a,* if

It is implicit that if the left-hand limit (L.H.L), right-hand limit (R.H.L), and the value of the function at *x*=*a* exist and these parameters are equal to each other, then the function *f* is said to be continuous at *x*=*a.*

$\lim_{x\to a}$ f(x) Exists, and

$\lim_{x\to a}$ f(x) = *f*(*a*)

If the function is unspecified or does not exist, then we say that the function is discontinuous.

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9. **Answer: c**

Explanation:

$$f(x)+f\left(\frac{1}{1-x}\right)=1+x$$
$$x=2\Rightarrow f(2)+f(-1)=3$$
$$x=-1\Rightarrow f(-1)+f\left(\frac{1}{2}\right)=0\ (2)$$
$$x=\frac{1}{2}\Rightarrow f\left(\frac{1}{2}\right)+f(2)=\frac{3}{2}\ (3)$$
$$(1)+(3)-(2)\Rightarrow 2f(2)=\frac{9}{2}$$
$$\therefore f(2)=\frac{9}{4}$$

Concepts:

1. **Continuity & Differentiability:**

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The position of the minima given by y (measured from the centre of the screen) is:

tanθ≈θ≈y/D

For small θ,

sin θ≈θ

⇒ λ = a sin θ≈aθ

⇒ θ = y/D = λa

⇒ y = λDa

The width of the central maximum is simply twice this value

⇒ **Width of central maximum** = 2λDa

⇒ **Angular width of central maximum** = 2θ = 2λa

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10. **Answer: 2039 - 2039**

Explanation:

The correct answer is 2039
Let fog(x)=h(x)

⇒h-1(x)=(2x-7)31

⇒h(x)= fog (x)=2x3+7
fog (x)=a(xb+c)-3

⇒a=2,b=3,c=5

⇒fog(ac)=fog(10)=2007
g(f(x)=(2x-3)3+5
⇒gof(b)=gof(3)=32
⇒sum=2039

Concepts:

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Definition of Differentiability

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⇒ $\theta = y/D = \lambda a$

⇒ $y = \lambda Da$

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