NEST 2023 Shift 2 Question Paper

Time Allowed :3 Hours	**Maximum Marks :**200	**Total questions :**68

General Instructions

Read the following instructions very carefully and strictly follow them:

1. **Conducting Bodies:** National Institute of Science Education and Research (NISER) and University of Mumbai - Department of Atomic Energy Centre for Excellence in Basic Sciences (UM-DAE CEBS).
2. **Exam Mode:** Online (CBT)
3. **Total Marks:** 200
4. **Total Questions:** 68

BIOLOGY

1. An example of multiple alleles is seen at a locus that determines the feather pattern of mallard ducks. One allele M produces the wild-type mallard pattern. A second allele M^R produces a different pattern called restricted, and a third allele m^d produces a pattern termed dusky. In this allelic series, the dominance pattern is $M^R > M > m^d$. In a cross between restricted and mallard ducks, it was found that only dusky ducks were absent in the F1 generation. This indicates that the genotypes of the parents most likely could be:

(1) $(M^RM \times Mm^d)$ and $(M^Rm^d \times Mm^d)$

(2) $(M^RM^R \times MM)$ and $(M^Rm^d \times Mm^d)$

(3) $(M^RM \times Mm^d)$ and $(M^Rm^d \times MM)$

(4) $(M^RM^R \times MM)$ only

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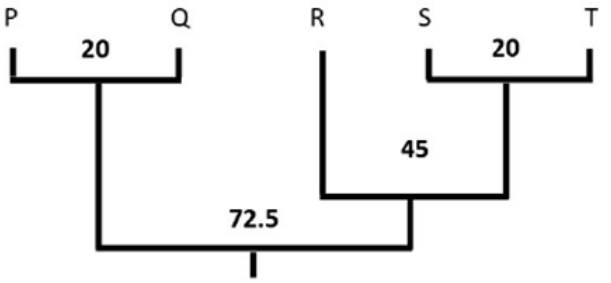
2. UPGMA is a method of constructing phylogenetic trees using distance matrices between organisms. The following matrix depicts distances (measured as the difference in characters) between five organisms (P, Q, R, S, and T). The distance between a pair of organisms (say, P and Q) and a third organism (R) is calculated as an average of their individual distances from the third organism (for example: New average distance between PQ and R is $\frac{60+50}{2} = 55$).

	P	Q	R	S	T
P	0				
Q	20	0			
R	60	50	0		
S	100	90	40	0	
T	90	80	50	20	0

Based on this distance matrix, the correct phylogenetic tree is:



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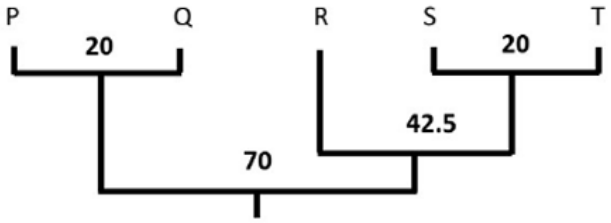

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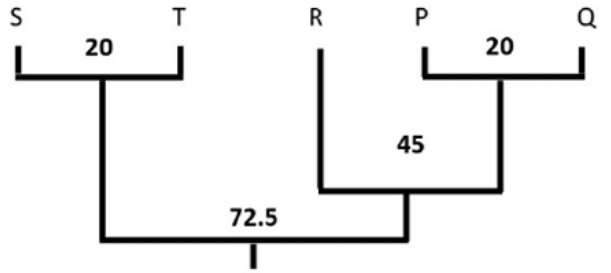

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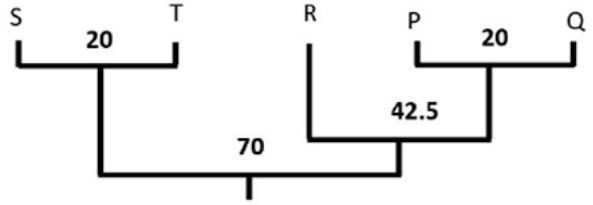

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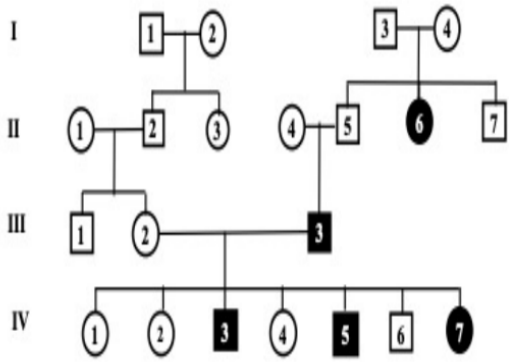
3. In the given pedigree, circles represent females and squares represent males. Filled shapes indicate affected individuals while unfilled shapes indicate unaffected individuals. Based on the pedigree information provided below, identify the inheritance pattern.



(1) Autosomal dominant

(2) Autosomal recessive

(3) X-linked dominant

(4) X-linked recessive

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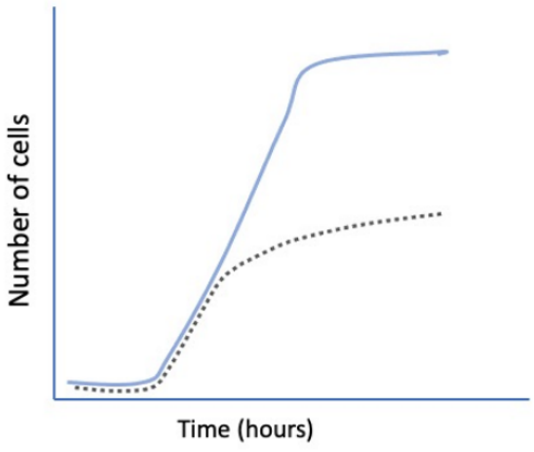
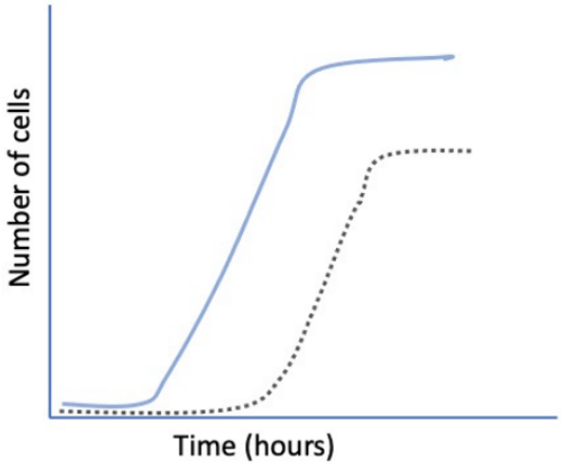
4. In an experiment with a facultative aerobic bacterial species, identical number of cells were inoculated in two 500 ml jars (M and N) with 250 ml volume of media in each. Both the jars contained the same concentration of glucose as the only energy source. Jar M was incubated in airtight conditions while N was maintained in aerobic conditions. Both the jars were kept in a sterile chamber and all other conditions of incubation were kept the same. The correct plot that depicts the growth patterns of these bacterial cultures in M (grey dotted line) and N (blue solid line) is:

(1)





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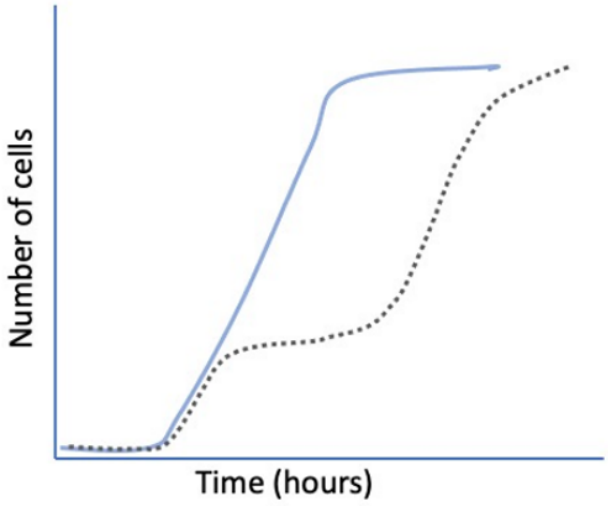

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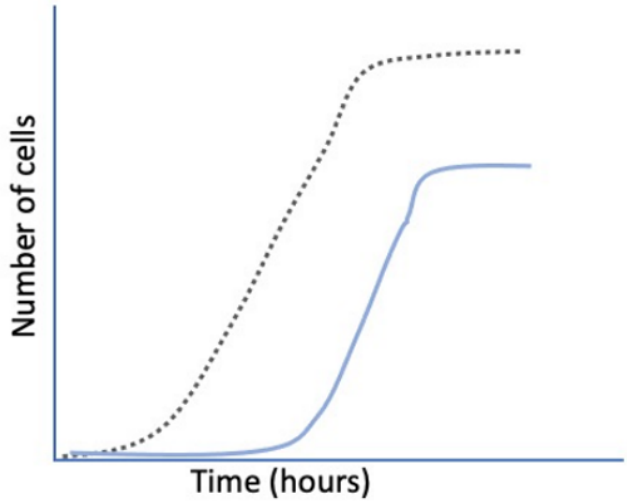

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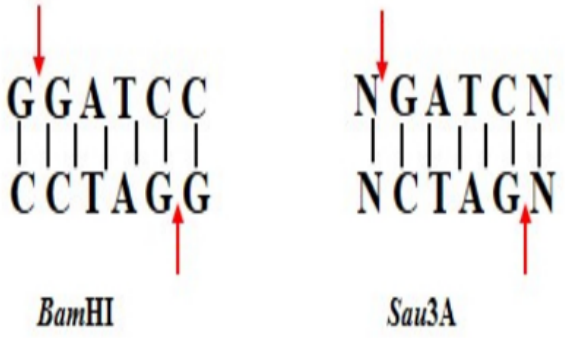
5. Restriction enzymes recognize certain sequences within the DNA and cleave them. If

a DNA fragment is cleaved with BamHI restriction enzyme, it generates sticky ends. If the same DNA fragment is cleaved with Sau3A restriction enzyme, it generates sticky ends. The cleaved fragments can be joined using DNA ligase. The recognition and cleavage site (red arrows) for BamHI and Sau3A are given below. N represents any of the nucleotides. Based on this information and assuming there is only a single cleavage site, choose the correct option.



(1) If a Sau3A cleaved end is ligated to a BamHI cleaved end, the ligated fragment can be further digested using Sau3A irrespective of the neighbouring sequence.
(2) If a BamHI cleaved end is ligated to a Sau3A cleaved end, the ligated fragment can be further digested using BamHI irrespective of the neighbouring sequence.
(3) If both the recognition sequences are reverse complemented then it cannot be cleaved using either BamHI or Sau3A.

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6. A mutant bacterial strain having a shorter glycolytic pathway was discovered. If the mutant bacteria are grown aerobically, the net ATP yield was lowered to 28 (compared to the net ATP yield of 34 from Krebs's cycle for wild type bacteria). Except for the reaction that is bypassed in the mutant, assume that the other reactions of the pathway remain unaffected. The step that is most likely bypassed is:

(1) phosphoenolpyruvate to pyruvate
(2) glyceraldehyde-3-phosphate to 1,3-bisphosphoglycerate
(3) fructose 6-phosphate to fructose 1,6-bisphosphate
(4) 1,3-bisphosphoglycerate to 3-phosphoglycerate

7. The table below presents the kinetic data obtained for an enzyme in the absence and presence of two different inhibitors P and Q, each at a concentration of 10.0 mM.

Substrate (1/S) $(mM)^{-1}$	Without Inhibitor $(1/V_0)$ $(\mu mol/mL.s)^{-1}$	With Inhibitor P $(1/V_0)$ $(\mu mol/mL.s)^{-1}$	With Inhibitor Q $(1/V_0)$ $(\mu mol/mL.s)^{-1}$
1.000	0.28	0.31	0.39
0.500	0.16	0.19	0.22
0.250	0.10	0.13	0.14
0.125	0.07	0.09	0.09
0.083	0.06	0.08	0.08

Consider that the total enzyme concentration $[E]_T$ is the same for all the experimental conditions. P and Q, respectively, are

(1) competitive and non-competitive inhibitors

(2) uncompetitive and competitive inhibitors

(3) uncompetitive and non-competitive inhibitors

(4) competitive and uncompetitive inhibitors

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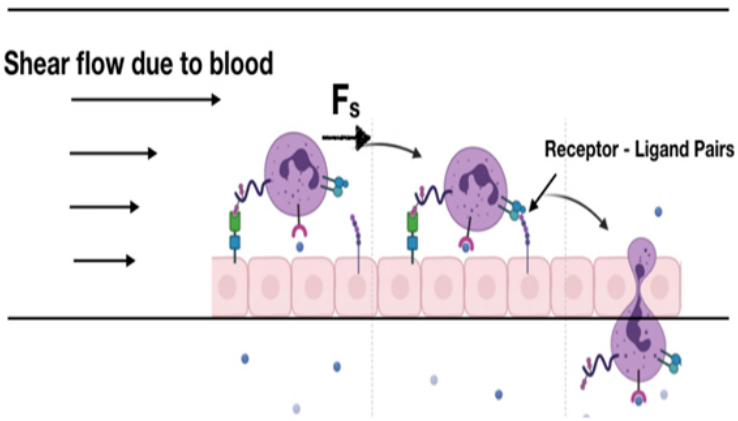
8. Tissue damage alters the surface-adhesive behaviour of leukocytes resulting in leukocyte rolling. This involves several cycles of attachment and detachment of leukocytes on the surface of endothelial cells. Given that the typical rupture force for a ligand-receptor pair is 25 pN, multiple bonds must be formed at the same time to provide the necessary counterbalance to the shear force exerted by the flowing blood. The general schematic is depicted below.



Consider the effective cross-sectional area of the cell that experiences the shear and the following parameters.

Leukocyte radius (approximated to be a sphere)	5 μm
Rolling velocity	10 μm/s
Shear Stress due to blood flow	1 N/m^2

Among the given options, the minimum number of ligand-receptor pairs (bonds) that need to form at the same time to provide the counterforce against the shear force to stop the leukocyte rolling is

(1) 15 ligand-receptor pair

(2) 10 ligand-receptor pair

(3) 2 ligand-receptor pair

(4) 5 ligand-receptor pair

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9. Clutch size in birds refers to the number of eggs laid in a single nesting attempt by a nesting pair of birds, while number in brood refers to the number of young hatched. The graphs below represent the relationship between these parameters (clutch size and number in brood) and fitness in bird populations of the Great tit.

(1) Average nest dimension; Average weight of young

(2) Average number of clutches; Average adult survival

(3) Average adult survival; Average nest dimension

(4) Average weight of young; Average number of clutches

10. A population has three genotypes, XX, XY, and YY, where X is dominant over the Y allele. The number of each genotype in the population is as follows: $XX = 1185$, $XY = 3045$, and $YY = 1300$ individuals. Consider that there is random mating, no gene flow, no mutation and selection, and the population size is sufficiently large. The correct statement is:

(1) The population is in Hardy-Weinberg equilibrium and will remain the same if random mating is allowed for one generation. (2) The population is not in Hardy-Weinberg equilibrium but will come to equilibrium if random mating is allowed for one generation. (3) The population is in Hardy-Weinberg equilibrium and will deviate from equilibrium if selection is acting against any one genotype. (4) The population is not in Hardy-Weinberg equilibrium and will come to equilibrium if selection is acting against the dominant genotype.

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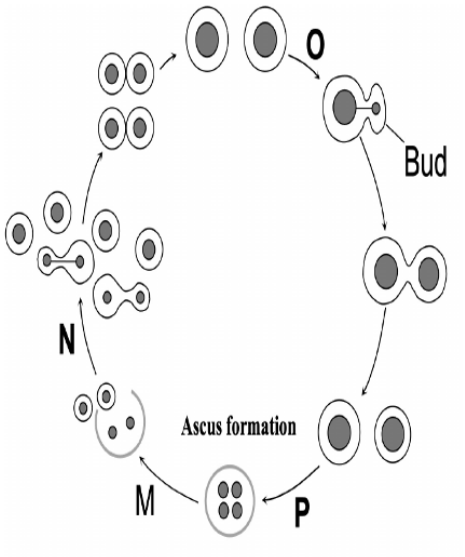
11. The life cycle of yeast *Saccharomyces cerevisiae* which reproduces both sexually as well as asexually is depicted below.



M, N, O and P represent

(1) M - Germination; N - Vegetative growth of haploid cells; O - Vegetative growth of diploid cells; P - Starvation

(2) M - Vegetative growth of haploid cells; N - Starvation; O - Germination; P - Vegetative growth of diploid cells

(3) M - Germination; N - Vegetative growth of diploid cells; O - Vegetative growth of haploid cells; P - Starvation

(4) M - Vegetative growth of diploid cells; N - Starvation; O - Germination; P - Vegetative growth of haploid cells

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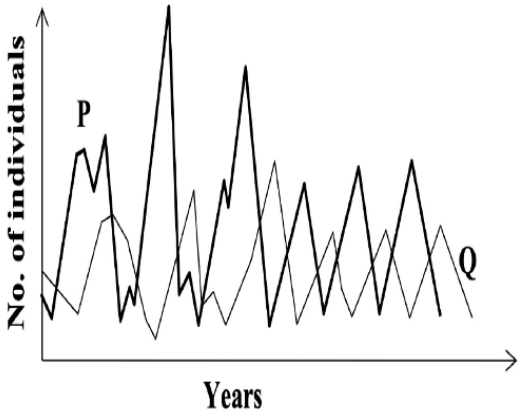
12. The population sizes of two organisms P and Q growing in a given habitat is shown.



If P and Q share ecological relationship, then they most likely represent

(1) P: Predator; Q: Prey

(2) P: Parasite; Q: Host

(3) P: Herbivore; Q: Carnivore

(4) P: Competitor of Q; Q: Competitor of P

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13. Female beetles are known to prefer males with bigger mandibles. In an experiment, a population of these beetles were picked and divided into two groups. For one group, only those males who had larger than average mandible size were allowed to mate to produce next-generation offspring (group 1). For the other group, the males and females were allowed to mate randomly (group 2). These populations were maintained using this regime for 50 generations. After this, it was found that the male mandible

size in group 1 was significantly larger than that of group 2. However, the females in group 1 produced fewer offspring than females in group 2. Possible explanation(s) of this observation is(are):

(1) In the experiment, as selection on female reproduction was not imposed in group 1, female reproductive capability declined over time.

(2) Group 1 males produced offspring with larger thorax (to support larger mandible) and hence smaller abdomen, which influenced the egg-carrying capacity in female offspring leading to a decline in female reproductive ability.

(3) Under unlimited food condition, males with larger mandibles in group 1, preferred females with lesser reproductive ability as that allowed dominant individual males to have more resources for themselves.

(4) Under limited food condition, females producing fitter offspring after mating with males with larger mandibles started producing fewer offspring to nourish them better.

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14. Autoradiography of a green leaf of summer squash (upper panel) showed import of ^{14}C carbon from the source over a period of time. A similar experiment was carried out with an albino tobacco leaf (lower panel) which do not photosynthesize. Shaded portions denote ^{14}C labelling. Based on these observations, the correct option(s) is(are):

(1) In the early stages of development, the leaf acts as a source.

(2) Mature leaf gains the ability to load and export sugar.

(3) The import to export transition is dependent on the developmental stage of leaves irrespective of photosynthesis.

(4) Import cessation and export initiation are two separate events.

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15. In a true-breeding homozygous lines of snapdragon, *Antirrhinum majus*, white coloured flower of personate shape was crossed with red coloured flower with peloric shape. The F1 flowers were pink and personate-shaped. Assuming that both these genes segregate independently, choose the correct option(s).

(1) F2 progeny will have $\frac{1}{4}$ probability of showing the parental phenotype.

(2) 50% of the progeny in the F2 generation will be pink in colour.

(3) In F2 progeny, peloric-shaped flowers with pink colour are expected to be in $\frac{1}{8}$ ratio.

(4) In F2 progeny, the ratio of red-coloured personate-shaped flowers would be $\frac{1}{4}$.

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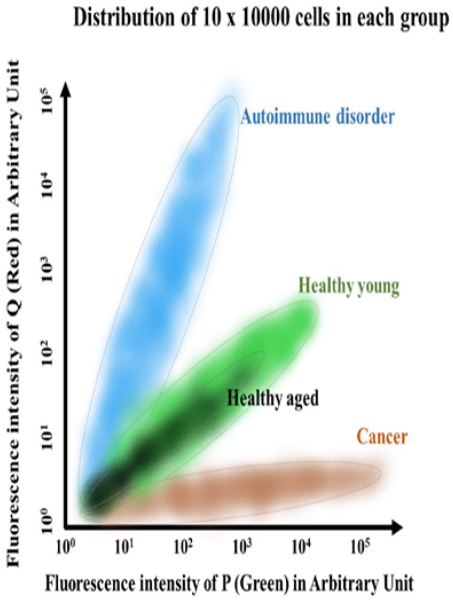
16. Specific fluorescence probes are used to label proteins present on the surface of specific immune cell type. A scientist labelled protein P with a green probe and protein Q with a red probe. A machine can provide quantitative information about the amount of these two proteins present on the surface of each cell by quantifying 10000 cells. This experiment is repeated for cells present in blood of multiple individuals who are healthy young, healthy aged, with cancer, and with auto-immune disorder. The data of 10 individuals per group is provided below.



If there are no other confounding factors, then based on this data, the correct inference(s) is(are)

(1) In healthy aged individuals, the expression of P reduces drastically as compared to Q

(2) Reduction of Q protein can be correlated with the development of cancer

(3) Increased expression of Q protein can be correlated with the auto-immune disorder

(4) In comparison to healthy individuals, the expression of Q in autoimmune condition is negatively regulated by expression of P

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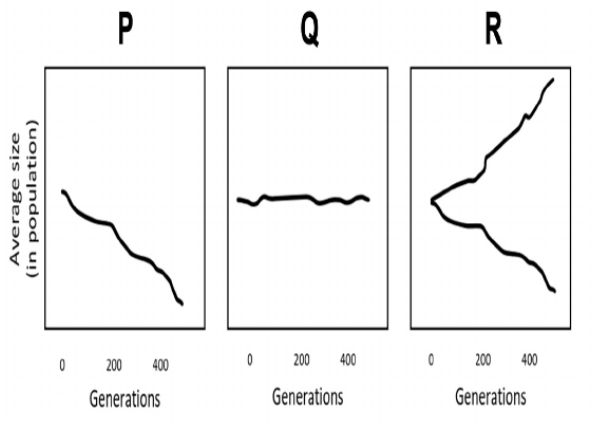
17. The following graphs depict three different scenarios where the average body size of a population (assuming a normal distribution with a single mean) of a study organism has been plotted over several generations. If body size is heritable and there is no genetic drift present in the population, the correct option(s) that can give rise to the observed patterns would be:



(1) P - Directional selection, Q - Stabilizing selection

(2) Q - No selection, R - Disruptive selection

(3) P - No selection, R - Disruptive selection

(4) P - No selection, Q - Directional selection

Chemistry

18. The correct statement regarding the halides and monoxides of the alkaline earth metals is:

(1) All the oxides and halides are ionic in nature.

(2) All the halides are always monomeric.

(3) Hydrated chlorides of all the alkaline earth metals give dehydrated products at high temperature.

(4) Beryllium monoxide reacts with water to give beryllium hydroxide which further reacts with an alkali metal hydroxide to give $[Be(OH)_4]^{2-}$.

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19. The reaction of methyl chloride with silicon at 573 K in the presence of copper as a catalyst produces substituted chlorosilanes. Hydrolysis of chlorosilanes produces silanols. The silanols with appropriate substitution, thus formed, are used to make silicone polymers. The correct statement in this context is:

(1) Only two different chlorosilanes are produced in the reaction of methyl chloride with silicon.

(2) A silicate is formed on condensation polymerization of silanols.

(3) The chain length of the silicone polymer can be controlled by adding trimethylchlorosilane.

(4) Trimethylsilanol upon condensation yields a straight chain polymer.

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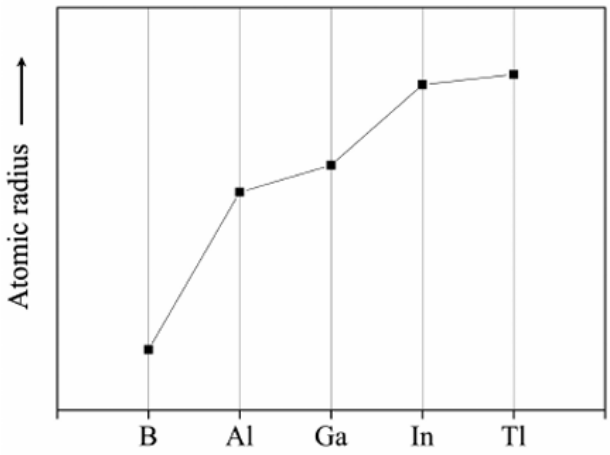
20. The correct graph representing the trend in the atomic radius of the boron family is

(1)



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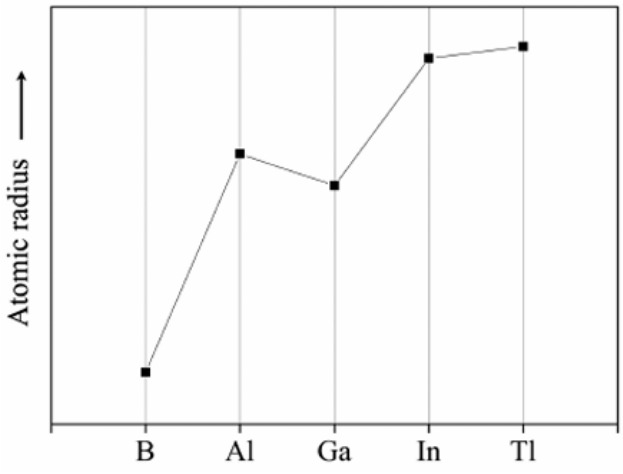

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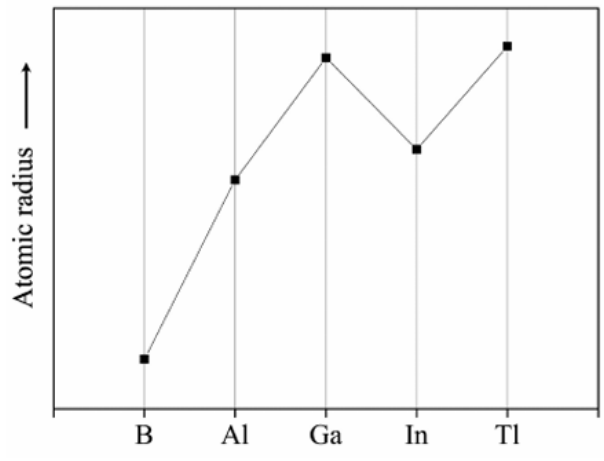

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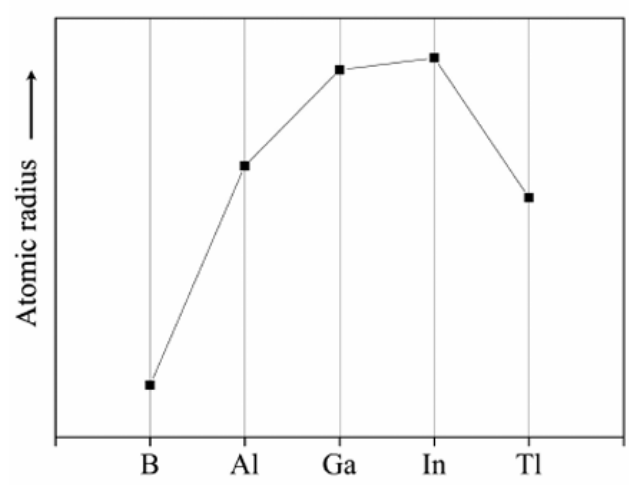

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21. The largest crystal field stabilization energy is for

(1) $[Cr(H_2O)_6]^{3+}$

(2) $[Ti(H_2O)_6]^{3+}$

(3) $[V(H_2O)_6]^{3+}$

(4) $[Fe(H_2O)_6]^{3+}$

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22. The equation appropriate for the exact calculation of pH of an aqueous solution of HCl at a concentration ($c_{\mathbf{HCl}}$), close to 10^{-6} M, is given by:

(1) $[\mathrm{H}^+] = c_{\mathrm{HCl}}$

(2) $[\mathrm{H}^+] = c_{\mathrm{HCl}} + K_w/[\mathrm{H}^+]$

(3) $[H^+] = c_{HCl} + \sqrt{K_w}$

(4) $[H^+] = c_{HCl} + K_w/(2c_{HCl})$

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23. Consider a hypothetical one-electron atom, where the nucleus and the electron interact with a force $F = -kr$. Here, r is the distance between the electron and the nucleus, and k is a constant. If this atom is studied using the Bohr model, the electron is assumed to move around the nucleus in selected stable orbits of fixed radii, characterized by quantum number n. The radius of the orbiting electron (of mass m_e) is

(1) $\left(\frac{n^2h^2}{4\pi^2km_e}\right)^{1/4}$

(2) $\left(\frac{n^2h^2}{4\pi^2km_e}\right)$

(3) $\left(\frac{n^2h^2}{4\pi^2km_e}\right)^{1/3}$

(4) $\left(\frac{n^2h^2}{4\pi^2km_e}\right)^{1/2}$

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24. Biological standard potential (E^*) is defined as the potential measured at $pH = 7.0$. The species nicotinamide adenine dinucleotide (NADH) and its oxidised form NAD^+ play an important role in the respiratory process. Given, the standard potential $E^0 = -0.099\,V$ for the reaction $NAD^+(aq) + H^+(aq) + 2e^- \rightarrow NADH(aq)$, the value of E^* for the conversion of NAD^+ (aq) to NADH (aq) in 1.0 M NAD^+ solution, at room temperature (25°C), is:

(1) $-0.31\,V$

(2) $-0.99\,V$

(3) $-0.51\,V$

(4) $-0.41\,V$

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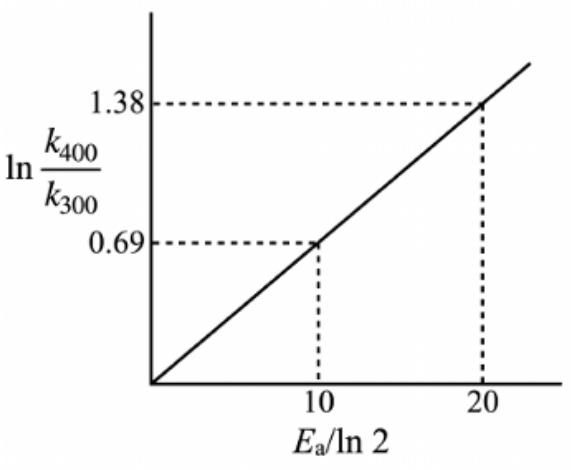
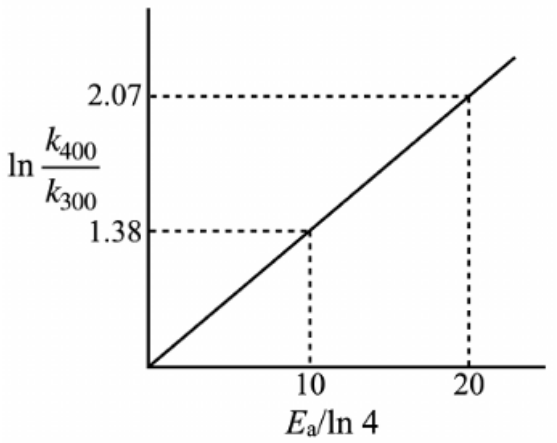
25. The temperature dependence of the rate constants (k) of a chemical reaction can be expressed in terms of the Arrhenius equation, which contains the corresponding

activation energy (E_a) term. The correct plot of the ratio of the rate constants (not drawn to scale) of different chemical reactions, at two temperatures 300 K and 400 K, as a function of their E_a values (in kJ mol^{-1}) is:



(1)



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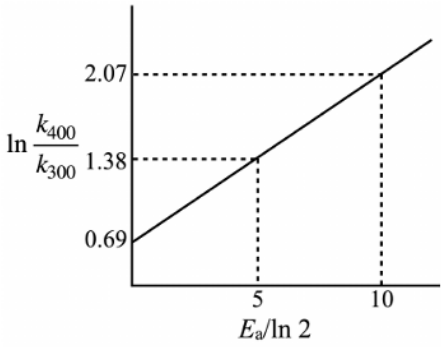

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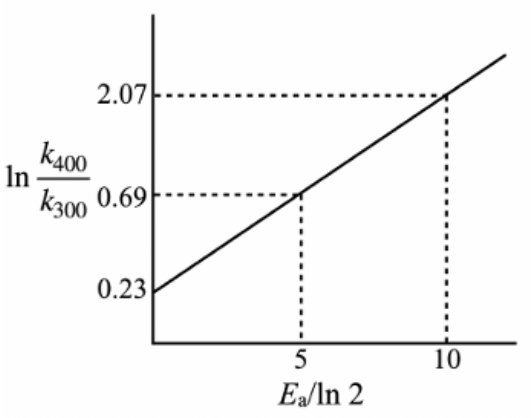

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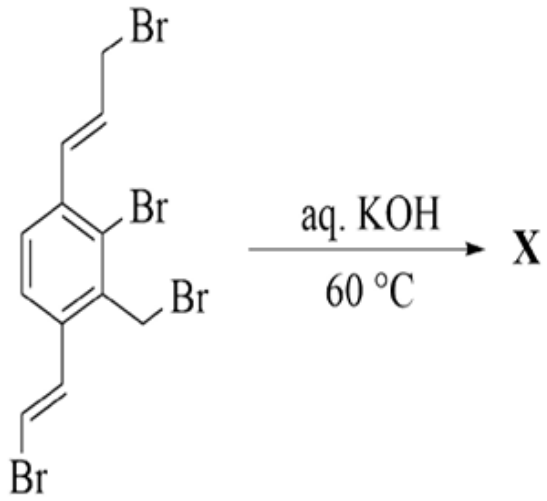
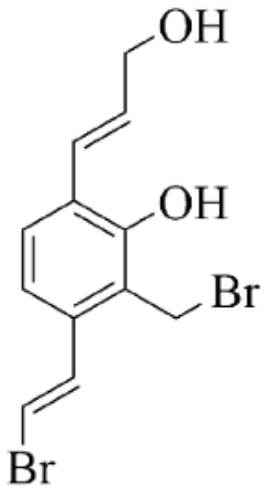
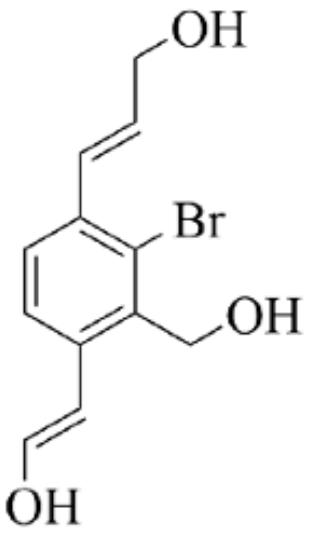
26. In the reaction shown below,



The major product X is:



(1)



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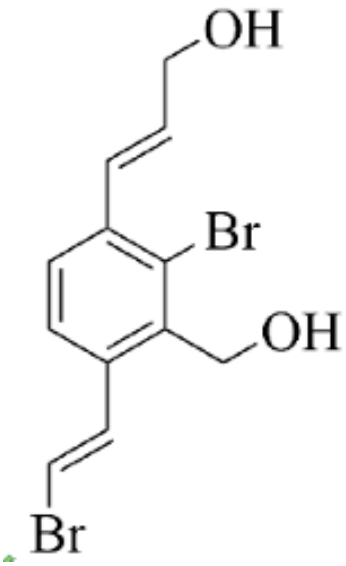

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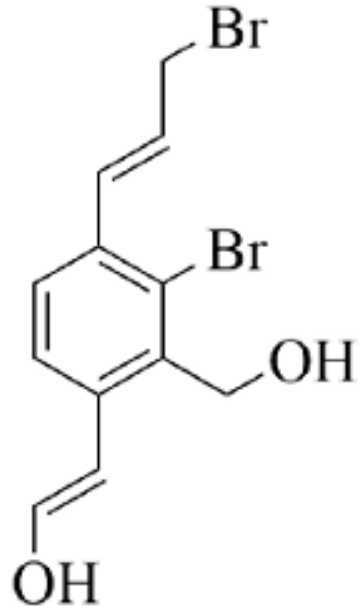
(4) 

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27. In an aqueous solution, glucose exists in cyclic and open-chain forms, in equilibrium, as shown below. Glucose solution does not give positive Schiff test. The correct statement is:

(1) α-D-(+)-glucose and β-D-(+)-glucose are enantiomers.

(2) α-D-(+)-glucose and β-D-(+)-glucose are anomers.

(3) In solution, the open-chain form predominates over the cyclic forms.

(4) Glucose reacts with sodium bisulphite to form an addition product.

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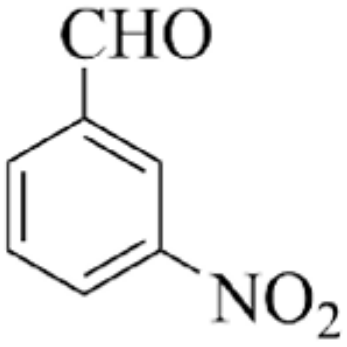
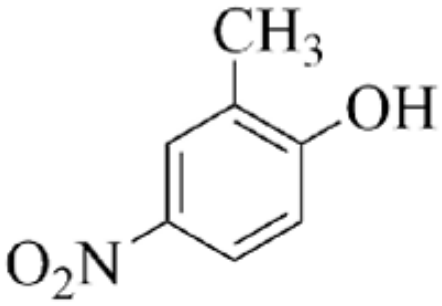
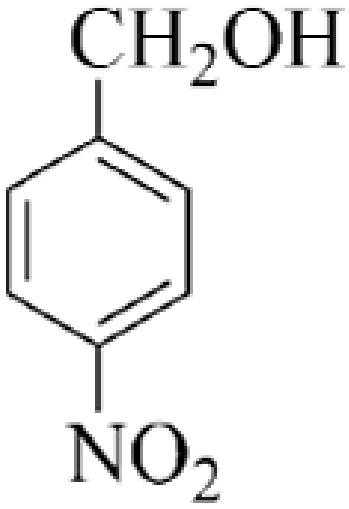
28. One mole of toluene on reaction with 2 moles of Cl_2 in the presence of light gives X, which on hydrolysis at 100°C gives Y. Y on reaction with conc. HNO_3/H_2SO_4 at 0-10°C provides Z as the major product. The compound Z is:



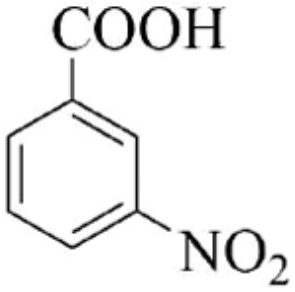
(1)



(2)

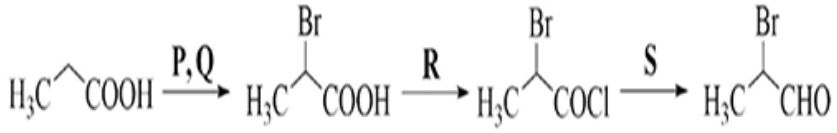


(3)



(4)

29. Consider the following sequence of reactions.



The correct reagents (P, Q, R and S) required are:

(1) P = Br_2/red phosphorous; Q = H_2O; R = $SOCl_2$; S = H_2, Pd-$BaSO_4$

(2) P = Br_2/red phosphorous; Q = H_3O^+; R = $SOCl_2$; S = $LiAlH_4$

(3) P = Br_2/NaOH; Q = H_2O; R = PCl_3; S = DIBAL-H

(4) P = PBr_3; Q = H_3O^+; R = $Cl_2/FeCl_3$; S = Pd-$BaSO_4$

---

30. Ammonium sulfate on reaction with sodium hydroxide produces compounds Q and R along with water. Catalytic oxidation of Q by atmospheric oxygen yields T (an oxide of nitrogen) and water. T reacts with oxygen to produce compound X, which dissolves in water giving Y and T. The correct statement(s) is(are):

(1) The geometry of compound X is bent.

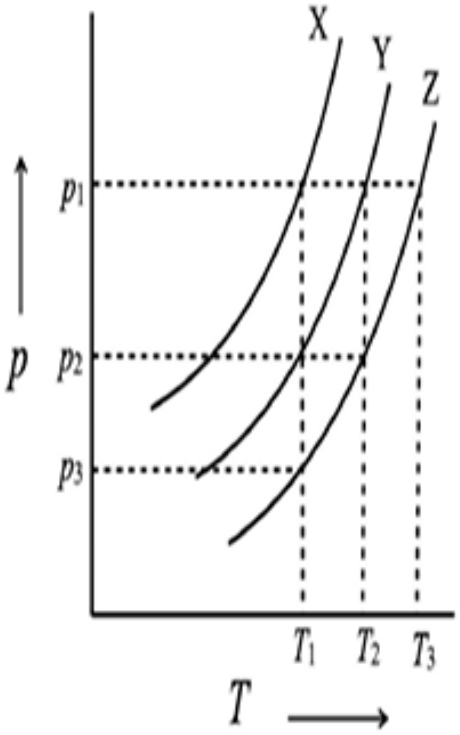
(2) Compound T on reaction with hexaaqua iron(II) complex gives brown color.

(3) The conversion of X to Y is a reduction process.

(4) Compound Y on reaction with carbon yields compound X, CO_2 and water.

---

31. Consider three liquids: water, dilute aqueous solution of glucose, and dilute aqueous solution of NaCl. The aqueous solutions of glucose and NaCl are of the same molal concentrations. The vapour pressures (p) of the three liquids are plotted (not drawn to scale) as a function of temperature (T) in the figure below.



(1) Curves X, Y, and Z correspond to pure water, glucose solution, and NaCl solution respectively.

(2) The temperatures T_1, T_2 and T_3 represent the boiling points of the solutions corresponding to the curves X, Y and Z, respectively.

(3) The pressures, p_1, p_2 and p_3 are related as $2p_2 = p_1 + p_3$.

(4) The temperatures, T_1, T_2 and T_3 are related as $3T_2 = 2T_1 + T_3$.

---

32. Consider the three electrodes Fe/Fe^{2+}, Fe^{2+}/Fe^{3+}, and Fe/Fe^{3+}, for which the standard electrode (oxidation) potentials are: E_1^0, E_2^0, and E_3^0, respectively. The standard EMF of the cell, $Fe/Fe^{2+}||Fe^{3+}/Fe$, is E_4^0. The correct expression(s) is (are):

(1) $E_3^0 = (3E_2^0 - 2E_1^0)$

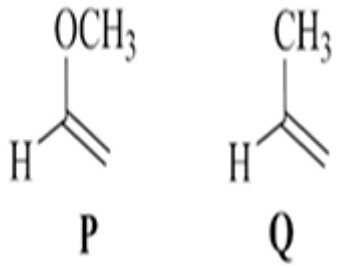
(2) $E_3^0 = (E_2^0 - E_1^0)$

(3) $E_4^0 = (2E_1^0 - 3E_2^0)$

(4) $E_4^0 = (E_1^0 - E_2^0)$

---

33. In an electrophilic addition reaction of olefins, the stability of carbocations plays a crucial role. Consider the compounds P and Q that undergo such reactions with different reagents.



P Q

With reference to the above reactions, the correct statement(s) is(are):

(1) In the HBr addition, the rate of the reaction is faster for Q than for P.

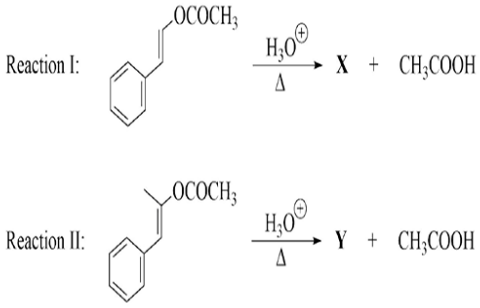
(2) HBr addition to P gives an equal mixture of enantiomers as a major product.

(3) P reacts with diborane followed by oxidation with H_2O_2/NaOH gives racemic alcohol as a major product.

(4) Reaction of P with O_3 followed by treatment with Zn/H_2O gives methyl formate and formaldehyde.

---

34. Consider the following acid hydrolysis of esters.



The correct statement(s) about X and Y is(are):

(1) Both X and Y on reaction with Lucas reagent ($ZnCl_2$ + conc. HCl) give turbid solutions.

(2) Y on reaction with Br_2/NaOH gives sodium salt of phenyl acetic acid.

(3) X forms silver mirror with ammonical silver nitrate solution.

(4) The reaction of Y with NH_2NH_2 followed by heating with KOH in ethylene glycol gives

n-propylbenzene.

Mathematics

35. Let f be the function on $\mathbb{R}$ defined by $f(x) = x^3 - 3x^2 + ax - 1$, where $a \in \mathbb{R}$. Then the set of all possible values of a for which f is strictly increasing is

(1) $[3, \infty)$

(2) $(-\infty, 3]$

(3) $[-3, 0]$

(4) $[0, 3]$

---

36. If $\frac{1}{(1+i)^{2023}} = te^{i\theta}$, where $t \in \mathbb{R}$ and $0 \leq \theta < 2\pi$, then the value of θ is:

(1) $\frac{\pi}{4}$

(2) $\frac{3\pi}{4}$

(3) $\frac{5\pi}{4}$

(4) $\frac{7\pi}{4}$

---

37. Let S be the set of all 4-digit natural numbers with the following properties:

(i) every digit of any element of S belongs to the set $\{0, 1, 3, 5, 7, 9\}$,

(ii) every element of S is divisible by 5, and

(iii) no element of S is divisible by 2.

Then the number of elements in S is

(1) 180

(2) 216

(3) 360

(4) 250

---

38. If a teacher assigns homework on the nth day, the probability that she will assign homework on the $(n+1)$th day is $\frac{1}{3}$. If she does not assign homework on the nth day, the probability that she will assign homework on the $(n+1)$th day is $\frac{2}{3}$. If she assigned homework on a Monday, then the probability that she will assign homework on the Thursday of the week is:

(1) $\frac{1}{3}$

(2) $\frac{7}{27}$

(3) $\frac{13}{27}$

(4) $\frac{2}{3}$

---

39. The value of the following sum is:

$$\sin\left(\frac{2\pi}{23}\right) + \sin\left(\frac{4\pi}{23}\right) + \cdots + \sin\left(\frac{44\pi}{23}\right)$$

(1) -1

(2) 0

(3) 1

(4) 2

---

40. Let $P = (a, b)$ be a point in the Euclidean plane, with a and b nonzero. For any point S on the x-axis, let T be the point of intersection of the line PS with the y-axis. Let M be the midpoint of the segment ST. Then the locus of M, as S varies on the x-axis, is given by:

(1) $xy = ab$

(2) $xy = \frac{ab}{4}$

(3) $xy = ay + bx$

(4) $2xy = ay + bx$

---

41. Let f be a differentiable function on $\mathbb{R}$ satisfying the conditions

(i) $f(x) = \int_0^x (f(t))^{\frac{1}{3}} dt$ for all $x \in \mathbb{R}$, and

(ii) $f(x) > 0$ for all $x > 0$.

Then the value of $f(3)$ is

(1) $2\sqrt{2}$

(2) $3\sqrt{3}$

(3) $\frac{1}{2}$

(4) $\frac{1}{3}$

---

42. Let $f(x) = \ln x - 2023x + 2023$ for all $x \in (0, \infty)$. Then the number of points at which the graph of f cuts the x axis is

(1) 0

(2) 2

(3) 3

(4) 1

---

43. Let N be the number of integers n such that:

$$(i) n = 2^a 3^b 5^c \tag{1}$$

$$\text{where} \quad a, b, c \text{ are non-negative integers} \leq 10, \tag{2}$$

$$\text{and} \quad (ii) n \text{ is neither a square nor a cube of a natural number.} \tag{3}$$

Then N is equal to:

(1) 848

(2) 849

(3) 1051

(4) 1059

---

44. Let ABC be a triangle and let a, b, and c denote the lengths of the sides BC, CA, and AB respectively. Let α and β be positive real numbers such that:

$$\alpha(\angle A) + \beta(\angle B) = (\alpha + \beta)(\angle C).$$

Then, the correct statement is:

(1) $\alpha a + \beta b = (\alpha + \beta)c$

(2) $\alpha a + \beta b = (\alpha + \beta)c$ implies $a = b$

(3) $\alpha a + \beta b > (\alpha + \beta)c$

(4) $\alpha a + \beta b = (\alpha + \beta)c$ implies $\alpha a = \beta b$

---

45. For $a, b \in \mathbb{R}$, with $a > 0$, let $N(a, b)$ denote the number of elements in the set

$$\{x \in \mathbb{R} \mid x + a \sin x = b\}.$$

Then the correct statement(s) is(are):

(1) $N(a, b) = 1$ for all a, b.

(2) There does not exist any a such that $N(a, b) = 1$ for all b.

(3) $N(a, b)$ is finite for all a, b.

(4) There exist a, b such that $N(a, b)$ is infinite.

---

46. Let N be the number of solutions of the equation

$$x_0 + 2x_1 + 2x_2 + 2x_3 + 2x_4 + x_5 = 6,$$

with $x_0, x_1, x_2, x_3, x_4, x_5$ taking non-negative integer values. Then the correct option(s) is(are):

(1) $N < 50$

(2) $50 \leq N < 100$

(3) $100 \leq N < 1000$

(4) $1000 \leq N$

---

47. For $a, b > 0$ let $F(a, b) = \int_a^b |\sin 2\pi x| dx$. Then

(1) $F(10, 11) = 2F(0, \frac{1}{2})$

(2) $F(\frac{41}{4}, \frac{43}{4}) = \frac{1}{2}F(\frac{1}{2}, 1)$

(3) $F(\frac{1}{8}, \frac{1}{4}) = F(1, 2)$

(4) $F(\frac{41}{4}, \frac{43}{4}) = \frac{2}{3}F(0, \frac{3}{4})$

---

48. Let $S = \{x, y, z\}$ and $f : S \to \mathbb{N}$ be a function. Let A be a subset of $\mathbb{N}$ such that the following conditions are satisfied: (i) if $f(x) \in A$ then $f(y) \in A$, and (ii) if $f(z) \notin A$ then $f(y) \notin A$. Then it follows that

(1) whenever $f(x) \in A$, $f(z) \in A$.

(2) whenever $f(x) \notin A$, $f(z) \notin A$.

(3) whenever $f(z) \in A$, $f(x) \in A$.

(4) whenever $f(z) \notin A$, $f(x) \notin A$.

---

49. Let a and b be non-zero vectors. Let S be the set of vectors v such that $a \times v = b$. Then:

(1) There exists a positive real number r such that $||v|| < r$ for all $v \in S$.

(2) S is non-empty if and only if $a \cdot b = 0$.

(3) S is contained in a plane.

(4) If v_1 and v_2 are in S, then there exists $\lambda \in \mathbb{R}$ such that $v_1 - v_2 = \lambda a$.

---

50. Let C_1, C_2, C_3 be three circles having the same radius r, which touch each other externally. Then:

(1) For any circle C which is touched internally by C_1 and C_2, C_3 lies within C.

(2) There is no circle C touched internally by C_1, C_2, and C_3.

(3) A circle C touched internally by C_1, C_2, and C_3 has radius $\left(1+\frac{2}{\sqrt{3}}\right)r$.

(4) The radius of any circle C touched internally by C_1 and C_2 is at least $2r$.

---

51. Let $S=\{(a,b)|a,b\in\mathbb{Z}\}$. Let R be the equivalence relation on S defined by $(a,b)R(c,d)$ if $a^2+b^2=c^2+d^2$. For $(a,b)\in S$, let $F(a,b)$ denote the equivalence class $\{(c,d)\in S|(a,b)R(c,d)\}$ of (a,b). Then, the correct statement(s) is(are):

(1) There exists $(a,b)\in S$ such that $F(a,b)$ has only one element.

(2) There exists $(a,b)\in S$ such that $F(a,b)$ has exactly 4 elements.

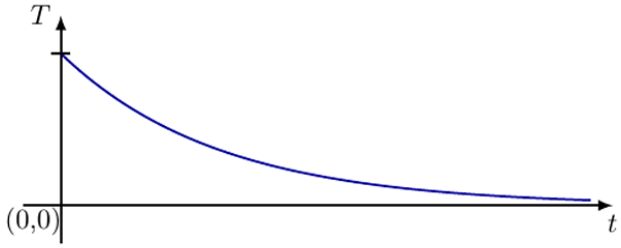
(3) There exists $(a,b)\in S$ such that $F(a,b)$ has exactly 6 elements.

(4) There exists $(a,b)\in S$ such that $F(a,b)$ has infinitely many elements.

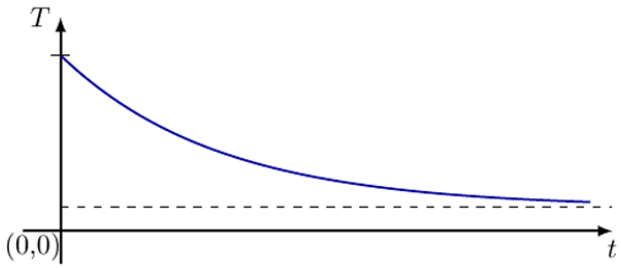
---

52. An arrow is released from a rigid bow at time $t=0$. The magnitude of the tension (T) in the bowstring as a function of time is best described by:

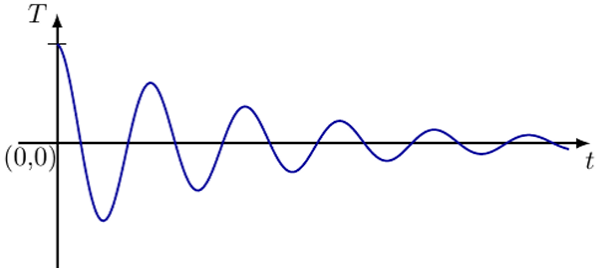
(1)

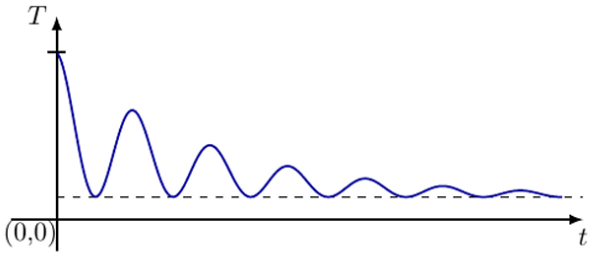


(2)



(3)





(4)

Topic - Oscillatory Motion and Energy Dissipation

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53. Two air bubbles of equal initial volume rise from the bottom of a lake to the surface. One bubble ascends and expands adiabatically while the other bubble ascends and expands isothermally. Let V_A and V_T be the final volumes of the bubbles with adiabatic and isothermal expansions, respectively. Consider an ideal gas behaviour and note that γ is the adiabatic constant. Then,

(1) $V_A > V_T$

(2) $V_A < V_T$

(3) $V_A = V_T$

(4) $V_A = \gamma V_T$

---

54. A current I flows through a regular hexagonal loop of side length l. The magnitude of the magnetic field at the centre is

(1) $\frac{\mu_0 I}{3\pi l}$

(2) $\frac{\mu_0 I}{2\sqrt{3}\pi l}$

(3) $\frac{\sqrt{3}\mu_0 I}{\pi l}$

(4) $\frac{3\mu_0 I}{\pi l}$

---

55. A planet of mass m is orbiting around a non-rotating star of mass αm ($\alpha \gg 1$) with an orbital radius r. The star ejects mass λm ($\lambda \ll 1$) radially outwards in a spherically

symmetric fashion. Neglecting any impact of ejected mass on the planet, the radius of the new circular orbit of the planet is

(1) $(1+\frac{\lambda}{\alpha})^{-1}r$

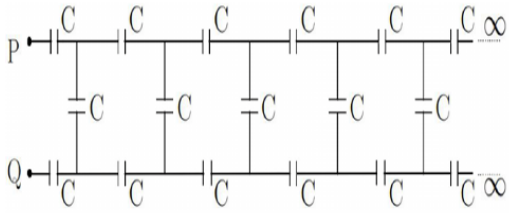
(2) $(1-\lambda\alpha)^{-1}r$

(3) $(1+\lambda\alpha)r$

(4) $(1-\frac{\lambda}{\alpha})^{-1}r$

---

56. The equivalent capacitance between P and Q for the infinite series of capacitors shown in the figure is



(1) $\frac{C}{2}(\sqrt{3}+1)$

(2) $\frac{C}{3}$

(3) $3C$

(4) $\frac{C}{2}(\sqrt{3}-1)$

---

57. The temperature and pressure at the summit of Mt. Everest is $-30°$C and 0.27×10^5N.m^{-2}, respectively. The corresponding values at sea-level are $27°$C and 1×10^5N.m^{-2}. Considering air to be an ideal gas, the ratio between the molecular number density at the summit of Mt. Everest to that at sea level is closest to

(1) 1:30

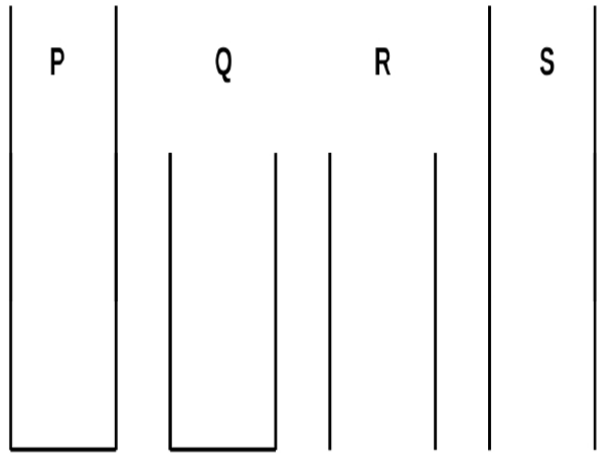
(2) 81:100

(3) 27:100

(4) 1:3

---

58. Consider the following four cylindrical tubes (P, Q, R, S) all of equal radii. The tubes Q and R are of length l. The tubes P and S are of length $1.5l$. If the fundamental frequencies are ν_P, ν_Q, ν_R and ν_S, respectively, then the correct option is:



(1) $\nu_R > \nu_S > \nu_P > \nu_Q$

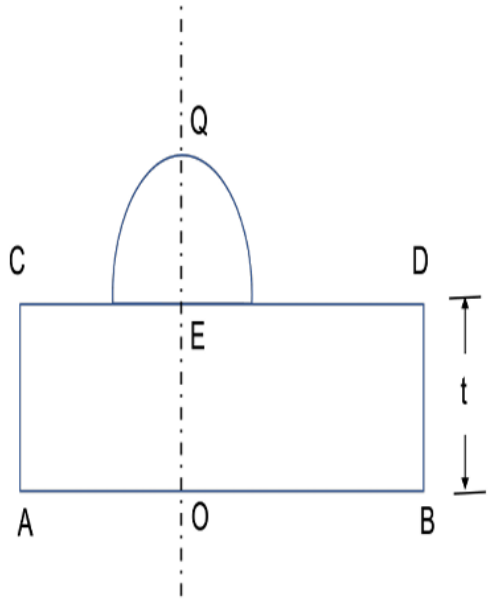
(2) $\nu_R > \nu_S > \nu_Q > \nu_P$

(3) $\nu_S > \nu_R > \nu_Q > \nu_P$

(4) $\nu_R > \nu_P > \nu_S > \nu_Q$

---

59. A transparent glass slab of thickness $t = 0.50$ cm is placed with its face AB on a horizontal table. A hemispherical water drop of radius $R = 0.33$ cm condenses on the glass slab as shown in figure. The refractive indices of the slab and the water drop are respectively 1.50 and 1.33. The image of the object at O on the face AB is viewed after refraction from the drop. Taking OEQ as the optical axis, the distance (cm) of the image from the point Q is:



(1) 1.40

(2) 0.60

(3) 0.72

(4) 2.00

---

60. A beam of monochromatic light is incident on one face of a prism of angle $75°$. If the angle of incidence is $60°$ and the refractive index of the prism is $\sqrt{3}$, then the correct option about the emergence of the beam from the opposite face is

(1) no emergence.

(2) grazing emergence.

(3) emergence with an angle of $60°$ from the normal.

(4) emergence with an angle of $30°$ from the normal.

---

61. In an isobaric process involving an ideal gas the mean distance between the molecules is quadrupled (four times). Then, the ratio of final to initial sound speeds is

(1) 1

(2) 2

(3) 8

(4) 4

---

62. Two radioactive samples X and Y have the same number of atoms initially [$N_X(t=0) = N_Y(t=0)$]. The half life $\tau_{1/2}^X$ of X is half the mean life τ_Y of Y. Then the ratio $N_Y(t)/N_X(t)$ when $t = \tau_{1/2}^X$ is close to

(1) 0.8

(2) 1.0

(3) 1.2

(4) 1.4

---

63. Consider the Bohr model of the hydrogen atom. Suppose that the charge of the proton were $+1.1e$ while the electron charge continued to be $-e$ but the masses for both remained unchanged. Then, the angular frequency of revolution ω_B of the electron would have

(1) remain unchanged.

(2) change to $\sqrt{1.1}\omega_B$.

(3) change to $1.1\omega_B$.

(4) change to $1.21\omega_B$.

---

64. A heavy disc of radius R and mass M is placed horizontally. A small coin of mass m is placed at a radial distance $R/2$ from the centre. The disc is now ($t = 0$) given a constant angular acceleration of magnitude α rad $\cdot$ s^{-2} about a vertical axis passing through its centre. If μ_s and μ_d are the coefficients of static and dynamic friction, respectively, between the coin and the rotating disc, then:

(1) at $t > 0$, the force due to static friction acts radially inwards.

(2) at $t > 0$, the magnitude of force due to static friction is always $F_s = \mu_s mg$.

(3) the coin starts sliding at time $t = \frac{1}{\alpha}\sqrt{\frac{2\mu_s g}{R}}$.

(4) the coin reaches the edge of the disc at time $t = \frac{2}{\alpha}\sqrt{\frac{(\mu_d - \mu_s)g}{R}}$.

---

65. An electromagnetic wave, travelling in vacuum, is represented by $\vec{E} = E_0 \cos(kz - \omega t)\hat{j}$ where E_0 is the amplitude of the electric field. A square loop of side a ($a \ll 2\pi/k$) is placed in its path. Then, the correct option(s) is (are)

(1) $\vec{B} = B_0 \cos(kz - \omega t)\hat{i}$ where $B_0 = -E_0/c$

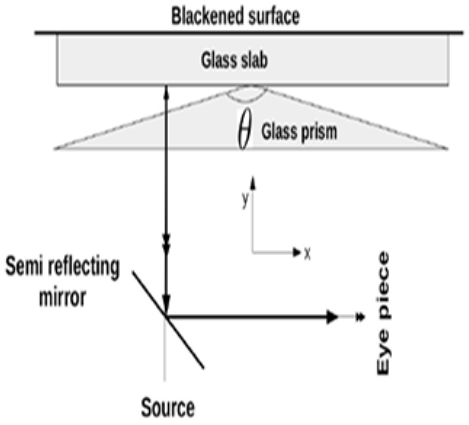
(2) The wave is travelling in the y-direction.

(3) The induced emf is zero if the loop lies in the xz plane.

(4) The induced emf is finite if the loop lies in the yz plane.

---

66. Consider the experimental set-up shown in the figure to observe the interference pattern. Note that the prism angle θ is close to π. The correct option(s) regarding this experiment is (are)



(1) fringe width will increase with increasing angle θ.

(2) fringe width will decrease with the refractive index of the lens.

(3) fringe width will increase if the glass slab is lifted along y direction.

(4) fringes will alternate between dark and bright if glass slab is lifted along y direction.

---

67. A current I is passing flowing through a thin copper slab placed on a diamond slab. The bottom surface of the diamond slab is maintained at 0°C and the remaining arrangement is thermally insulated from the surroundings. Note that diamond is an excellent thermal conductor but a poor electrical conductor. Then, the correct option(s) is(are)

(1) the steady-state temperature of the copper slab is directly proportional to the thickness of the diamond slab.

(2) the steady-state temperature of the copper slab depends upon the specific heat of the copper.

(3) if the current is supplied from a constant voltage source, the steady-state temperature of the copper slab will double when the its thickness is doubled.

(4) if the current is held constant, the steady-state temperature of the copper slab will be halved if its thickness is doubled.

---

68. The pair(s) with same dimensions is(are)

(1) Pressure and Young's modulus

(2) Power and energy flux

(3) Gravitational potential and latent heat

(4) Rotational impulse and Planck's constant
