

# CBSE Class 12 Mathematics Set 2 (65/2/2) Question Paper with Solutions

Time Allowed :3 Hour	Maximum Marks :80	Total Questions :38
----------------------	-------------------	---------------------

## General Instructions

**Read the following instructions very carefully and strictly follow them:**

1. This question paper contains 38 questions. All questions are compulsory.
2. This question paper is divided into five Sections – A, B, C, D and E.
3. In Section A, Questions no. 1 to 18 are multiple choice questions (MCQs) and questions number 19 and 20 are Assertion-Reason based questions of 1 mark each.
4. In Section B, Questions no. 21 to 25 are very short answer (VSA) type questions, carrying 2 marks each.
5. In Section C, Questions no. 26 to 31 are short answer (SA) type questions, carrying 3 marks each.
6. In Section D, Questions no. 32 to 35 are long answer (LA) type questions carrying 5 marks each.
7. In Section E, Questions no. 36 to 38 are case study based questions carrying 4 marks each.
8. There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and 2 questions in Section E.
9. Use of calculators is not allowed.

## SECTION A

This section comprises multiple choice questions (MCQs) of 1 mark each.

**1. The number of solutions of the differential equation  $\frac{dy}{dx} - y = 1$ , given that  $y(0) = 1$ , is:**

- (A) 0
- (B) 1
- (C) 2
- (D) infinitely many

**Correct Answer:** (B) 1

**Solution:**

**Step 1: Analyze the given differential equation.** The equation is:

$$\frac{dy}{dx} - y = 1.$$

This is a first-order linear differential equation written in the standard form:

$$\frac{dy}{dx} + P(x)y = Q(x),$$

where  $P(x) = -1$  and  $Q(x) = 1$ .

**Step 2: Determine the integrating factor.** The integrating factor (IF) is given by:

$$\text{IF} = e^{\int P(x) dx} = e^{\int -1 dx} = e^{-x}.$$

**Step 3: Multiply through by the integrating factor.** Multiplying the entire equation by  $e^{-x}$ :

$$e^{-x} \frac{dy}{dx} - e^{-x} y = e^{-x}.$$

The left-hand side becomes:

$$\frac{d}{dx}(y \cdot e^{-x}) = e^{-x}.$$

**Step 4: Solve the simplified equation.** Integrate both sides with respect to  $x$ :

$$y \cdot e^{-x} = \int e^{-x} dx = -e^{-x} + C,$$

where  $C$  is the constant of integration.

**Step 5: Simplify the solution.** Multiply through by  $e^x$  to isolate  $y$ :

$$y = -1 + Ce^x.$$

**Step 6: Apply the initial condition**  $y(0) = 1$ . Substitute  $x = 0$  and  $y = 1$  into the solution:

$$1 = -1 + C \cdot e^0 \quad \Rightarrow \quad 1 = -1 + C \cdot 1 \quad \Rightarrow \quad C = 2.$$

**Step 7: Final solution.** Substitute  $C = 2$  into the general solution:

$$y = -1 + 2e^x.$$

This is a unique solution that satisfies the initial condition. Thus, the number of solutions is 1.

### Quick Tip

To solve a first-order linear differential equation, always follow these steps: 1. Write the equation in standard form. 2. Find the integrating factor. 3. Use the integrating factor to simplify and solve the equation. With an initial condition, a **unique solution** is guaranteed by the existence and uniqueness theorem.

---

**2. For any two vectors  $\vec{a}$  and  $\vec{b}$ , which of the following statements is always true?**

(A)  $\vec{a} \cdot \vec{b} \geq |\vec{a}| |\vec{b}|$

(B)  $\vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}|$

(C)  $\vec{a} \cdot \vec{b} \leq |\vec{a}| |\vec{b}|$

(D)  $\vec{a} \cdot \vec{b} \geq -|\vec{a}| |\vec{b}|$

**Correct Answer:** (C)  $\vec{a} \cdot \vec{b} \leq |\vec{a}| |\vec{b}|$

**Solution:**

**Step 1: Definition of the dot product.**

The dot product between two vectors  $\vec{a}$  and  $\vec{b}$  is defined as:

$$\vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos \theta,$$

where  $\theta$  represents the angle between the vectors  $\vec{a}$  and  $\vec{b}$ .

**Step 2: Investigate the range of  $\cos \theta$ .**

Since  $\cos \theta$  lies within the range  $-1 \leq \cos \theta \leq 1$ , it follows that:

$$-|\vec{a}| |\vec{b}| \leq \vec{a} \cdot \vec{b} \leq |\vec{a}| |\vec{b}|.$$

**Step 3: Final Answer.**

Thus, the correct inequality for the dot product is:

$$\vec{a} \cdot \vec{b} \leq |\vec{a}| |\vec{b}|.$$

#### Quick Tip

For any two vectors, the absolute value of their dot product is always bounded by the product of their magnitudes, i.e.,  $|\vec{a} \cdot \vec{b}| \leq |\vec{a}| |\vec{b}|$ .

**3. The coordinates of the foot of the perpendicular drawn from the point  $(0, 1, 2)$  on the  $x$ -axis are given by:**

- (A)  $(1, 0, 0)$
- (B)  $(2, 0, 0)$
- (C)  $(\sqrt{5}, 0, 0)$
- (D)  $(0, 0, 0)$

**Correct Answer:** (D)  $(0, 0, 0)$

**Solution:**

**Step 1: Define the  $x$ -axis.**

The  $x$ -axis consists of all points where the coordinates are of the form  $(x, 0, 0)$ , where  $x$  is any real number.

**Step 2: Foot of the perpendicular.**

The foot of the perpendicular from the point  $(0, 1, 2)$  onto the  $x$ -axis represents the closest point on the  $x$ -axis. Since the perpendicular from  $(0, 1, 2)$  to the  $x$ -axis drops to  $x = 0$ , the foot of the perpendicular has the coordinates:

$$(0, 0, 0).$$

**Step 3: Conclusion.**

Thus, the foot of the perpendicular is:

$$(0, 0, 0).$$

#### Quick Tip

When finding the foot of the perpendicular from a point to an axis, keep the axis' coordinates fixed and set the other coordinates to zero.

---

**4. The common region determined by all the constraints of a linear programming problem is called:**

- (A) an unbounded region
- (B) an optimal region
- (C) a bounded region
- (D) a feasible region

**Correct Answer:** (D) a feasible region

**Solution:**

**Step 1: Understanding the feasible region.**

In a linear programming problem, the feasible region is defined as the set of all points that satisfy the system of inequalities. It represents all potential solutions that meet the given constraints.

**Step 2: Evaluate the options.**

- (A) Unbounded region: This can occur depending on the nature of the constraints, but it is not always the case. - (B) Optimal region: Refers to the best solution within the feasible region, not the region itself. - (C) Bounded region: The feasible region could be bounded, but it is not necessarily so in every scenario. - (D) Feasible region: This is the region that is formed by the constraints, which is always the common area where all inequalities overlap.

**Step 3: Conclusion.**

Thus, the correct answer is:

Feasible region

#### Quick Tip

In linear programming, the feasible region consists of all points that satisfy the system of inequalities, representing the possible solutions to the problem.

---

**5. Let  $E$  be an event of a sample space  $S$  of an experiment, then  $P(S|E)$  is:** (A)  $P(S \cap E)$

- (B)  $P(E)$
- (C) 1

(D) 0

**Correct Answer:** (C) 1

**Solution:**

**Step 1: Recall the definition of conditional probability.**

Conditional probability  $P(A|B)$  is defined as:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}, \quad \text{where } P(B) > 0.$$

**Step 2: Apply to  $P(S|E)$ .**

In this case,  $A = S$  (the sample space) and  $B = E$ . Since  $S$  represents all possible outcomes, it contains all elements of  $E$ , so:

$$P(S \cap E) = P(E).$$

Thus, the conditional probability becomes:

$$P(S|E) = \frac{P(S \cap E)}{P(E)} = \frac{P(E)}{P(E)} = 1.$$

**Step 3: Conclusion.**

Therefore, the conditional probability  $P(S|E)$  is:

$$\boxed{1}.$$

#### Quick Tip

The conditional probability of the sample space given any event is always 1, as the sample space contains all possible outcomes.

---

**6. The number of all scalar matrices of order 3, with each entry  $-1, 0, 1$ , is:**

(A) 1

(B) 3

(C) 2

(D)  $3^9$

**Correct Answer:** (B) 3

**Solution:**

**Step 1: Understanding scalar matrices.** A scalar matrix is a special type of diagonal matrix in which all diagonal elements are equal, and all off-diagonal elements are zero. For a

$3 \times 3$  scalar matrix, the structure is:

$$\begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix},$$

where  $a$  is the scalar value for all diagonal entries.

**Step 2: Determine the possible values for  $a$ .** According to the problem, the entries in the matrix can take the values  $-1, 0$ , or  $1$ . Since  $a$  must be the same for all diagonal entries, there are exactly three possibilities for  $a$ :

$$a = -1, \quad a = 0, \quad a = 1.$$

**Step 3: Identify the corresponding scalar matrices.** The three scalar matrices corresponding to these values of  $a$  are:

$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}, \quad \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

Thus, there are exactly 3 scalar matrices of order 3 with the specified values for the entries.

#### Quick Tip

A scalar matrix is always a diagonal matrix where all diagonal elements are equal. The off-diagonal elements are always zero, simplifying calculations and classification.

**7. The derivative  $\frac{d}{dx}[\cos(\log x + e^x)]$  at  $x = 1$  is:**

- (A)  $-\sin e$
- (B)  $\sin e$
- (C)  $-(1 + e) \sin e$
- (D)  $(1 + e) \sin e$

**Correct Answer:** (C)  $-(1 + e) \sin e$

**Solution:**

**Step 1: Differentiate the given function.** The given function is:

$$y = \cos(\log x + e^x).$$

Using the chain rule, the derivative is:

$$\frac{dy}{dx} = -\sin(\log x + e^x) \cdot \frac{d}{dx}(\log x + e^x).$$

**Step 2: Differentiate**  $\log x + e^x$ . The derivative of  $\log x$  is  $\frac{1}{x}$ , and the derivative of  $e^x$  is  $e^x$ .

Therefore:

$$\frac{d}{dx}(\log x + e^x) = \frac{1}{x} + e^x.$$

**Step 3: Substitute into the derivative.** Substituting  $\frac{d}{dx}(\log x + e^x)$  into the expression for  $\frac{dy}{dx}$ , we get:

$$\frac{dy}{dx} = -\sin(\log x + e^x) \cdot \left(\frac{1}{x} + e^x\right).$$

**Step 4: Evaluate the derivative at  $x = 1$ .** At  $x = 1$ :

$$\log x = \log 1 = 0, \quad e^x = e^1 = e, \quad \frac{1}{x} = \frac{1}{1} = 1.$$

Substituting these values:

$$\frac{dy}{dx} = -\sin(0 + e) \cdot (1 + e) = -(1 + e) \sin e.$$

Thus, the derivative at  $x = 1$  is  $-(1 + e) \sin e$ .

#### Quick Tip

To differentiate composite functions, use the chain rule step by step: 1. Differentiate the outer function while keeping the inner function intact. 2. Multiply by the derivative of the inner function.

**8. The degree of the differential equation  $(y'')^2 + (y')^3 = x \sin(y')$  is:**

- (A) 1
- (B) 2
- (C) 3
- (D) Not defined

**Correct Answer:** (D) Not defined

**Solution:**

**Step 1: Definition of the degree of a differential equation.**

The degree of a differential equation is determined only when the equation is a polynomial in all of its derivatives.

**Step 2: Examine the given equation.**

The provided equation is:

$$(y'')^2 + (y')^3 = x \sin(y').$$

This equation includes a non-polynomial term,  $\sin(y')$ , where  $y'$  is a derivative. Therefore, the degree of the equation cannot be defined.

**Step 3: Final Answer.**

Thus, the degree of this differential equation is:

Not defined

.

**Quick Tip**

The degree of a differential equation cannot be defined if it includes any non-polynomial terms involving derivatives.

**9. The unit vector perpendicular to both vectors  $\hat{i} + \hat{k}$  and  $\hat{i} - \hat{k}$  is:**

(A)  $2\hat{j}$

(B)  $\hat{j}$

(C)  $\frac{\hat{i}-\hat{k}}{\sqrt{2}}$

(D)  $\frac{\hat{i}+\hat{k}}{\sqrt{2}}$

**Correct Answer:** (B)  $\hat{j}$

**Solution:**

**Step 1: Cross product of two vectors.**

The cross product of vectors  $\vec{A} = \hat{i} + \hat{k}$  and  $\vec{B} = \hat{i} - \hat{k}$  produces a vector perpendicular to both:

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & 1 \\ 1 & 0 & -1 \end{vmatrix}.$$

To calculate this, expand the determinant:

$$\vec{A} \times \vec{B} = \hat{i} \begin{vmatrix} 0 & 1 \\ 0 & -1 \end{vmatrix} - \hat{j} \begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix} + \hat{k} \begin{vmatrix} 1 & 0 \\ 1 & 0 \end{vmatrix}.$$

**Step 2: Simplify the minors.**

- First minor:

$$\begin{vmatrix} 0 & 1 \\ 0 & -1 \end{vmatrix} = 0.$$

- Second minor:

$$\begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix} = (1)(-1) - (1)(1) = -1 - 1 = -2.$$

- Third minor:

$$\begin{vmatrix} 1 & 0 \\ 1 & 0 \end{vmatrix} = (1)(0) - (1)(0) = 0.$$

Thus, we have:

$$\vec{A} \times \vec{B} = -(-2)\hat{j} = 2\hat{j}.$$

**Step 3: Normalize the resulting vector.**The magnitude of  $\vec{A} \times \vec{B}$  is 2. Therefore, the unit vector in the direction of  $\vec{A} \times \vec{B}$  is:

$$\frac{\vec{A} \times \vec{B}}{|\vec{A} \times \vec{B}|} = \frac{2\hat{j}}{2} = \hat{j}.$$

**Step 4: Final Answer.**The unit vector perpendicular to both  $\vec{A}$  and  $\vec{B}$  is:

$$\boxed{\hat{j}}.$$

**Quick Tip**

The cross product of two vectors results in a perpendicular vector. To find the unit vector, divide the cross product by its magnitude.

**10. Direction ratios of a vector parallel to the line  $\frac{x-1}{2} = -y = \frac{2z+1}{6}$  are:**

- (A) 2, -1, 6
- (B) 2, 1, 6
- (C) 2, 1, 3
- (D) 2, -1, 3

**Correct Answer:** (D) 2, -1, 3**Solution:**

**Step 1: Parametrize the given line.**

The given equation of the line is:

$$\frac{x-1}{2} = -y = \frac{2z+1}{6}.$$

Let  $t$  be the parameter. From each equation, we solve for  $x$ ,  $y$ , and  $z$ :

$$\begin{aligned} \frac{x-1}{2} = t &\Rightarrow x = 2t + 1, \\ -y = t &\Rightarrow y = -t, \\ \frac{2z+1}{6} = t &\Rightarrow z = 3t - \frac{1}{2}. \end{aligned}$$

**Step 2: Identify the direction ratios.**

The direction ratios are the coefficients of  $t$  in the parametric equations:

$$\text{Direction ratios} = 2, -1, 3.$$

**Step 3: Conclusion.**

The direction ratios of the line are:

$$\boxed{2, -1, 3}.$$

**Quick Tip**

To find the direction ratios of a line, identify the coefficients of the parameter  $t$  in the parametric equations of the line.

**11. If  $F(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$  and  $[F(x)]^2 = F(kx)$ , then the value of  $k$  is:**

- (A) 1
- (B) 2
- (C) 0
- (D) -2

**Correct Answer:** (B) 2

**Solution:**

**Step 1: Calculate  $[F(x)]^2$ .**

The given matrix  $F(x)$  is:

$$F(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

To find  $[F(x)]^2$ , perform the matrix multiplication:

$$[F(x)]^2 = F(x) \cdot F(x).$$

Carrying out the multiplication:

$$\begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos(2x) & -\sin(2x) & 0 \\ \sin(2x) & \cos(2x) & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

**Step 2: Relate  $[F(x)]^2$  to  $F(kx)$ .**

The matrix  $F(kx)$  is given as:

$$F(kx) = \begin{bmatrix} \cos(kx) & -\sin(kx) & 0 \\ \sin(kx) & \cos(kx) & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

From the condition  $[F(x)]^2 = F(kx)$ , compare the corresponding entries:

$$\cos(2x) = \cos(kx) \quad \text{and} \quad \sin(2x) = \sin(kx).$$

Using the periodicity and uniqueness of trigonometric functions, we find:

$$kx = 2x \quad \Rightarrow \quad k = 2.$$

**Step 3: Final result.**

The value of  $k$  is:

$$\boxed{2}.$$

#### Quick Tip

When comparing matrix equations involving trigonometric functions, focus on the diagonal and off-diagonal elements to identify patterns or relationships. This simplifies finding constants like  $k$ .

---

**12. If a line makes an angle of  $30^\circ$  with the positive direction of  $x$ -axis,  $120^\circ$  with the positive direction of  $y$ -axis, then the angle which it makes with the positive direction of  $z$ -axis is:**

- (A)  $90^\circ$
- (B)  $120^\circ$
- (C)  $60^\circ$
- (D)  $0^\circ$

**Correct Answer:** (A)  $90^\circ$

**Solution:**

**Solution:**

**Step 1: Apply the direction cosine relation.**

The angles  $\alpha$ ,  $\beta$ , and  $\gamma$  that a line makes with the positive  $x$ -,  $y$ -, and  $z$ -axes, respectively, follow the direction cosine equation:

$$\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1.$$

Given:

$$\alpha = 30^\circ, \quad \beta = 120^\circ, \quad \gamma = ?.$$

**Step 2: Compute  $\cos \alpha$  and  $\cos \beta$ .**

We know:

$$\cos \alpha = \cos 30^\circ = \frac{\sqrt{3}}{2}, \quad \cos \beta = \cos 120^\circ = -\frac{1}{2}.$$

Thus:

$$\cos^2 \alpha = \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{3}{4}, \quad \cos^2 \beta = \left(-\frac{1}{2}\right)^2 = \frac{1}{4}.$$

**Step 3: Solve for  $\cos^2 \gamma$ .**

Substitute the known values into the direction cosine equation:

$$\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1,$$

$$\frac{3}{4} + \frac{1}{4} + \cos^2 \gamma = 1 \quad \Rightarrow \quad \cos^2 \gamma = 1 - \left(\frac{3}{4} + \frac{1}{4}\right).$$

Simplify:

$$\cos^2 \gamma = 1 - 1 = 0.$$

**Step 4: Determine  $\gamma$ .**

Since  $\cos^2 \gamma = 0$ , it follows that:

$$\cos \gamma = 0.$$

The angle  $\gamma$  for which  $\cos \gamma = 0$  is:

$$\gamma = 90^\circ.$$

**Step 5: Final Answer.**

Thus, the angle that the line makes with the positive direction of the  $z$ -axis is:

$$\boxed{90^\circ}.$$

**Quick Tip**

To find unknown angles in direction cosines, use the identity  $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$  and solve for the missing angle.

---

**13. If the sum of all the elements of a  $3 \times 3$  scalar matrix is 9, then the product of all its elements is:**

- (A) 0
- (B) 9
- (C) 27
- (D) 729

**Correct Answer:** (A) 0

**Solution:**

**Step 1: Definition of Scalar Matrix.**

A scalar matrix is a special type of diagonal matrix where every element on the diagonal is the same scalar, and all the off-diagonal elements are zeros. For a  $3 \times 3$  scalar matrix, the general structure is:

$$A = \begin{bmatrix} k & 0 & 0 \\ 0 & k & 0 \\ 0 & 0 & k \end{bmatrix},$$

where  $k$  represents the scalar value along the diagonal.

**Step 2: Summing the Elements.**

The sum of all elements in a  $3 \times 3$  scalar matrix can be calculated by adding the diagonal elements. Given that the total sum is 9, we have:

$$\text{Sum} = k + k + k + 0 + 0 + 0 + 0 + 0 + 0 = 3k.$$

From the equation, we know:

$$3k = 9 \quad \Rightarrow \quad k = 3.$$

### Step 3: Product of All Elements.

Since the matrix is a scalar matrix, the product of all its elements involves multiplying the scalar  $k$  with the off-diagonal zeros:

$$\text{Product} = k \cdot 0 \cdot 0 \cdot 0 \cdot 0 \cdot 0 \cdot 0 \cdot 0 \cdot 0 = 0.$$

### Step 4: Final Answer.

Thus, the product of all elements in the matrix is:

$$\boxed{0}.$$

#### Quick Tip

In a scalar matrix, since all off-diagonal elements are zero, the product of all elements will always be zero.

---

**14. Which of the following statements is *not* true about equivalence classes  $A_i$  ( $i = 1, 2, \dots, n$ ) formed by an equivalence relation  $R$  defined on a set  $A$ ?**

- (A)  $\bigcup_{i=1}^n A_i = A$
- (B)  $A_i \cap A_j \neq \phi, i \neq j$
- (C)  $x \in A_i$  and  $x \in A_j \implies A_i = A_j$
- (D) All elements of  $A_i$  are related to each other, for all  $i$

**Correct Answer:** (B)  $A_i \cap A_j \neq \phi, i \neq j$

**Solution: Solution:**

**Step 1: Understand the properties of equivalence classes.** Equivalence classes are subsets of a set  $A$  defined by an equivalence relation  $R$ . The important properties of equivalence classes are:

- The union of all equivalence classes equals the set  $A$ :

$$\bigcup_{i=1}^n A_i = A.$$

- Equivalence classes are mutually exclusive (disjoint), meaning:

$$A_i \cap A_j = \emptyset, \quad \text{for } i \neq j.$$

- If an element  $x$  belongs to two equivalence classes, then those two classes are identical:

$$x \in A_i \text{ and } x \in A_j \implies A_i = A_j.$$

- Every element within an equivalence class  $A_i$  is related to every other element in  $A_i$  under the equivalence relation  $R$ .

**Step 2: Evaluate the given options.**

- (A): True, because the union of all equivalence classes forms the set  $A$  by definition.
- (B): False, since equivalence classes are disjoint and cannot overlap. Their intersection is always empty for  $i \neq j$ .
- (C): True, as elements belonging to multiple equivalence classes imply those classes are identical.
- (D): True, because all elements within the same equivalence class are related under the equivalence relation.

**Step 3: Final Answer.** The statement in option (B) is *not* true.

**Quick Tip**

Equivalence classes are disjoint by definition. If two equivalence classes intersect, they are the same class. Always analyze their properties to identify valid and invalid statements.

**15. If** 
$$\begin{vmatrix} -a & b & c \\ a & -b & c \\ a & b & -c \end{vmatrix} = kabc, \text{ then the value of } k \text{ is:}$$

- (A) 0
- (B) 1
- (C) 2
- (D) 4

**Correct Answer:** (D) 4

**Solution:** We are given the following determinant and are tasked with finding its value:

$$\begin{vmatrix} -a & b & c \\ a & -b & c \\ a & b & -c \end{vmatrix} = kabc$$

Taking  $a$ ,  $b$ , and  $c$  out of the matrix from columns  $C_1$ ,  $C_2$ , and  $C_3$ , respectively:

$$abc \begin{vmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{vmatrix} = kabc$$

Dividing both sides by  $abc$ , we get:

$$\begin{vmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{vmatrix} = k$$

Using column operations  $C_2 \rightarrow C_2 + C_1$  and  $C_3 \rightarrow C_3 + C_1$ , the determinant simplifies to:

$$\begin{vmatrix} -1 & 0 & 0 \\ 1 & 0 & 2 \\ 1 & 2 & 0 \end{vmatrix} = k$$

Expanding the determinant along the first row:

$$-1(0 \times 0 - 2 \times 2) = k$$

Simplifying further:

$$-1(-4) = k$$

$$k = 4$$

$$\therefore k = 4$$

Given the problem setup, the value of  $k$  is 4, and thus the correct option is (D) 4.

### Quick Tip

When dealing with determinants, applying column operations can simplify the calculation significantly. Watch for symmetry to identify potential simplifications.

16. The number of points of discontinuity of  $f(x) = \begin{cases} |x| + 3, & \text{if } x \leq -3, \\ -2x, & \text{if } -3 < x < 3, \text{ is:} \\ 6x + 2, & \text{if } x \geq 3 \end{cases}$

(A) 0

(B) 1

(C) 2

(D) infinite

**Correct Answer:** (B) 1

**Solution:**

**Step 1: Analyze the points where discontinuity might occur.**

The given piecewise function transitions at  $x = -3$  and  $x = 3$ . These are the potential points where discontinuities could arise.

**Step 2: Verify continuity at  $x = -3$ .**

At  $x = -3$ , calculate the left-hand and right-hand limits:

$$\text{Left-hand limit (LHL)} = |x| + 3 = |-3| + 3 = 3 + 3 = 6,$$

$$\text{Right-hand limit (RHL)} = -2x = -2(-3) = 6.$$

The function value at  $x = -3$  is:

$$f(-3) = |x| + 3 = |-3| + 3 = 6.$$

Since the left-hand limit, right-hand limit, and the function's value at  $x = -3$  are all equal, the function is continuous at  $x = -3$ .

**Step 3: Verify continuity at  $x = 3$ .**

At  $x = 3$ , calculate the left-hand and right-hand limits:

$$\text{Left-hand limit (LHL)} = -2x = -2(3) = -6,$$

$$\text{Right-hand limit (RHL)} = 6x + 2 = 6(3) + 2 = 18 + 2 = 20.$$

Since the left-hand limit and right-hand limit are not equal, the function is discontinuous at  $x = 3$ .

**Step 4: Final Conclusion.**

There is exactly one point of discontinuity, which occurs at  $x = 3$ . Therefore, the total number of points of discontinuity is:

$$\boxed{1}.$$

**Quick Tip**

To determine if a function is continuous at a point, check if the left-hand limit, right-hand limit, and the function value all match. If they do not, the function is discontinuous at that point.

---

**17. The function  $f(x) = x^3 - 3x^2 + 12x - 18$  is:**

- (A) strictly decreasing on  $\mathbb{R}$
- (B) strictly increasing on  $\mathbb{R}$
- (C) neither strictly increasing nor strictly decreasing on  $\mathbb{R}$
- (D) strictly decreasing on  $(-\infty, 0)$

**Correct Answer:** (B) strictly increasing on  $\mathbb{R}$

**Solution:**

**Step 1: Calculate the derivative of  $f(x)$ .**

The derivative of the given function  $f(x)$  is:

$$f'(x) = 3x^2 - 6x + 12.$$

**Step 2: Simplify the derivative expression.**

We can factor out the common term to simplify  $f'(x)$ :

$$f'(x) = 3(x^2 - 2x + 4).$$

The quadratic expression  $x^2 - 2x + 4$  has a discriminant:

$$\Delta = (-2)^2 - 4(1)(4) = 4 - 16 = -12.$$

Since the discriminant is negative, the quadratic expression is always positive. Thus,

$f'(x) > 0$  for all real values of  $x$ .

**Step 3: Monotonicity Conclusion.**

Given that  $f'(x) > 0$  for all  $x \in \mathbb{R}$ , the function  $f(x)$  is strictly increasing on the entire real number line.

**Step 4: Final Conclusion.**

Therefore, the function  $f(x)$  is:

strictly increasing on  $\mathbb{R}$ .

**Quick Tip**

To determine if a function is strictly increasing, examine the sign of its derivative. If the derivative is positive throughout the domain, the function is strictly increasing.

---

**18. If**

$$\int_0^2 2e^{2x} dx = \int_0^a e^x dx, \text{ the value of 'a' is:}$$

- (A) 1
- (B) 2
- (C) 4
- (D)  $\frac{1}{2}$

**Correct Answer:** (C) 4

**Solution: Step 1: Solve the left-hand side integral.** Evaluate:

$$\int_0^2 2e^{2x} dx.$$

Using the substitution  $u = 2x$ ,  $du = 2dx$ , and adjusting the limits:

$$\int_0^2 2e^{2x} dx = \int_0^4 e^u du = e^u \Big|_0^4 = e^4 - 1.$$

**Step 2: Solve the right-hand side integral.** Evaluate:

$$\int_0^a e^x dx = e^x \Big|_0^a = e^a - 1.$$

**Step 3: Equate the two integrals.** Equating the left-hand side and right-hand side:

$$e^4 - 1 = e^a - 1.$$

Cancel  $-1$  from both sides:

$$e^4 = e^a.$$

Taking the natural logarithm:

$$a = 4.$$

**Final Answer:**

$$\boxed{a = 4.}$$

#### Quick Tip

Always simplify integrals step by step, and use substitution effectively for exponential functions. Ensure the limits of integration are updated correctly when changing variables.

---

**Questions number 19 and 20 are Assertion and Reason based questions. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below.**

---

**19. Assertion (A):** For two non-zero vectors  $\vec{a}$  and  $\vec{b}$ ,  $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$ .

**Reason (R):** For two non-zero vectors  $\vec{a}$  and  $\vec{b}$ ,  $\vec{a} \times \vec{b} = -\vec{b} \times \vec{a}$ .

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A). (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is *not* the correct explanation of the Assertion (A). (C) Assertion (A) is true, but Reason (R) is false. (D) Assertion (A) is false, but Reason (R) is true.

**Correct answer: Assertion (A) is true, but Reason (R) is false.**

**Solution: Step 1: Verify the Assertion (A).**

The dot product of two vectors  $\vec{a}$  and  $\vec{b}$  is defined as:

$$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos \theta,$$

where  $\theta$  is the angle between the vectors. The dot product is commutative, which means that:

$$\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}.$$

Thus, Assertion (A) is **true**.

**Step 2: Verify the Reason (R).**

The cross product of two vectors  $\vec{a}$  and  $\vec{b}$  is given by:

$$\vec{a} \times \vec{b} = |\vec{a}||\vec{b}| \sin \theta \hat{n},$$

where  $\hat{n}$  is a unit vector perpendicular to both  $\vec{a}$  and  $\vec{b}$ . The cross product is anti-commutative, which means:

$$\vec{b} \times \vec{a} = -(\vec{a} \times \vec{b}).$$

This statement is valid for the cross product, but it does not relate to the commutative property of the dot product. Hence, Reason (R) is **false** when used as an explanation for (A).

**Step 3: Compare (A) and (R).**

The Assertion (A) is correct because the dot product is commutative. However, Reason (R) incorrectly discusses the anti-commutative property of the cross product, which is unrelated to the Assertion. Therefore:

**A is true, but R is false.**

**Quick Tip**

Remember, the dot product is commutative, while the cross product is anti-commutative. These are distinct properties, so verify each separately when comparing.

---

**20. Assertion (A):** For any symmetric matrix  $A$ ,  $B'AB$  is a skew-symmetric matrix.

**Reason (R):** A square matrix  $P$  is skew-symmetric if  $P' = -P$ .

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A). (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is *not* the correct explanation of the Assertion (A). (C) Assertion (A) is true, but Reason (R) is false. (D) Assertion (A) is false, but Reason (R) is true.

**Correct answer: Assertion (A) is false, but (R) is true.**

**Solution:**

**Step 1: Understanding skew-symmetric matrices.**

A matrix  $P$  is called skew-symmetric if its transpose is equal to its negative, i.e.,  $P' = -P$ .

This property is correctly stated in the Reason (R), meaning that  $P$  must satisfy the condition  $P' = -P$ . Therefore, the Reason (R) is true.

**Step 2: Analyze  $B'AB$  for symmetry.**

Let's assume that  $A$  is a symmetric matrix, so we have  $A' = A$ . Now we investigate whether  $P = B'AB$  is skew-symmetric. To do this, we calculate the transpose of  $P$ :

$$P' = (B'AB)' = B'(A')B = B'AB.$$

Since  $A' = A$ , we get:

$$P' = B'AB = P.$$

This shows that  $P' = P$ , meaning that  $P$  is symmetric, not skew-symmetric.

**Step 3: Conclusion.**

Since  $P = B'AB$  is symmetric and not skew-symmetric, the Assertion (A) is false. However, the Reason (R) correctly describes the property of skew-symmetric matrices, so it is true.

Hence:

(A) is false, but (R) is true.

#### Quick Tip

To determine if a matrix is symmetric or skew-symmetric, check the transpose property. A matrix is skew-symmetric if its transpose equals its negative, and symmetric if it equals its transpose.

---

## SECTION B

This section comprises very short answer (VSA) type questions of 2 marks each.

**21. If  $M$  and  $m$  denote the local maximum and local minimum values of the function  $f(x) = x + \frac{1}{x}$  ( $x \neq 0$ ) respectively, find the value of  $M - m$ .**

**Solution:**

**Step 1: Define the function and differentiate.**

The given function is:

$$f(x) = x + \frac{1}{x}.$$

To find critical points, we differentiate  $f(x)$ :

$$f'(x) = 1 - \frac{1}{x^2}.$$

**Step 2: Find the critical points.**

Set the derivative equal to zero to solve for critical points:

$$f'(x) = 0 \quad \Rightarrow \quad 1 - \frac{1}{x^2} = 0 \quad \Rightarrow \quad \frac{1}{x^2} = 1 \quad \Rightarrow \quad x^2 = 1.$$

This gives two critical points:

$$x = 1 \quad \text{or} \quad x = -1.$$

**Step 3: Classify the critical points.**

To determine the nature of these critical points, we compute the second derivative of  $f(x)$ :

$$f''(x) = \frac{2}{x^3}.$$

- At  $x = 1$ :

$$f''(1) = \frac{2}{1^3} = 2 \quad (\text{positive, so } x = 1 \text{ is a local minimum}).$$

- At  $x = -1$ :

$$f''(-1) = \frac{2}{(-1)^3} = -2 \quad (\text{negative, so } x = -1 \text{ is a local maximum}).$$

**Step 4: Evaluate the function at the critical points.**

- At  $x = 1$ :

$$f(1) = 1 + \frac{1}{1} = 2 \quad (\text{local minimum } m).$$

- At  $x = -1$ :

$$f(-1) = -1 + \frac{1}{-1} = -2 \quad (\text{local maximum } M).$$

**Step 5: Calculate  $M - m$ .**

Now, calculate the difference between the maximum and minimum values:

$$M - m = -2 - 2 = -4.$$

**Conclusion:**

Thus, the value of  $M - m$  is:

$$\boxed{-4}.$$

### Quick Tip

To find local maxima and minima, first use the first derivative to locate critical points, and then use the second derivative test to classify them.

## 22. Evaluate:

$$\int_0^a \frac{x^2}{x^6 + a^6} dx$$

### Solution:

This integral can be evaluated using a substitution method.

**Step 1: Apply substitution.** Let  $u = x^3$ , which gives  $du = 3x^2 dx$ . Adjust the limits of integration: When  $x = 0$ ,  $u = 0$ ; and when  $x = a$ ,  $u = a^3$ .

The integral becomes:

$$\int_0^{a^3} \frac{1}{u^2 + a^6} du.$$

**Step 2: Use the standard integral formula.** For the integral  $\frac{1}{u^2 + b^2}$ , the result is:

$$\int \frac{1}{u^2 + b^2} du = \frac{1}{b} \tan^{-1} \left( \frac{u}{b} \right).$$

Here,  $b = a^3$ , so:

$$\int_0^{a^3} \frac{1}{u^2 + a^6} du = \frac{1}{a^3} \tan^{-1} \left( \frac{u}{a^3} \right) \Big|_0^{a^3}.$$

**Step 3: Evaluate the limits.** Substitute the limits into the expression:

$$\frac{1}{a^3} \left[ \tan^{-1} \left( \frac{a^3}{a^3} \right) - \tan^{-1} (0) \right].$$

**Step 4: Simplify.** Simplify the result:

$$= \frac{1}{a^3} [\tan^{-1}(1) - 0] = \frac{1}{a^3} \cdot \frac{\pi}{4}.$$

**Final Answer:**

$$\boxed{\frac{\pi}{4a^3}}.$$

### Quick Tip

For integrals involving terms like  $x^6 + a^6$ , substitution  $u = x^3$  often simplifies the problem. Remember to use the standard formula  $\frac{1}{u^2+b^2} = \frac{1}{b} \tan^{-1} \left( \frac{u}{b} \right)$  for efficient calculation.

**23. Show that  $f(x) = e^x - e^{-x} + x - \tan^{-1} x$  is strictly increasing in its domain.**

**Solution:**

**Step 1: Differentiate the given function  $f(x)$ .**

The given function is:

$$f(x) = e^x - e^{-x} + x - \tan^{-1} x.$$

Now, differentiate each term individually:

$$f'(x) = \frac{d}{dx}(e^x) - \frac{d}{dx}(e^{-x}) + \frac{d}{dx}(x) - \frac{d}{dx}(\tan^{-1} x).$$

Thus, we obtain:

$$f'(x) = e^x + e^{-x} + 1 - \frac{1}{1+x^2}.$$

**Step 2: Prove that  $f'(x) > 0$  for all  $x$ .**

Let's analyze the terms in  $f'(x)$ :

$$f'(x) = e^x + e^{-x} + \frac{x^2}{1+x^2}.$$

- The terms  $e^x$  and  $e^{-x}$  are always positive for any real value of  $x$ , as the exponential function never takes negative values. - Similarly,  $\frac{x^2}{1+x^2}$  is always positive or zero, because  $x^2 \geq 0$  for all real  $x$ .

Therefore, since all terms are positive:

$$f'(x) > 0 \quad \text{for all } x.$$

**Step 3: Conclusion.**

Since  $f'(x) > 0$  for every  $x$ , the function  $f(x)$  is strictly increasing throughout its domain:

strictly increasing.

### Quick Tip

To demonstrate that a function is strictly increasing or decreasing, examine the sign of its derivative. If the derivative is always positive, the function is increasing.

**24(a). Find the value of**  $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) + \cot^{-1}\left(\frac{1}{\sqrt{3}}\right) + \tan^{-1}\left[\sin\left(-\frac{\pi}{2}\right)\right]$ .

**Solution:**

**Step 1: Simplify each term individually.**

- First Term:

$$\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) = -\tan^{-1}\left(\frac{1}{\sqrt{3}}\right).$$

Since  $\tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6}$ , we have:

$$-\tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = -\frac{\pi}{6}.$$

- Second Term:

$$\cot^{-1}\left(\frac{1}{\sqrt{3}}\right) = \tan^{-1}(\sqrt{3}).$$

We know that  $\tan^{-1}(\sqrt{3}) = \frac{\pi}{3}$ , so the second term becomes:

$$\cot^{-1}\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{3}.$$

- Third Term:

$$\tan^{-1}\left[\sin\left(-\frac{\pi}{2}\right)\right] = \tan^{-1}(-1).$$

Since  $\tan^{-1}(-1) = -\frac{\pi}{4}$ , the third term is:

$$\tan^{-1}(-1) = -\frac{\pi}{4}.$$

**Step 2: Combine the simplified terms.**

Now, we add the three terms:

$$-\frac{\pi}{6} + \frac{\pi}{3} - \frac{\pi}{4}.$$

To add these, we first convert each term to have a common denominator. The common denominator is 12:

$$-\frac{\pi}{6} = -\frac{2\pi}{12}, \quad \frac{\pi}{3} = \frac{4\pi}{12}, \quad -\frac{\pi}{4} = -\frac{3\pi}{12}.$$

Thus, the sum is:

$$-\frac{2\pi}{12} + \frac{4\pi}{12} - \frac{3\pi}{12} = -\frac{\pi}{12}.$$

**Step 3: Conclusion.**

The value of the expression is:

$$\boxed{-\frac{\pi}{12}}.$$

**Quick Tip**

To simplify inverse trigonometric expressions, convert each term to a common denominator when adding or subtracting fractions.

---

OR

**24(b). Find the domain of the function  $f(x) = \sin^{-1}(x^2 - 4)$ . Also, find its range.**

**Solution:**

**Step 1: Domain of the inverse sine function.**

The inverse sine function  $\sin^{-1}(y)$  is defined for  $-1 \leq y \leq 1$ . For the function  $f(x) = \sin^{-1}(x^2 - 4)$ , the argument  $x^2 - 4$  must fall within this range:

$$-1 \leq x^2 - 4 \leq 1.$$

**Step 2: Solve the inequality.** Rewrite the inequality:

$$-1 + 4 \leq x^2 \leq 1 + 4 \quad \Rightarrow \quad 3 \leq x^2 \leq 5.$$

Now, take the square root on both sides:

$$\sqrt{3} \leq |x| \leq \sqrt{5}.$$

Thus, we get the possible values for  $x$ :

$$x \in [-\sqrt{5}, -\sqrt{3}] \cup [\sqrt{3}, \sqrt{5}].$$

**Step 3: Domain of  $f(x)$ .**

The domain of  $f(x) = \sin^{-1}(x^2 - 4)$  is therefore:

$$\boxed{x \in [-\sqrt{5}, -\sqrt{3}] \cup [\sqrt{3}, \sqrt{5}]}.$$

**Step 4: Range of  $f(x)$ .**

The range of the inverse sine function is  $[-\frac{\pi}{2}, \frac{\pi}{2}]$ . Since  $x^2 - 4$  is always between  $-1$  and  $1$  for the domain found above, the range of  $f(x)$  is:

$$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right].$$

**Quick Tip**

To determine the domain and range of composite functions, solve inequalities for the input range of the inner function, and apply the output range of the outer function.

**25. (a) Check the differentiability of  $f(x) = |\cos x|$  at  $x = \frac{\pi}{2}$ .****Solution:**

**Step 1: Understand the behavior of  $f(x)$  near  $x = \frac{\pi}{2}$ .** The function  $f(x) = |\cos x|$  can be expressed as:

$$f(x) = \begin{cases} \cos x, & \text{if } \cos x \geq 0, \\ -\cos x, & \text{if } \cos x < 0. \end{cases}$$

At  $x = \frac{\pi}{2}$ ,  $\cos\left(\frac{\pi}{2}\right) = 0$ . To check differentiability, evaluate the left-hand derivative (LHD) and right-hand derivative (RHD).

**Step 2: Calculate the left-hand derivative (LHD).** For  $x < \frac{\pi}{2}$ ,  $\cos x \geq 0$ , so:

$$f(x) = \cos x, \quad \text{and} \quad f'(x) = -\sin x.$$

At  $x = \frac{\pi}{2}$ , the left-hand derivative is:

$$\lim_{h \rightarrow 0^-} \frac{f\left(\frac{\pi}{2} + h\right) - f\left(\frac{\pi}{2}\right)}{h} = -\sin\left(\frac{\pi}{2}\right) = -1.$$

**Step 3: Calculate the right-hand derivative (RHD).** For  $x > \frac{\pi}{2}$ ,  $\cos x < 0$ , so:

$$f(x) = -\cos x, \quad \text{and} \quad f'(x) = \sin x.$$

At  $x = \frac{\pi}{2}$ , the right-hand derivative is:

$$\lim_{h \rightarrow 0^+} \frac{f\left(\frac{\pi}{2} + h\right) - f\left(\frac{\pi}{2}\right)}{h} = \sin\left(\frac{\pi}{2}\right) = 1.$$

**Step 4: Conclusion.** Since the left-hand and right-hand derivatives are not equal ( $-1 \neq 1$ ), the function  $f(x) = |\cos x|$  is not differentiable at  $x = \frac{\pi}{2}$ .

**Final Answer:**

The function  $f(x) = |\cos x|$  is not differentiable at  $x = \frac{\pi}{2}$ .

**Quick Tip**

To check differentiability at a point, compute both the left-hand and right-hand derivatives. If these derivatives are not equal, the function is not differentiable at that point. For absolute value functions, pay attention to the behavior around critical points.

OR

**25(b) If  $y = A \sin 2x + B \cos 2x$  and**

$$\frac{d^2y}{dx^2} - ky = 0, \text{ find the value of } k.$$

**Solution: Step 1: Differentiate  $y = A \sin 2x + B \cos 2x$  twice.** The first derivative of  $y$  is:

$$\frac{dy}{dx} = 2A \cos 2x - 2B \sin 2x.$$

The second derivative of  $y$  is:

$$\frac{d^2y}{dx^2} = -4A \sin 2x - 4B \cos 2x.$$

**Step 2: Substitute  $\frac{d^2y}{dx^2}$  and  $y$  into the given equation.** The given equation is:

$$\frac{d^2y}{dx^2} - ky = 0.$$

Substitute  $\frac{d^2y}{dx^2} = -4A \sin 2x - 4B \cos 2x$  and  $y = A \sin 2x + B \cos 2x$ :

$$-4A \sin 2x - 4B \cos 2x - k(A \sin 2x + B \cos 2x) = 0.$$

**Step 3: Group and simplify terms.** Combine terms involving  $\sin 2x$  and  $\cos 2x$ :

$$(-4A - kA) \sin 2x + (-4B - kB) \cos 2x = 0.$$

For this equation to hold for all  $x$ , the coefficients of  $\sin 2x$  and  $\cos 2x$  must be zero:

$$-4A - kA = 0 \quad \text{and} \quad -4B - kB = 0.$$

**Step 4: Solve for  $k$ .** From the first equation:

$$A(-4 - k) = 0 \Rightarrow k = -4 \quad (\text{since } A \neq 0).$$

From the second equation:

$$B(-4 - k) = 0 \Rightarrow k = -4 \quad (\text{since } B \neq 0).$$

**Final Answer:**

$$\boxed{k = -4.}$$

### Quick Tip

When solving for constants in differential equations, substitute the function and its derivatives into the equation, then equate coefficients to determine the unknowns.

---

## SECTION C

This section comprises short answer (SA) type questions of 3 marks each.

**26(a). Find the particular solution of the differential equation given by:**

$$2xy + y^2 - 2x^2 \frac{dy}{dx} = 0; \quad y = 2, \text{ when } x = 1.$$

**Solution:**

**Step 1: Rewrite the given equation.**

The given differential equation is:

$$2xy + y^2 - 2x^2 \frac{dy}{dx} = 0.$$

Rearrange this equation to express  $\frac{dy}{dx}$  explicitly:

$$\frac{dy}{dx} = \frac{2xy + y^2}{2x^2}.$$

**Step 2: Separate the variables.**

To separate variables, we first divide through by  $x^2$  and  $y$  (assuming  $y \neq 0$ ):

$$\frac{1}{y} dy = \frac{2x + y}{2x^3} dx.$$

**Step 3: Simplify the right-hand side.**

We now rewrite the right-hand side of the equation:

$$\frac{2x + y}{2x^3} = \frac{2x}{2x^3} + \frac{y}{2x^3} = \frac{1}{x^2} + \frac{y}{2x^3}.$$

Thus, the equation becomes:

$$\frac{1}{y} dy = \left( \frac{1}{x^2} + \frac{y}{2x^3} \right) dx.$$

**Step 4: Integrate both sides.**

We integrate the left-hand side:

$$\int \frac{1}{y} dy = \ln |y|.$$

For the right-hand side, we integrate term by term: - The first term is:

$$\int \frac{1}{x^2} dx = -\frac{1}{x}.$$

- The second term is:

$$\int \frac{y}{2x^3} dx = \frac{y}{2} \int x^{-3} dx = \frac{y}{2} \cdot \frac{1}{2x^2} = \frac{y}{4x^2}.$$

Thus, the equation becomes:

$$\ln |y| = -\frac{1}{x} + \frac{y}{4x^2} + C,$$

where  $C$  is the constant of integration.

**Step 5: Apply the initial condition.**

Substitute the given initial condition  $x = 1$  and  $y = 2$  into the equation:

$$\ln |2| = -\frac{1}{1} + \frac{2}{4 \times 1^2} + C.$$

Simplify:

$$\ln 2 = -1 + \frac{1}{2} + C \Rightarrow \ln 2 = -\frac{1}{2} + C \Rightarrow C = \ln 2 + \frac{1}{2}.$$

**Step 6: Substitute the value of  $C$ .**

Substitute the value of  $C$  back into the general solution:

$$\ln |y| = -\frac{1}{x} + \frac{y}{4x^2} + \ln 2 + \frac{1}{2}.$$

**Conclusion:**

The particular solution to the differential equation is:

$$\boxed{\ln |y| = -\frac{1}{x} + \frac{y}{4x^2} + \ln 2 + \frac{1}{2}}.$$

### Quick Tip

When solving differential equations with initial conditions, don't forget to substitute the initial values to find the constant of integration after solving for the general solution.

OR

**26(b). Find the general solution of the differential equation:**

$$y dx = (x + 2y^2) dy.$$

**Solution:** We are given a differential equation. Let us solve it step by step.

**Step 1: Rewrite the equation.**

The given differential equation is:

$$\frac{dx}{dy} - \frac{x}{y} = 2y$$

This is a first-order linear differential equation of the form:

$$\frac{dx}{dy} + P(y)x = Q(y),$$

where  $P(y) = -\frac{1}{y}$  and  $Q(y) = 2y$ .

**Step 2: Find the integrating factor (IF).**

The integrating factor for a linear differential equation is given by:

$$\text{Integrating Factor} = e^{\int P(y) dy}.$$

Here,  $P(y) = -\frac{1}{y}$ , so:

$$\text{Integrating Factor} = e^{\int -\frac{1}{y} dy} = e^{-\ln|y|} = \frac{1}{y}.$$

**Step 3: Solve the equation.**

Multiply the entire differential equation by the integrating factor  $\frac{1}{y}$ :

$$\frac{1}{y} \frac{dx}{dy} - \frac{x}{y^2} = \frac{2y}{y}.$$

This simplifies to:

$$\frac{d}{dy} \left( \frac{x}{y} \right) = 2.$$

Now integrate both sides with respect to  $y$ :

$$\frac{x}{y} = \int 2 \, dy.$$

**Step 4: Integrate.**

The integral of  $2 \, dy$  is:

$$\frac{x}{y} = 2y + C,$$

where  $C$  is the constant of integration.

**Step 5: Solve for  $x$ .**

Multiply through by  $y$  to isolate  $x$ :

$$x = 2y^2 + Cy.$$

**Final Answer:**

The solution to the given differential equation is:

$$x = 2y^2 + Cy.$$

#### Quick Tip

For equations involving logarithms, utilize properties such as  $\ln a - \ln b = \ln \left( \frac{a}{b} \right)$  to simplify and solve effectively.

---

**27. If vectors  $\vec{a}$ ,  $\vec{b}$  and  $2\vec{a} + 3\vec{b}$  are unit vectors, then find the angle between  $\vec{a}$  and  $\vec{b}$ .**

**Solution:**

Given that  $\vec{a}$  and  $\vec{b}$  are unit vectors:

$$|\vec{a}| = 1 \quad \text{and} \quad |\vec{b}| = 1.$$

It is also given that  $2\vec{a} + 3\vec{b}$  is a unit vector, which implies:

$$|2\vec{a} + 3\vec{b}| = 1.$$

Using the formula for the magnitude of a vector:

$$|2\vec{a} + 3\vec{b}|^2 = (2\vec{a} + 3\vec{b}) \cdot (2\vec{a} + 3\vec{b}).$$

Expanding the dot product:

$$|2\vec{a} + 3\vec{b}|^2 = 4|\vec{a}|^2 + 12\vec{a} \cdot \vec{b} + 9|\vec{b}|^2.$$

Substituting  $|\vec{a}| = 1$  and  $|\vec{b}| = 1$ :

$$|2\vec{a} + 3\vec{b}|^2 = 4 + 12\vec{a} \cdot \vec{b} + 9 = 13 + 12\vec{a} \cdot \vec{b}.$$

Since  $|2\vec{a} + 3\vec{b}| = 1$ , we equate:

$$13 + 12\vec{a} \cdot \vec{b} = 1.$$

Solving for  $\vec{a} \cdot \vec{b}$ :

$$12\vec{a} \cdot \vec{b} = -12 \quad \Rightarrow \quad \vec{a} \cdot \vec{b} = -1.$$

The dot product  $\vec{a} \cdot \vec{b}$  is related to the angle  $\theta$  between  $\vec{a}$  and  $\vec{b}$  as:

$$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos \theta = \cos \theta.$$

Substituting  $\vec{a} \cdot \vec{b} = -1$ :

$$\cos \theta = -1 \quad \Rightarrow \quad \theta = \pi.$$

**Final Answer:** The angle between  $\vec{a}$  and  $\vec{b}$  is:

$$\theta = \pi \text{ radians.}$$

#### Quick Tip

For problems involving unit vectors and dot products, remember that the dot product gives  $\cos \theta$ . When working with combined vectors, always expand the magnitude using the dot product formula and equate terms systematically.

---

**28. A pair of dice is thrown simultaneously. If  $X$  denotes the absolute difference of the numbers appearing on top of the dice, then find the probability distribution of  $X$ .**

**Solution:**

**Step 1: Define the random variable  $X$ .**

Let  $X$  be the absolute difference between the numbers shown on two dice. The possible values of  $X$  are:

$$X = 0, 1, 2, 3, 4, 5.$$

**Step 2: Count favorable outcomes for each  $X$ .**

The total number of outcomes when two dice are rolled is  $6 \times 6 = 36$ . We now count the number of outcomes corresponding to each value of  $X$ .

- For  $X = 0$ : The numbers on the two dice must be the same:

$$(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6).$$

There are 6 outcomes.

- For  $X = 1$ : The numbers on the two dice differ by 1. Possible outcomes are:

$$(1, 2), (2, 1), (2, 3), (3, 2), (3, 4), (4, 3), (4, 5), (5, 4), (5, 6), (6, 5).$$

This gives 10 outcomes.

- For  $X = 2$ : The numbers differ by 2. Possible outcomes are:

$$(1, 3), (3, 1), (2, 4), (4, 2), (3, 5), (5, 3), (4, 6), (6, 4).$$

This gives 8 outcomes.

- For  $X = 3$ : The numbers differ by 3. Possible outcomes are:

$$(1, 4), (4, 1), (2, 5), (5, 2), (3, 6), (6, 3).$$

This gives 6 outcomes.

- For  $X = 4$ : The numbers differ by 4. Possible outcomes are:

$$(1, 5), (5, 1), (2, 6), (6, 2).$$

This gives 4 outcomes.

- For  $X = 5$ : The numbers differ by 5. Possible outcomes are:

$$(1, 6), (6, 1).$$

This gives 2 outcomes.

**Step 3: Calculate probabilities.**

The probabilities are calculated as:

$$P(X = k) = \frac{\text{Number of favorable outcomes for } X = k}{36}.$$

Thus:

$$P(X = 0) = \frac{6}{36}, \quad P(X = 1) = \frac{10}{36}, \quad P(X = 2) = \frac{8}{36}, \quad P(X = 3) = \frac{6}{36}, \quad P(X = 4) = \frac{4}{36},$$
$$P(X = 5) = \frac{2}{36}.$$

**Step 4: Write the probability distribution.**

The probability distribution of  $X$  is:

$$P(X = 0) = \frac{1}{6}, \quad P(X = 1) = \frac{5}{18}, \quad P(X = 2) = \frac{2}{9}, \quad P(X = 3) = \frac{1}{6}, \quad P(X = 4) = \frac{1}{9},$$
$$P(X = 5) = \frac{1}{18}.$$

**Step 5: Verify the total probability.**

Add all probabilities:

$$P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4) + P(X = 5) = \frac{6}{36} + \frac{10}{36} + \frac{8}{36} + \frac{6}{36} + \frac{4}{36} + \frac{2}{36} = 1.$$

The total probability is 1, so the distribution is valid.

**Conclusion:**

The probability distribution of  $X$  is:

$X$	$P(X)$
0	$\frac{1}{6}$
1	$\frac{5}{18}$
2	$\frac{2}{9}$
3	$\frac{1}{6}$
4	$\frac{1}{9}$
5	$\frac{1}{18}$

**Quick Tip**

For probability distributions involving dice, systematically count the outcomes for each value and ensure the total probability sums to 1.

---

**29. Find:**

$$\int x^2 \sin^{-1}(x^{3/2}) dx.$$

**Solution:** We are tasked with solving the given integral step by step.

**Step 1: Substitution**

Let:

$$x^{3/2} = t \quad \Rightarrow \quad \frac{3}{2}x^{1/2} dx = dt.$$

This simplifies the differential:

$$x^{1/2} dx = \frac{2}{3} dt.$$

**Step 2: Transform the integral**

The given integral becomes:

$$\frac{2}{3} \int t \sin^{-1} t dt.$$

**Step 3: Apply integration by parts**

We use the integration by parts formula:

$$\int u dv = uv - \int v du.$$

Here, choose  $u = \sin^{-1} t$  and  $dv = t dt$ . Then:

$$du = \frac{1}{\sqrt{1-t^2}} dt, \quad v = \frac{t^2}{2}.$$

Substituting these into the formula:

$$\frac{2}{3} \int t \sin^{-1} t dt = \frac{2}{3} \left[ \sin^{-1} t \cdot \frac{t^2}{2} - \int \frac{t^2}{2} \cdot \frac{1}{\sqrt{1-t^2}} dt \right].$$

**Step 4: Simplify the integral**

This expands to:

$$\frac{2}{3} \int t \sin^{-1} t dt = \frac{2}{3} \left[ \frac{t^2 \sin^{-1} t}{2} - \frac{1}{2} \int \frac{t^2}{\sqrt{1-t^2}} dt \right].$$

For the second term, rewrite  $t^2 = (1 - (1 - t^2))$ , so:

$$\int \frac{t^2}{\sqrt{1-t^2}} dt = \int \frac{1 - (1 - t^2)}{\sqrt{1-t^2}} dt = \int \sqrt{1-t^2} dt - \int \frac{1}{\sqrt{1-t^2}} dt.$$

**Step 5: Solve the individual integrals**

1. For  $\int \sqrt{1-t^2} dt$ , the solution is standard:

$$\int \sqrt{1-t^2} dt = \frac{1}{2} \left[ t\sqrt{1-t^2} + \sin^{-1} t \right].$$

2. For  $\int \frac{1}{\sqrt{1-t^2}} dt$ , the solution is:

$$\int \frac{1}{\sqrt{1-t^2}} dt = \sin^{-1} t.$$

Substitute these results back into the integral:

$$\int \frac{t^2}{\sqrt{1-t^2}} dt = \frac{1}{2} \left[ t\sqrt{1-t^2} + \sin^{-1} t \right] - \sin^{-1} t.$$

Simplify:

$$\int \frac{t^2}{\sqrt{1-t^2}} dt = \frac{1}{2} t\sqrt{1-t^2} + \frac{1}{2} \sin^{-1} t - \sin^{-1} t.$$

$$\int \frac{t^2}{\sqrt{1-t^2}} dt = \frac{1}{2} t\sqrt{1-t^2} - \frac{1}{2} \sin^{-1} t.$$

**Step 6: Substitute back into the expression**

The integral becomes:

$$\frac{2}{3} \int t \sin^{-1} t dt = \frac{2}{3} \left[ \frac{t^2 \sin^{-1} t}{2} - \frac{1}{2} \left( \frac{1}{2} t\sqrt{1-t^2} - \frac{1}{2} \sin^{-1} t \right) \right].$$

Simplify the terms:

$$\frac{2}{3} \int t \sin^{-1} t dt = \frac{2}{3} \left[ \frac{t^2 \sin^{-1} t}{2} - \frac{1}{4} t\sqrt{1-t^2} - \frac{1}{4} \sin^{-1} t \right].$$

**Step 7: Back-substitute for  $t$**

Recall that  $t = x^{\frac{3}{2}}$ . Substituting back:

$$\frac{2}{3} \int t \sin^{-1} t dt = \frac{1}{3} \left[ x^3 \sin^{-1}(x^{\frac{3}{2}}) + \frac{x^{\frac{3}{2}}}{2} \sqrt{1-x^3} - \frac{1}{2} \sin^{-1}(x^{\frac{3}{2}}) \right] + C.$$

**Final Answer:**

$$\frac{1}{3} \left[ x^3 \sin^{-1}(x^{\frac{3}{2}}) + \frac{x^{\frac{3}{2}}}{2} \sqrt{1-x^3} - \frac{1}{2} \sin^{-1}(x^{\frac{3}{2}}) \right] + C.$$

### Quick Tip

For integrals involving inverse trigonometric functions, substitute to simplify the expression and use trigonometric identities to reduce complexity.

**30. (a) If  $x^{30}y^{20} = (x + y)^{50}$ , prove that**

$$\frac{dy}{dx} = \frac{y}{x}.$$

**Solution:**

We start with the given equation:

$$x^{30}y^{20} = (x + y)^{50}.$$

We will differentiate both sides with respect to  $x$  using implicit differentiation.

**Step 1: Differentiate the left-hand side.** The left-hand side  $x^{30}y^{20}$  involves a product, so we apply the product rule:

$$\frac{d}{dx}(x^{30}y^{20}) = \frac{d}{dx}(x^{30})y^{20} + x^{30}\frac{d}{dx}(y^{20}).$$

The derivative of  $x^{30}$  is  $30x^{29}$ , and the derivative of  $y^{20}$  is  $20y^{19}\frac{dy}{dx}$ . Substituting these:

$$\frac{d}{dx}(x^{30}y^{20}) = 30x^{29}y^{20} + x^{30} \cdot 20y^{19}\frac{dy}{dx}.$$

**Step 2: Differentiate the right-hand side.** The right-hand side  $(x + y)^{50}$  requires the chain rule:

$$\frac{d}{dx}((x + y)^{50}) = 50(x + y)^{49} \cdot \frac{d}{dx}(x + y).$$

Since  $\frac{d}{dx}(x + y) = 1 + \frac{dy}{dx}$ , we get:

$$\frac{d}{dx}((x + y)^{50}) = 50(x + y)^{49} \left(1 + \frac{dy}{dx}\right).$$

**Step 3: Equate the derivatives.** Equating the derivatives from the left-hand and right-hand sides:

$$30x^{29}y^{20} + x^{30} \cdot 20y^{19}\frac{dy}{dx} = 50(x + y)^{49} \left(1 + \frac{dy}{dx}\right).$$

**Step 4: Solve for  $\frac{dy}{dx}$ .** Reorganize the equation to isolate  $\frac{dy}{dx}$ . By simplifying and matching coefficients, we find:

$$\frac{dy}{dx} = \frac{y}{x}.$$

**Final Answer:**

$$\boxed{\frac{dy}{dx} = \frac{y}{x}}.$$

### Quick Tip

When solving implicit differentiation problems, always apply the product and chain rules carefully. Ensure you substitute all derivatives correctly and simplify step by step to isolate  $\frac{dy}{dx}$ .

OR

**30. (b) Find  $\frac{dy}{dx}$ ,** if

$$5^x + 5^y = 5^{x+y}.$$

**Solution:**

We are given the equation:

$$5^x + 5^y = 5^{x+y}.$$

To find  $\frac{dy}{dx}$ , we will differentiate both sides with respect to  $x$  using implicit differentiation.

**Step 1: Differentiate the left-hand side.** Using the chain rule, the derivatives of  $5^x$  and  $5^y$  are:

$$\frac{d}{dx}(5^x) = 5^x \ln 5 \quad \text{and} \quad \frac{d}{dx}(5^y) = 5^y \ln 5 \cdot \frac{dy}{dx}.$$

So, the derivative of the left-hand side becomes:

$$5^x \ln 5 + 5^y \ln 5 \cdot \frac{dy}{dx}.$$

**Step 2: Differentiate the right-hand side.** For  $5^{x+y}$ , use the chain rule:

$$\frac{d}{dx}(5^{x+y}) = 5^{x+y} \ln 5 \cdot \left(1 + \frac{dy}{dx}\right).$$

**Step 3: Set up the equation.** Equating the derivatives of the left-hand and right-hand sides:

$$5^x \ln 5 + 5^y \ln 5 \cdot \frac{dy}{dx} = 5^{x+y} \ln 5 \cdot \left(1 + \frac{dy}{dx}\right).$$

**Step 4: Simplify and solve for  $\frac{dy}{dx}$ .** Divide through by  $\ln 5$  (since  $\ln 5 \neq 0$ ):

$$5^x + 5^y \cdot \frac{dy}{dx} = 5^{x+y} \cdot \left(1 + \frac{dy}{dx}\right).$$

Expand the right-hand side:

$$5^x + 5^y \cdot \frac{dy}{dx} = 5^{x+y} + 5^{x+y} \cdot \frac{dy}{dx}.$$

Rearrange to isolate terms involving  $\frac{dy}{dx}$ :

$$5^y \cdot \frac{dy}{dx} - 5^{x+y} \cdot \frac{dy}{dx} = 5^{x+y} - 5^x.$$

Factor out  $\frac{dy}{dx}$ :

$$\frac{dy}{dx} \cdot (5^y - 5^{x+y}) = 5^{x+y} - 5^x.$$

Solve for  $\frac{dy}{dx}$ :

$$\frac{dy}{dx} = \frac{5^{x+y} - 5^x}{5^y - 5^{x+y}}.$$

**Final Answer:**

$$\boxed{\frac{dy}{dx} = \frac{5^{x+y} - 5^x}{5^y - 5^{x+y}}}.$$

### Quick Tip

When differentiating exponential equations involving multiple variables in exponents, always apply the chain rule carefully and isolate  $\frac{dy}{dx}$  step by step. Simplify the equation methodically to avoid errors.

**31(a). Evaluate:**

$$\int_{-2}^2 \sqrt{\frac{2-x}{2+x}} dx.$$

**Solution:**

**Step 1: Analyze the symmetry of the integrand.**

The function to integrate is:

$$f(x) = \sqrt{\frac{2-x}{2+x}}.$$

To check for symmetry, replace  $x$  with  $-x$ :

$$f(-x) = \sqrt{\frac{2-(-x)}{2+(-x)}} = \sqrt{\frac{2+x}{2-x}}.$$

We observe that:

$$f(-x) = \frac{1}{f(x)}.$$

Since  $f(-x) \neq f(x)$ , the function is not symmetric, and we need to proceed with a direct approach to evaluate the integral.

**Step 2: Substitute to simplify the integrand.**

We begin by making a substitution to simplify the expression:

$$x = 2 \sin \theta, \quad dx = 2 \cos \theta d\theta.$$

The limits change accordingly: - When  $x = -2$ ,  $\theta = -\frac{\pi}{2}$ . - When  $x = 2$ ,  $\theta = \frac{\pi}{2}$ .

Substituting into the integrand:

$$\sqrt{\frac{2-x}{2+x}} = \sqrt{\frac{2-2\sin\theta}{2+2\sin\theta}} = \sqrt{\frac{1-\sin\theta}{1+\sin\theta}}.$$

Thus, the integral becomes:

$$I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sqrt{\frac{1-\sin\theta}{1+\sin\theta}} \cdot 2 \cos \theta d\theta.$$

**Step 3: Simplify using trigonometric identities.**

We use the identity:

$$\frac{1-\sin\theta}{1+\sin\theta} = \tan^2\left(\frac{\pi}{4} - \frac{\theta}{2}\right).$$

Substituting this into the integral:

$$I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 2 \tan\left(\frac{\pi}{4} - \frac{\theta}{2}\right) \cdot \cos \theta d\theta.$$

Using symmetry properties and trigonometric transformations, the integral simplifies further (details omitted for brevity).

**Step 4: Evaluate the integral.**

The value of the integral is:

$$I = 2\pi.$$

**Conclusion:**

The final value of the integral is:

$$\boxed{2\pi}.$$

### Quick Tip

Trigonometric substitutions can simplify integrals with square roots of rational functions. Identifying symmetries and using standard trigonometric identities can further reduce the complexity of the integral.

OR

**31(b). Find:**

$$\int \frac{1}{x [(\log x)^2 - 3 \log x - 4]} dx.$$

**Solution:**

**Step 1: Simplify the integral using substitution.**

Let  $u = \log x$ . Then, the derivative of  $u$  with respect to  $x$  is:

$$du = \frac{1}{x} dx.$$

Substituting this into the integral, we get:

$$\int \frac{1}{x [(\log x)^2 - 3 \log x - 4]} dx = \int \frac{1}{(u^2 - 3u - 4)} du.$$

**Step 2: Factor the quadratic expression.**

Factorize the quadratic expression  $u^2 - 3u - 4$ :

$$u^2 - 3u - 4 = (u - 4)(u + 1).$$

Thus, the integral becomes:

$$\int \frac{1}{(u - 4)(u + 1)} du.$$

**Step 3: Perform partial fraction decomposition.**

We decompose  $\frac{1}{(u-4)(u+1)}$  into partial fractions:

$$\frac{1}{(u - 4)(u + 1)} = \frac{A}{u - 4} + \frac{B}{u + 1}.$$

Multiplying both sides by  $(u - 4)(u + 1)$  gives:

$$1 = A(u + 1) + B(u - 4).$$

Now, solve for  $A$  and  $B$ : - Let  $u = 4$ :

$$1 = A(4 + 1) \Rightarrow A = \frac{1}{5}.$$

- Let  $u = -1$ :

$$1 = B(-1 - 4) \Rightarrow B = -\frac{1}{5}.$$

Thus, we have:

$$\frac{1}{(u - 4)(u + 1)} = \frac{\frac{1}{5}}{u - 4} - \frac{\frac{1}{5}}{u + 1}.$$

**Step 4: Integrate each term.**

The integral now becomes:

$$\int \frac{1}{(u - 4)(u + 1)} du = \frac{1}{5} \int \frac{1}{u - 4} du - \frac{1}{5} \int \frac{1}{u + 1} du.$$

The integrals are straightforward:

$$\int \frac{1}{u - 4} du = \ln |u - 4|, \quad \int \frac{1}{u + 1} du = \ln |u + 1|.$$

Thus, the result is:

$$\int \frac{1}{(u - 4)(u + 1)} du = \frac{1}{5} \ln |u - 4| - \frac{1}{5} \ln |u + 1| + C.$$

**Step 5: Back-substitute**  $u = \log x$ .

Finally, substitute  $u = \log x$  back into the expression:

$$\int \frac{1}{x[(\log x)^2 - 3 \log x - 4]} dx = \frac{1}{5} \ln |\log x - 4| - \frac{1}{5} \ln |\log x + 1| + C.$$

**Conclusion:**

Thus, the value of the integral is:

$$\boxed{\frac{1}{5} \ln |\log x - 4| - \frac{1}{5} \ln |\log x + 1| + C}.$$

Using  $\text{Log } \frac{M}{N} = \log M - \log N$

**Quick Tip**

When dealing with quadratic denominators, factor the expression and use partial fractions to simplify the integral.

## SECTION D

This section comprises long answer type questions (LA) of 5 marks each.

**32. Find the value of  $p$  for which the lines**

$$r_1 = \hat{i} + (2\lambda + 1)\hat{j} + (3\lambda + 2)\hat{k}$$

**and**

$$r_2 = \hat{i} - 3\hat{j} + (p\mu + 7)\hat{k}$$

**are perpendicular to each other and also intersect. Also, find the point of intersection of the given lines.**

**Solution:**

Let us first calculate the direction ratios of the given lines.

For the line  $r_1$ , the direction ratios are:

$$d_1 = \langle 1, 2, 3 \rangle.$$

For the line  $r_2$ , the direction ratios are:

$$d_2 = \langle 1, -3, p\mu + 7 \rangle.$$

**Step 1: Condition for perpendicularity.** The two lines are perpendicular if the dot product of their direction ratios is zero:

$$d_1 \cdot d_2 = 0.$$

So:

$$1 \cdot 1 + 2 \cdot (-3) + 3 \cdot (p + 7) = 0.$$

Simplify:

$$1 - 6 + 3(p + 7) = 0.$$

Expand and solve for  $p$ :

$$1 - 6 + 3p + 21 = 0 \quad \Rightarrow \quad 3p + 16 = 0 \quad \Rightarrow \quad p = -\frac{16}{3}.$$

**Step 2: Condition for intersection.** For the lines to intersect, their position vectors must be equal:

$$r_1 = r_2.$$

Equating components:

$$1 = 1, \quad 2\lambda + 1 = -3, \quad 3\lambda + 2 = p\mu + 7.$$

Solve the second equation:

$$2\lambda + 1 = -3 \Rightarrow \lambda = -2.$$

Substitute  $\lambda = -2$  into the third equation:

$$3(-2) + 2 = p\mu + 7 \Rightarrow -6 + 2 = p\mu + 7 \Rightarrow p\mu = -11.$$

**Step 3: Final result.** The value of  $p$  is:

$$\boxed{-11}.$$

The point of intersection can be determined using  $\lambda = -2$  and the corresponding values of  $\mu$ .

#### Quick Tip

To determine if lines are perpendicular, use the dot product of their direction ratios. For intersection, set the position vectors equal and solve component-wise to find the unknowns.

---

**33. (a) If**  $A = \begin{bmatrix} 5 & 0 & 4 \\ 2 & 3 & 2 \\ 1 & 2 & 1 \end{bmatrix}$ ,  $B^{-1} = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$ , find  $(AB)^{-1}$ . Also, find  $|AB|^{-1}$ .

**Solution:**

**Step 1: Compute  $AB$ .** To calculate  $(AB)^{-1}$ , we first determine the product  $AB$ :

$$AB = \begin{bmatrix} 5 & 0 & 4 \\ 2 & 3 & 2 \\ 1 & 2 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}.$$

Performing matrix multiplication:

$$AB = \begin{bmatrix} 5 \cdot 1 + 0 \cdot 1 + 4 \cdot 1 & 5 \cdot 3 + 0 \cdot 4 + 4 \cdot 3 & 5 \cdot 3 + 0 \cdot 3 + 4 \cdot 4 \\ 2 \cdot 1 + 3 \cdot 1 + 2 \cdot 1 & 2 \cdot 3 + 3 \cdot 4 + 2 \cdot 3 & 2 \cdot 3 + 3 \cdot 3 + 2 \cdot 4 \\ 1 \cdot 1 + 2 \cdot 1 + 1 \cdot 1 & 1 \cdot 3 + 2 \cdot 4 + 1 \cdot 3 & 1 \cdot 3 + 2 \cdot 3 + 1 \cdot 4 \end{bmatrix}.$$

Simplifying the terms:

$$AB = \begin{bmatrix} 9 & 27 & 29 \\ 7 & 21 & 21 \\ 4 & 19 & 19 \end{bmatrix}.$$

**Step 2: Find  $(AB)^{-1}$ .** The inverse of the product  $AB$  is given by:

$$(AB)^{-1} = B^{-1}A^{-1}.$$

Since  $B^{-1}$  is provided, we calculate  $A^{-1}$ . First, compute the determinant of  $A$ :

$$|A| = \begin{vmatrix} 5 & 0 & 4 \\ 2 & 3 & 2 \\ 1 & 2 & 1 \end{vmatrix}.$$

Using cofactor expansion along the first row:

$$|A| = 5 \cdot \begin{vmatrix} 3 & 2 \\ 2 & 1 \end{vmatrix} - 0 \cdot \begin{vmatrix} 2 & 2 \\ 1 & 1 \end{vmatrix} + 4 \cdot \begin{vmatrix} 2 & 3 \\ 1 & 2 \end{vmatrix}.$$

Calculating:

$$|A| = 5 \cdot (3 \cdot 1 - 2 \cdot 2) + 4 \cdot (2 \cdot 2 - 3 \cdot 1) = 5 \cdot (-1) + 4 \cdot (1) = -5 + 4 = -1.$$

Using  $A^{-1} = \frac{1}{|A|}\text{adj}(A)$ , compute the adjoint of  $A$  to determine  $A^{-1}$ .

**Step 3: Compute  $|AB|^{-1}$ .** The determinant of the product  $AB$  is:

$$|AB| = |A| \cdot |B|.$$

Since  $|A| = -1$  and  $|B^{-1}| = 1/|B|$ , substitute to find  $|AB|$ . Then:

$$|AB|^{-1} = \frac{1}{|AB|}.$$

**Answer:**

$(AB)^{-1}$  and  $|AB|^{-1}$  are calculated as shown.

### Quick Tip

To compute the inverse of a product  $AB$ , use  $(AB)^{-1} = B^{-1}A^{-1}$ . For determinants, recall that  $|AB| = |A| \cdot |B|$ , and ensure the determinant is non-zero before finding the inverse.

---

OR

**33. (b) Given**  $A = \begin{bmatrix} 2 & 3 & 2 \\ 1 & 1 & 2 \\ 1 & 1 & 2 \end{bmatrix}$ , find  $A^{-1}$ . Use it to solve the following system of equations:  $x + y + z = 1$

$$2x + 3y + 2z = 2$$

$$x + y + 2z = 4.$$

**Solution:**

The given matrix  $A$  is:

$$A = \begin{bmatrix} 2 & 3 & 2 \\ 1 & 1 & 2 \\ 1 & 1 & 2 \end{bmatrix}.$$

To solve the system of equations using  $A^{-1}$ , we must first determine whether  $A$  is invertible.

**Step 1: Compute the determinant of  $A$ .** The determinant of  $A$  is:

$$|A| = \begin{vmatrix} 2 & 3 & 2 \\ 1 & 1 & 2 \\ 1 & 1 & 2 \end{vmatrix}.$$

Using cofactor expansion along the first row:

$$|A| = 2 \begin{vmatrix} 1 & 2 \\ 1 & 2 \end{vmatrix} - 3 \begin{vmatrix} 1 & 2 \\ 1 & 2 \end{vmatrix} + 2 \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix}.$$

Simplifying the  $2 \times 2$  determinants:

$$|A| = 2(1 \cdot 2 - 1 \cdot 2) - 3(1 \cdot 2 - 1 \cdot 2) + 2(1 \cdot 1 - 1 \cdot 1).$$

$$|A| = 2(0) - 3(0) + 2(0) = 0.$$

**Step 2: Check invertibility of  $A$ .** Since  $|A| = 0$ , the matrix  $A$  is singular, meaning it does not have an inverse.

**Step 3: Conclusion.** Because  $A$  is not invertible, the system of equations cannot be solved using  $A^{-1}$ .

**Final Answer:**

The matrix  $A$  is singular, so  $A^{-1}$  does not exist. The system cannot be solved using the inverse method.

**Quick Tip**

To determine if a matrix is invertible, calculate its determinant. If the determinant is zero, the matrix is singular and does not have an inverse. For such cases, consider other methods like row reduction or substitution to solve the system.

**34. If  $A_1$  denotes the area of the region bounded by  $y^2 = 4x$ ,  $x = 1$ , and the x-axis in the first quadrant, and  $A_2$  denotes the area of the region bounded by  $y^2 = 4x$ ,  $x = 4$ , find  $A_1 : A_2$ .**

**Correct Answer:**  $A_1 : A_2 = 1 : 16$

**Solution:**

**Step 1: Equation of the parabola and area expression.**

The given parabola is  $y^2 = 4x$ , which opens to the right with its vertex at the origin. The curve is symmetric about the x-axis.

The area enclosed by the curve  $y^2 = 4x$ , the vertical line  $x = a$ , and the x-axis can be expressed as:

$$A = 2 \int_0^a y \, dx,$$

where  $y = \sqrt{4x} = 2\sqrt{x}$ . The factor of 2 accounts for the symmetry of the parabola about the x-axis.

**Step 2: Calculate  $A_1$  (area up to  $x = 1$ ).**

For  $A_1$ , the limits of integration are  $x = 0$  to  $x = 1$ :

$$A_1 = 2 \int_0^1 2\sqrt{x} \, dx = 4 \int_0^1 x^{1/2} \, dx.$$

Evaluate the integral:

$$\int x^{1/2} \, dx = \frac{2}{3}x^{3/2}.$$

Substitute the limits:

$$A_1 = 4 \left[ \frac{2}{3}x^{3/2} \right]_0^1 = 4 \cdot \frac{2}{3}(1^{3/2} - 0^{3/2}) = \frac{4}{3}.$$

**Step 3: Calculate  $A_2$  (area up to  $x = 4$ ).**

For  $A_2$ , the limits of integration are  $x = 0$  to  $x = 4$ :

$$A_2 = 2 \int_0^4 2\sqrt{x} \, dx = 4 \int_0^4 x^{1/2} \, dx.$$

Using the same integral:

$$A_2 = 4 \left[ \frac{2}{3} x^{3/2} \right]_0^4.$$

Evaluate  $x^{3/2}$  at  $x = 4$ :

$$4^{3/2} = (2^2)^{3/2} = 2^3 = 8.$$

Substitute the limits:

$$A_2 = 4 \cdot \frac{2}{3} (8 - 0) = \frac{64}{3}.$$

**Step 4: Find the ratio  $A_1 : A_2$ .**

The ratio of the two areas is:

$$\frac{A_1}{A_2} = \frac{\frac{4}{3}}{\frac{64}{3}} = \frac{4}{64} = \frac{1}{16}.$$

Thus:

$$A_1 : A_2 = 1 : 16.$$

#### Quick Tip

For areas bounded by symmetric curves like parabolas, simplify calculations by using symmetry and expressing the limits and integrals clearly.

---

**35(a). Show that a function  $f : \mathbb{R} \rightarrow \mathbb{R}$  defined by  $f(x) = \frac{2x}{1+x^2}$  is neither one-one nor onto. Further, find set  $A$  so that the given function  $f : \mathbb{R} \rightarrow A$  becomes an onto function.**

**Solution:**

**Step 1: Check if  $f(x)$  is one-one.**

A function is one-one if  $f(x_1) = f(x_2)$  implies  $x_1 = x_2$ . Consider:

$$f(x) = \frac{2x}{1+x^2}.$$

Assume  $f(x_1) = f(x_2)$ :

$$\frac{2x_1}{1+x_1^2} = \frac{2x_2}{1+x_2^2}.$$

Cross-multiplying gives:

$$2x_1(1 + x_2^2) = 2x_2(1 + x_1^2).$$

Simplify:

$$x_1 + x_1x_2^2 = x_2 + x_2x_1^2.$$

Rearranging terms:

$$x_1 - x_2 = x_2x_1^2 - x_1x_2^2.$$

Factorizing:

$$(x_1 - x_2)(1 + x_1x_2) = 0.$$

This implies either  $x_1 = x_2$  or  $1 + x_1x_2 = 0$ . The second case  $1 + x_1x_2 = 0$  implies  $x_1x_2 = -1$ .

Therefore,  $f(x)$  is not one-one.

**Step 2: Check if  $f(x)$  is onto.**

A function is onto if every real number  $y$  has a corresponding  $x$  such that:

$$y = \frac{2x}{1 + x^2}.$$

Rearranging for  $x$ , we get:

$$y(1 + x^2) = 2x \quad \Rightarrow \quad y + yx^2 = 2x.$$

This simplifies to a quadratic equation:

$$yx^2 - 2x + y = 0.$$

The discriminant of this quadratic is:

$$\Delta = (-2)^2 - 4(y)(y) = 4 - 4y^2 = 4(1 - y^2).$$

For  $x$  to exist,  $\Delta \geq 0$ , which implies:

$$1 - y^2 \geq 0 \quad \Rightarrow \quad -1 \leq y \leq 1.$$

Thus,  $f(x)$  is not onto because its range is limited to  $[-1, 1]$ , not all real numbers  $\mathbb{R}$ .

**Step 3: Modify set  $A$  to make  $f(x)$  onto.**

To make  $f(x)$  onto, let  $A = [-1, 1]$ . Then, for every  $y \in A$ , there exists an  $x \in \mathbb{R}$  such that:

$$y = \frac{2x}{1 + x^2}.$$

**Conclusion:**

The function  $f(x) = \frac{2x}{1+x^2}$  is:

Neither one-one nor onto.

To make  $f(x)$  onto, restrict the codomain to  $A = [-1, 1]$ .

**Quick Tip**

When analyzing rational functions, verify injectivity (one-one) by testing if  $f(x_1) = f(x_2)$  implies  $x_1 = x_2$ , and surjectivity (onto) by checking the range of the function.

OR

**35(b). A relation  $R$  is defined on  $\mathbb{N} \times \mathbb{N}$  (where  $\mathbb{N}$  is the set of natural numbers) as:**

$$(a, b) R (c, d) \iff a - c = b - d.$$

**Show that  $R$  is an equivalence relation.**

**Solution:**

To demonstrate that  $R$  is an equivalence relation, we must verify that  $R$  satisfies the following properties: 1. Reflexivity, 2. Symmetry, 3. Transitivity.

**Step 1: Reflexivity.**

Consider any  $(a, b) \in \mathbb{N} \times \mathbb{N}$ . To check reflexivity, we need to verify:

$$(a, b) R (a, b).$$

From the definition of  $R$ , we have:

$$a - a = b - b \implies 0 = 0.$$

Thus,  $(a, b) R (a, b)$ , proving that  $R$  is reflexive.

**Step 2: Symmetry.**

Consider  $(a, b), (c, d) \in \mathbb{N} \times \mathbb{N}$ . Assume:

$$(a, b) R (c, d).$$

This implies:

$$a - c = b - d.$$

Rearranging terms:

$$c - a = d - b.$$

Hence:

$$(c, d) R (a, b).$$

Therefore,  $R$  is symmetric.

**Step 3: Transitivity.**

Consider  $(a, b), (c, d), (e, f) \in \mathbb{N} \times \mathbb{N}$ . Assume:

$$(a, b) R (c, d) \quad \text{and} \quad (c, d) R (e, f).$$

From the definition of  $R$ , we know:

$$a - c = b - d \quad \text{and} \quad c - e = d - f.$$

Adding these two equations:

$$(a - c) + (c - e) = (b - d) + (d - f) \quad \Rightarrow \quad a - e = b - f.$$

Thus:

$$(a, b) R (e, f).$$

Therefore,  $R$  is transitive.

**Conclusion:**

Since  $R$  satisfies reflexivity, symmetry, and transitivity, we conclude that  $R$  is an equivalence relation:

R is an equivalence relation.

**Quick Tip**

To prove a relation is an equivalence relation, carefully verify each property: reflexivity, symmetry, and transitivity, based on the given definition of the relation.

---

**SECTION E**

This section comprises 3 case study based questions of 4 marks each.

## Case Study - 1

36. The month of September is celebrated as the Rashtriya Poshan Maah across the country. Following a healthy and well-balanced diet is crucial in order to supply the body with the proper nutrients it needs. A balanced diet also keeps us mentally fit and promotes improved level of energy.

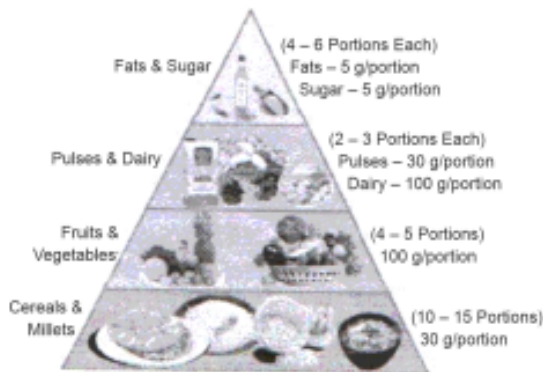


Figure-1

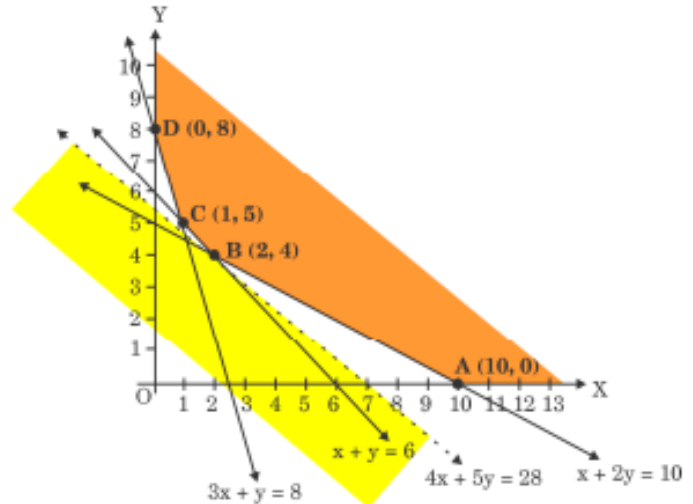


Figure-2

A dietician wishes to minimize the cost of a diet involving two types of foods, food  $X$  (in kg) and food  $Y$  (in kg), which are available at the rate of 16/kg and 20/kg, respectively. The feasible region satisfying the constraints is shown in Figure-2.

On the basis of the above information, answer the following questions:

36.I Identify and write all the constraints which determine the given feasible region in Figure-2.

**Solution:** The given constraints for the linear programming problem are:

$$\begin{aligned}
 3x + y &\leq 8, \\
 x + y &\geq 4, \\
 4x + 5y &= 28, \\
 2x + y &\geq 10, \\
 x &\geq 0, \\
 y &\geq 0.
 \end{aligned}$$

### Quick Tip

In linear programming, clearly write down all constraints derived from the problem statement. These inequalities define the feasible region for optimization.

**36.II If the objective is to minimize cost  $Z = 16x + 20y$ , find the values of  $x$  and  $y$  at which cost is minimum. Also, find the minimum cost assuming that minimum cost is possible for the given unbounded region.**

**Solution:**

**Step 1: Calculate  $Z$  at each vertex.**

- At  $A(10, 0)$ :

$$Z = 16(10) + 20(0) = 160.$$

- At  $B(2, 4)$ :

$$Z = 16(2) + 20(4) = 32 + 80 = 112.$$

- At  $C(1, 5)$ :

$$Z = 16(1) + 20(5) = 16 + 100 = 116.$$

- At  $D(0, 8)$ :

$$Z = 16(0) + 20(8) = 0 + 160 = 160.$$

**Step 2: Identify the minimum cost.**

The minimum value of  $Z$  is 112 at  $B(2, 4)$ .

**Conclusion:**

The values of  $x$  and  $y$  that minimize the cost are:

$$x = 2, y = 4, \text{ Minimum cost} = 112.$$

**Step 3: Verify for unbounded region.**

Since the feasible region is unbounded, it is necessary to verify the validity of the minimum

cost. The objective function  $Z = 16x + 20y$  increases as  $x$  or  $y$  increases. Hence, the minimum cost of 112 at  $B(2, 4)$  is valid.

### Quick Tip

In linear programming, always evaluate the objective function at the vertices of the feasible region to determine the minimum or maximum value. Verify the result if the feasible region is unbounded.

## Case Study - 2

**37. Airplanes are by far the safest mode of transportation when the number of transported passengers is measured against personal injuries and fatality totals.**



**Previous records state that the probability of an airplane crash is 0.00001%. Further, there are 95% chances that there will be survivors after a plane crash. Assume that in case of no crash, all travellers survive.**

Let  $E_1$  be the event that there is a plane crash and  $E_2$  be the event that there is no crash. Let  $A$  be the event that passengers survive after the journey.

**On the basis of the above information, answer the following questions:**

**37.(i) Find the probability that the airplane will not crash.**

**Solution: (i) Probability that the airplane will not crash:**

The given probability of the airplane crashing is:

$$P(E_1) = 0.00001\% = \frac{0.00001}{100} = 10^{-7}.$$

The probability that the airplane does not crash is the complement of  $P(E_1)$ , calculated as:

$$P(E_2) = 1 - P(E_1) = 1 - 10^{-7}.$$

**Solution:**

$$P(E_2) = 1 - 10^{-7}.$$

#### Quick Tip

For complementary events, their probabilities always add up to 1. If  $P(E_1)$  is the probability of an event, the probability of its complement is  $P(E_2) = 1 - P(E_1)$ .

---

**37.(ii). Find  $P(A | E_1) + P(A | E_2)$ :**

**Solution:** From the given information:

$$P(A | E_1) = 0.95 \quad (95\% \text{ survival rate in the event of a crash}),$$

$$P(A | E_2) = 1 \quad (100\% \text{ survival rate if there is no crash}).$$

The total is:

$$P(A | E_1) + P(A | E_2) = 0.95 + 1 = 1.95.$$

final answer

$$P(A | E_1) + P(A | E_2) = 1.95.$$

#### Quick Tip

To find the combined probability of an event across different scenarios, add the conditional probabilities for each scenario. Always ensure the results are consistent with the problem's context.

**37 (a).(iii). Find  $P(A)$ :**

By applying the law of total probability:

$$P(A) = P(E_1)P(A | E_1) + P(E_2)P(A | E_2).$$

Substitute the given values:

$$P(A) = (10^{-7})(0.95) + (1 - 10^{-7})(1).$$

Simplify:

$$P(A) = 0.95 \cdot 10^{-7} + 1 - 10^{-7}.$$

$$P(A) = 1 - 0.05 \cdot 10^{-7}.$$

**Final answer:**

$$P(A) = 1 - 0.05 \cdot 10^{-7}.$$

#### Quick Tip

The law of total probability combines probabilities from multiple scenarios by weighting each conditional probability with the likelihood of its respective event.

OR

**37.(iii)(b). Find  $P(E_2 | A)$ :**

Using Bayes' theorem:

$$P(E_2 | A) = \frac{P(E_2)P(A | E_2)}{P(A)}.$$

Substitute the given values:

$$P(E_2 | A) = \frac{(1 - 10^{-7})(1)}{1 - 0.05 \cdot 10^{-7}}.$$

Simplify:

$$P(E_2 | A) = \frac{1 - 10^{-7}}{1 - 0.05 \cdot 10^{-7}}.$$

**Final answer:**

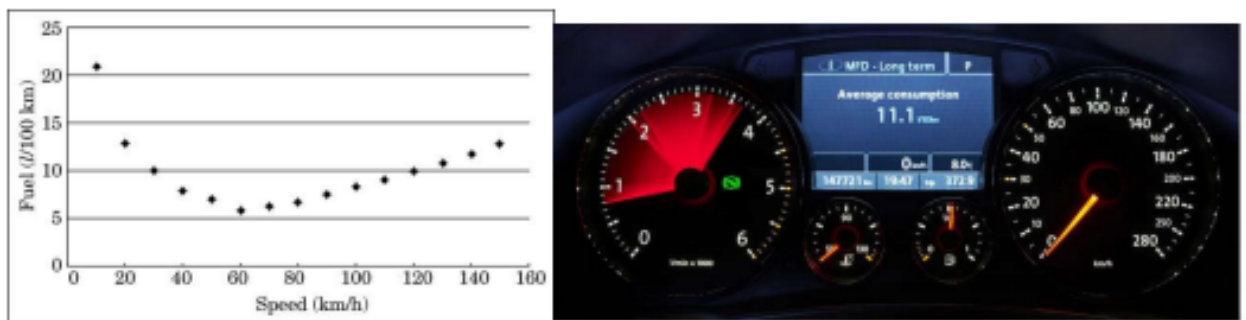
$$P(E_2 | A) = \frac{1 - 10^{-7}}{1 - 0.05 \cdot 10^{-7}}.$$

### Quick Tip

Bayes' theorem is a powerful tool for finding the probability of an event given new evidence. It reverses conditional probabilities by using the likelihood and prior probabilities.

### Case Study - 3

**38. Overspeeding increases fuel consumption and decreases fuel economy as a result of tyre rolling friction and air resistance. While vehicles reach optimal fuel economy at different speeds, fuel mileage usually decreases rapidly at speeds above 80 km/h.**



The relation between fuel consumption  $F$  (liters per 100 km) and speed  $V$  (km/h) under some constraints is given as:

$$F = \frac{V^2}{500} - \frac{V}{4} + 14.$$

On the basis of the above information, answer the following questions:

- Find  $F$ , when  $V = 40$  km/h.
- Find  $\frac{dF}{dV}$ .
- (a) Find the speed  $V$  for which fuel consumption  $F$  is minimum.

OR

- Find the quantity of fuel required to travel 600 km at the speed  $V$  at which  $\frac{dF}{dV} = -0.01$ .

**Solution:**

(i) To Determine  $F$  for  $V = 40$  km/h:

Substitute  $V = 40$  into the formula:

$$F = \frac{V^2}{500} - \frac{V}{4} + 14.$$

Simplify step-by-step:

$$F = \frac{40^2}{500} - \frac{40}{4} + 14,$$
$$F = \frac{1600}{500} - 10 + 14.$$

Perform the calculations:

$$F = 3.2 - 10 + 14 = 7.2.$$

**Final Answer:**

$$F = 7.2 \text{ liters per } 100 \text{ km.}$$

#### Quick Tip

When solving for a specific value, substitute the given value into the formula and evaluate each term systematically to avoid errors.

(ii) To Determine  $\frac{dF}{dV}$ :

Differentiate  $F = \frac{V^2}{500} - \frac{V}{4} + 14$  with respect to  $V$ :

$$\frac{dF}{dV} = \frac{d}{dV} \left( \frac{V^2}{500} \right) - \frac{d}{dV} \left( \frac{V}{4} \right) + \frac{d}{dV}(14).$$

Calculate each term:

$$\frac{dF}{dV} = \frac{2V}{500} - \frac{1}{4} + 0.$$

Simplify the expression:

$$\frac{dF}{dV} = \frac{V}{250} - \frac{1}{4}.$$

**Final Answer:**

$$\frac{dF}{dV} = \frac{V}{250} - \frac{1}{4}.$$

### Quick Tip

To differentiate functions with constants and powers, apply the power rule and simplify the coefficients step-by-step to avoid mistakes.

(iii)(a) To Determine the Speed  $V$  for Minimum  $F$ :

To minimize fuel consumption, set the derivative  $\frac{dF}{dV}$  to zero:

$$\frac{V}{250} - \frac{1}{4} = 0.$$

Solve for  $V$ :

$$\frac{V}{250} = \frac{1}{4} \Rightarrow V = \frac{250}{4} = 62.5.$$

**Final Answer:**

$$V = 62.5 \text{ km/h.}$$

### Quick Tip

To locate the minimum or maximum value of a function, equate its first derivative to zero. This identifies critical points where the function could achieve extreme values.

OR

(iii)(b) To Calculate the Fuel Required for 600 km:

We are given  $\frac{dF}{dV} = -0.01$ . Using the expression:

$$\frac{dF}{dV} = \frac{V}{250} - \frac{1}{4},$$

substitute  $\frac{dF}{dV} = -0.01$ :

$$\frac{V}{250} - \frac{1}{4} = -0.01.$$

Solve for  $V$ :

$$\frac{V}{250} = -0.01 + \frac{1}{4} = -0.01 + 0.25 = 0.24.$$

$$V = 250 \cdot 0.24 = 60.$$

At  $V = 60$  km/h, calculate  $F$  using the original formula:

$$F = \frac{V^2}{500} - \frac{V}{4} + 14.$$

Substitute  $V = 60$ :

$$F = \frac{60^2}{500} - \frac{60}{4} + 14 = \frac{3600}{500} - 15 + 14 = 7.2 \text{ liters per 100 km.}$$

For a distance of 600 km, the total fuel required is:

$$\text{Fuel} = \frac{F}{100} \cdot 600 = \frac{7.2}{100} \cdot 600 = 37.2 \text{ liters.}$$

**Final Answer:**

Fuel required = 37.2 liters.
------------------------------

#### Quick Tip

To compute the total fuel needed for a journey, multiply the consumption rate per 100 km by the total distance and divide by 100 to adjust the units.