

CBSE Class 12 Mathematics Set 4 (65/4/1) Question Paper with Solutions

Time Allowed :3 Hour	Maximum Marks :80	Total Questions :38
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General Instructions

Read the following instructions very carefully and strictly follow them:

1. This question paper contains 38 questions. All questions are compulsory.
2. This question paper is divided into five Sections – A, B, C, D and E.
3. In Section A, Questions no. 1 to 18 are multiple choice questions (MCQs) and questions number 19 and 20 are Assertion-Reason based questions of 1 mark each.
4. In Section B, Questions no. 21 to 25 are very short answer (VSA) type questions, carrying 2 marks each.
5. In Section C, Questions no. 26 to 31 are short answer (SA) type questions, carrying 3 marks each.
6. In Section D, Questions no. 32 to 35 are long answer (LA) type questions carrying 5 marks each.
7. In Section E, Questions no. 36 to 38 are case study based questions carrying 4 marks each.
8. There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and 2 questions in Section E.
9. Use of calculators is not allowed.

SECTION A

This section comprises multiple choice questions (MCQs) of 1 mark each.

1. If $\begin{bmatrix} a & c & 0 \\ b & d & 0 \\ 0 & 0 & 5 \end{bmatrix}$ is a scalar matrix, then the value of $a + 2b + 3c + 4d$ is:

(A) 0

(B) 5

(C) 10

(D) 25

Correct Answer: (D) 25

Solution:

Step 1: Definition of a scalar matrix

A scalar matrix is a diagonal matrix where all diagonal elements are equal. This means $a = d = 5$, and all off-diagonal elements (b, c) are zero.

Step 2: Substitute the values

Using the scalar matrix properties:

$$a + 2b + 3c + 4d = 5 + 2(0) + 3(0) + 4(5) = 5 + 0 + 0 + 20 = 25.$$

Step 3: Verify the options

The correct value is 25, which corresponds to option (D).

Quick Tip

In scalar matrices, diagonal elements are constant, and off-diagonal elements are zero.

2. Given that $A^{-1} = \frac{1}{7} \begin{bmatrix} 2 & 1 \\ -3 & 2 \end{bmatrix}$, matrix A is:

(A) $7 \begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$

(B) $\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$

$$(C) \frac{1}{7} \begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$$

$$(D) \frac{1}{49} \begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$$

Correct Answer: (A) $7 \begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$

Solution:

Given that $A^{-1} = \frac{1}{7} \begin{bmatrix} 2 & 1 \\ -3 & 2 \end{bmatrix}$, we need to find matrix A .

The inverse of a 2x2 matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is given by:

$$\frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Applying this to A^{-1} :

$$A^{-1} = \frac{1}{7} \begin{bmatrix} 2 & 1 \\ -3 & 2 \end{bmatrix}$$

The determinant of A^{-1} is:

$$\det(A^{-1}) = \left(\frac{1}{7}\right)^2 (2 \cdot 2 - 1 \cdot (-3)) = \frac{1}{49}(4 + 3) = \frac{7}{49} = \frac{1}{7}$$

Now, the inverse of A^{-1} (which is A) is:

$$A = \frac{1}{\det(A^{-1})} \begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix} = 7 \begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$$

Therefore, the correct answer is:

\boxed{A}

So, matrix A is:

$$7 \begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$$

Quick Tip

To find A from A^{-1} , multiply the inverse by the scalar reciprocal.

3. If $A = \begin{bmatrix} 2 & 1 \\ -4 & -2 \end{bmatrix}$, then the value of $I - A + A^2 - A^3 + \dots$ is:

(A) $\begin{bmatrix} -1 & -1 \\ 4 & 3 \end{bmatrix}$

(B) $\begin{bmatrix} 3 & 1 \\ -4 & -1 \end{bmatrix}$

(C) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

(D) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Correct Answer: (A) $\begin{bmatrix} -1 & -1 \\ 4 & 3 \end{bmatrix}$

Solution: Step 1: Understand the series S : The given series is $S = I - A + A^2 - A^3 + \dots$, which is an infinite series. It can be expressed as:

$$S = (I - A)^{-1}$$

if the matrix $(I - A)$ is invertible.

Step 2: Compute $I - A$: The identity matrix I is:

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

So, $I - A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 1 \\ -4 & -2 \end{bmatrix} = \begin{bmatrix} -1 & -1 \\ 4 & 3 \end{bmatrix}$.

Step 3: Check if $I - A$ is invertible: The determinant of $I - A$ is:

$$\det(I - A) = (-1)(3) - (-1)(4) = -3 + 4 = 1 \neq 0$$

Since the determinant is non-zero, $I - A$ is invertible.

Step 4: Verify the series sum: The inverse of $I - A$ is:

$$(I - A)^{-1} = \begin{bmatrix} -1 & -1 \\ 4 & 3 \end{bmatrix}$$

Thus, the sum of the series $S = I - A + A^2 - A^3 + \dots$ is:

$$S = \begin{bmatrix} -1 & -1 \\ 4 & 3 \end{bmatrix}$$

Conclusion: The correct option is (A) $\begin{bmatrix} -1 & -1 \\ 4 & 3 \end{bmatrix}$.

Quick Tip

For infinite geometric series $I - A + A^2 - A^3 + \dots$, check if $A^2 = 0$.

4. If $A = \begin{bmatrix} -2 & 0 & 0 \\ 1 & 2 & 3 \\ 5 & 1 & -1 \end{bmatrix}$, then the value of $|A \cdot \text{adj}(A)|$ is:

(A) 100 I

(B) 10 I

(C) 10

(D) 1000

Correct Answer: (D) 1000

Solution:

Step 1: Property of determinants

For a square matrix A , $|A \cdot \text{adj}(A)| = |A|^n$, where n is the size of A .

Step 2: Compute $|A|$

Using cofactor expansion:

$$|A| = -2 \cdot \begin{vmatrix} 2 & 3 \\ 1 & -1 \end{vmatrix} = -2((-2) - 3) = 10.$$

Step 3: Calculate $|A \cdot \text{adj}(A)|$

Since $n = 3$:

$$|A \cdot \text{adj}(A)| = |A|^3 = 10^3 = 1000.$$

Step 4: Verify the options

The correct value is 1000, which corresponds to option (D).

Quick Tip

To compute $|A \cdot \text{adj}(A)|$, use the property $|A|^n$ for $n \times n$ matrices.

5. Given that $\begin{bmatrix} 1 & x \end{bmatrix} \begin{bmatrix} 4 & 0 \\ -2 & 0 \end{bmatrix} = 0$, **then the value of x is:**

(A) -4

(B) -2

(C) 2

(D) 4

Correct Answer: (C) 2

Solution:

Step 1: Matrix multiplication

Perform the matrix multiplication:

$$\begin{bmatrix} 1 & x \end{bmatrix} \begin{bmatrix} 4 & 0 \\ -2 & 0 \end{bmatrix} = \begin{bmatrix} 4 - 2x & 0 \end{bmatrix}.$$

Step 2: Set the result to zero

For the equation to hold, $4 - 2x = 0$.

Step 3: Solve for x

Rearranging:

$$4 - 2x = 0 \implies x = \frac{4}{2} = 2.$$

Step 4: Verify the options

The value $x = 2$ matches option (C).

Quick Tip

For matrix equations, ensure each resulting element matches the given condition.

6. Derivative of e^{2x} with respect to e^x is:

- (A) e^x
- (B) $2e^x$
- (C) $2e^{2x}$
- (D) $2e^{3x}$

Correct Answer: (B) $2e^x$

Solution:

Step 1: Chain rule

The derivative of e^{2x} with respect to x is:

$$\frac{d}{dx}e^{2x} = 2e^{2x}.$$

Step 2: Substitute $e^x = u$

Let $u = e^x$, so $e^{2x} = u^2$. Differentiate u^2 with respect to u :

$$\frac{d}{du}u^2 = 2u.$$

Step 3: Final result

Since $u = e^x$, the result is $2e^x$.

Quick Tip

Use substitution to simplify differentiation of exponential functions.

7. For what value of k , the function given below is continuous at $x = 0$?

$$f(x) = \begin{cases} \frac{\sqrt{4+x}-2}{x}, & x \neq 0 \\ k, & x = 0 \end{cases}$$

- (A) 0
- (B) $\frac{1}{4}$
- (C) 1
- (D) 4

Correct Answer: (B) $\frac{1}{4}$

Solution:

Step 1: Condition for continuity

For continuity at $x = 0$, $\lim_{x \rightarrow 0} f(x) = f(0) = k$.

Step 2: Evaluate the limit

For $x \neq 0$:

$$f(x) = \frac{\sqrt{4+x} - 2}{x}.$$

Multiply numerator and denominator by $\sqrt{4+x} + 2$:

$$f(x) = \frac{(\sqrt{4+x} - 2)(\sqrt{4+x} + 2)}{x(\sqrt{4+x} + 2)} = \frac{4+x-4}{x(\sqrt{4+x} + 2)} = \frac{1}{\sqrt{4+x} + 2}.$$

Step 3: Substitute $x = 0$

$$\lim_{x \rightarrow 0} f(x) = \frac{1}{\sqrt{4+0} + 2} = \frac{1}{4}.$$

Step 4: Set $k = \lim_{x \rightarrow 0} f(x)$

Thus, $k = \frac{1}{4}$, which matches option (B).

Quick Tip

To find continuity, equate $\lim_{x \rightarrow a} f(x)$ and $f(a)$.

8. The value of $\int_0^3 \frac{dx}{\sqrt{9-x^2}}$ is:

(A) $\frac{\pi}{6}$

(B) $\frac{\pi}{4}$

(C) $\frac{\pi}{2}$

(D) $\frac{\pi}{18}$

Correct Answer: (C) $\frac{\pi}{2}$

Solution:

Step 1: Integral of the given form

The integral $\int \frac{dx}{\sqrt{a^2-x^2}}$ is a standard result:

$$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\left(\frac{x}{a}\right) + C.$$

Step 2: Substitute limits

Here, $a = 3$, so:

$$\int_0^3 \frac{dx}{\sqrt{9-x^2}} = \arcsin\left(\frac{x}{3}\right) \Big|_0^3.$$

Step 3: Evaluate the bounds

$$\arcsin\left(\frac{3}{3}\right) - \arcsin\left(\frac{0}{3}\right) = \arcsin(1) - \arcsin(0) = \frac{\pi}{2} - 0 = \frac{\pi}{2}.$$

Step 4: Verify the options

The correct value is $\frac{\pi}{2}$, which corresponds to option (C).

Quick Tip

For integrals of $\frac{1}{\sqrt{a^2-x^2}}$, use the arcsine formula.

9. The general solution of the differential equation $x dy + y dx = 0$ is:

- (A) $xy = c$
- (B) $x + y = c$
- (C) $x^2 + y^2 = c^2$
- (D) $\log y = \log x + c$

Correct Answer: (A) $xy = c$

Solution:

Step 1: Rewrite the equation

The given equation $x dy + y dx = 0$ can be written as:

$$\frac{dy}{y} + \frac{dx}{x} = 0.$$

Step 2: Integrate both sides

$$\int \frac{dy}{y} + \int \frac{dx}{x} = 0 \implies \ln|y| + \ln|x| = C.$$

Step 3: Simplify the solution

Combine logarithms:

$$\ln|xy| = C \implies xy = e^C = c.$$

Step 4: Verify the options

The solution is $xy = c$, which matches option (A).

Quick Tip

For separable differential equations, rearrange terms and integrate both sides.

10. The integrating factor of the differential equation $(x + 2y^2)\frac{dy}{dx} = y$ ($y > 0$) is:

- (A) $\frac{1}{x}$
- (B) x
- (C) y
- (D) $\frac{1}{y}$

Correct Answer: (D) $\frac{1}{y}$

Solution:

Step 1: Rewriting the equation

Divide through by y :

$$\frac{1}{y}(x + 2y^2)\frac{dy}{dx} = 1.$$

Step 2: Find the integrating factor

The integrating factor $\mu(y)$ is determined by identifying the dependency on y and multiplying the equation by $\frac{1}{y}$.

Step 3: Verify integrating factor

After multiplying, the left-hand side becomes exact. The integrating factor is $\frac{1}{y}$, which matches option (D).

Quick Tip

Integrating factors simplify differential equations by making them exact.

11. If \vec{a} and \vec{b} are two vectors such that $|\vec{a}| = 1$, $|\vec{b}| = 2$, and $\vec{a} \cdot \vec{b} = \sqrt{3}$, then the angle between $2\vec{a}$ and $-\vec{b}$ is:

- (A) $\frac{\pi}{6}$
- (B) $\frac{\pi}{3}$
- (C) $\frac{5\pi}{6}$
- (D) $\frac{11\pi}{6}$

Correct Answer: (C) $\frac{5\pi}{6}$

Solution:

Step 1: Find the angle between \vec{a} and \vec{b}

The dot product formula gives:

$$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos \theta \implies \sqrt{3} = (1)(2) \cos \theta \implies \cos \theta = \frac{\sqrt{3}}{2}.$$

Thus, $\theta = \frac{\pi}{6}$.

Step 2: Angle between $2\vec{a}$ and $-\vec{b}$

Since $2\vec{a}$ and $-\vec{b}$ involve a scalar multiplication, the angle becomes:

$$\pi - \frac{\pi}{6} = \frac{5\pi}{6}.$$

Step 3: Verify the options

The correct angle is $\frac{5\pi}{6}$, matching option (C).

Quick Tip

The angle between scaled vectors depends only on the original vectors' angle.

12. The vectors $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$, $\vec{b} = \hat{i} - 3\hat{j} - 5\hat{k}$, and $\vec{c} = -3\hat{i} + 4\hat{j} + 4\hat{k}$ represent the sides of:

- (A) an equilateral triangle
- (B) an obtuse-angled triangle
- (C) an isosceles triangle
- (D) a right-angled triangle

Correct Answer: (D) a right-angled triangle

Solution:

The magnitude of a vector $\mathbf{v} = x\hat{i} + y\hat{j} + z\hat{k}$ is given by:

$$|\mathbf{v}| = \sqrt{x^2 + y^2 + z^2}$$

1. Magnitude of \mathbf{a} :

$$|\mathbf{a}| = \sqrt{(2)^2 + (-1)^2 + (1)^2} = \sqrt{6}$$

2. Magnitude of \mathbf{b} :

$$|\mathbf{b}| = \sqrt{(1)^2 + (-3)^2 + (-5)^2} = \sqrt{35}$$

3. Magnitude of \mathbf{c} :

$$|\mathbf{c}| = \sqrt{(-3)^2 + (4)^2 + (4)^2} = \sqrt{41}$$

A triangle is a right-angled triangle if the dot product between two vectors representing two sides of the triangle is zero. Let's calculate the dot product of the vectors \mathbf{a} and \mathbf{b} , \mathbf{b} and \mathbf{c} , and \mathbf{c} and \mathbf{a} .

1. Dot product $\mathbf{a} \cdot \mathbf{b}$:

$$\mathbf{a} \cdot \mathbf{b} = (2)(1) + (-1)(-3) + (1)(-5) = 0$$

Since $\mathbf{a} \cdot \mathbf{b} = 0$, the vectors \mathbf{a} and \mathbf{b} are perpendicular, meaning the angle between them is 90° .

Quick Tip

Use dot products to identify right angles in vector triangles.

13. Let \vec{a} be any vector such that $|\vec{a}| = a$. The value of $|\vec{a} \times \hat{i}|^2 + |\vec{a} \times \hat{j}|^2 + |\vec{a} \times \hat{k}|^2$ is:

(A) a^2

(B) $2a^2$

(C) $3a^2$

(D) 0

Correct Answer: (B) $2a^2$

Solution:

Step 1: Recall the formula for cross product magnitudes

The magnitude of the cross product is:

$$|\vec{a} \times \hat{i}| = |\vec{a}||\hat{i}| \sin \theta.$$

Step 2: Evaluate each term

For $\vec{a} \times \hat{i}$, $\vec{a} \times \hat{j}$, and $\vec{a} \times \hat{k}$, the contributions along two directions add up, giving:

$$|\vec{a} \times \hat{i}|^2 + |\vec{a} \times \hat{j}|^2 + |\vec{a} \times \hat{k}|^2 = 2a^2.$$

Step 3: Verify the options

The correct result is $2a^2$, matching option (B).

Quick Tip

Cross product magnitudes depend on sine of the angle between vectors.

14. The vector equation of a line passing through the point $(1, -1, 0)$ and parallel to the y -axis is:

(A) $\vec{r} = \hat{i} - \hat{j} + \lambda(\hat{i} - \hat{j})$

(B) $\vec{r} = \hat{i} - \hat{j} + \lambda\hat{j}$

(C) $\vec{r} = \hat{i} - \hat{j} + \lambda\hat{k}$

(D) $\vec{r} = \lambda\hat{j}$

Correct Answer: (B) $\vec{r} = \hat{i} - \hat{j} + \lambda\hat{j}$

Solution:

Step 1: Identify the direction vector

The direction vector is along the y -axis, so it is \hat{j} .

Step 2: Write the line equation

The vector equation of a line passing through $(1, -1, 0)$ is:

$$\vec{r} = \vec{r}_0 + \lambda\vec{d},$$

where $\vec{r}_0 = \hat{i} - \hat{j}$ and $\vec{d} = \hat{j}$.

Step 3: Substitute values

$$\vec{r} = (\hat{i} - \hat{j}) + \lambda\hat{j} = \hat{i} - \hat{j} + \lambda\hat{j}.$$

Step 4: Verify the options

The correct equation is $\vec{r} = \hat{i} - \hat{j} + \lambda\hat{j}$, which matches option (B).

Quick Tip

Vector equations of lines require a point and a direction vector.

15. The lines $\frac{1-x}{2} = \frac{y-1}{3} = \frac{z}{1}$ and $\frac{2x-3}{2p} = \frac{y}{-1} = \frac{z-4}{7}$ are perpendicular to each other for p equal to:

- (A) $-\frac{1}{2}$
- (B) $\frac{1}{2}$
- (C) 2
- (D) 3

Correct Answer: (C) 2

Solution: Step 1: Represent the lines in symmetric form: For the first line (L_1), the symmetric equation is given as:

$$\frac{1-x}{2} = \frac{y-1}{3} = \frac{z}{1}$$

Rewriting this in parametric form:

$$x = 1 - 2t, \quad y = 1 + 3t, \quad z = t$$

The direction ratios of L_1 are:

$$a_1 = -2, \quad b_1 = 3, \quad c_1 = 1$$

For the second line (L_2), the symmetric equation is given as:

$$\frac{2x-3}{2p} = \frac{y}{-1} = \frac{z-4}{7}$$

Rewriting this in parametric form:

$$x = \frac{3}{2} + pt, \quad y = -t, \quad z = 4 + 7t$$

The direction ratios of L_2 are:

$$a_2 = p, \quad b_2 = -1, \quad c_2 = 7$$

Step 2: Apply the condition for perpendicularity: Two lines are perpendicular if the dot product of their direction ratios is zero:

$$a_1a_2 + b_1b_2 + c_1c_2 = 0$$

Substituting the direction ratios of L_1 and L_2 :

$$(-2)(p) + (3)(-1) + (1)(7) = 0$$

Simplify the equation:

$$-2p - 3 + 7 = 0$$

$$-2p + 4 = 0$$

$$p = 2$$

Step 3: Verify the result: For $p = 2$, the direction ratios of L_2 become:

$$a_2 = 2, b_2 = -1, c_2 = 7$$

The dot product with L_1 is:

$$(-2)(2) + (3)(-1) + (1)(7) = -4 - 3 + 7 = 0$$

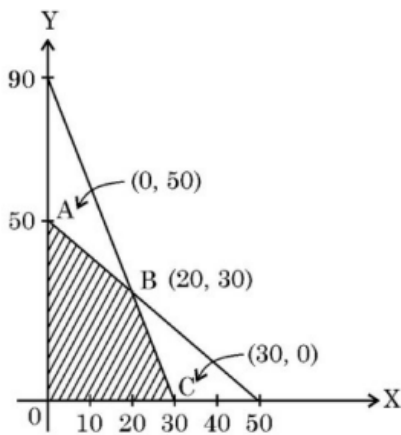
Thus, the lines are perpendicular.

Conclusion: The value of p is 2.

Quick Tip

To check if two lines are perpendicular, calculate the dot product of their direction ratios. If the dot product is zero, the lines are perpendicular. Always express the lines in symmetric or parametric form to extract direction ratios easily.

16. The maximum value of $Z = 4x + y$ for a L.P.P. whose feasible region is given below is:



- (A) 50
- (B) 110
- (C) 120
- (D) 170

Correct Answer: (C) 120

Solution:

Step 1: Identify the corner points of the feasible region

From the graph, the vertices of the feasible region are:

$$A(0, 50), B(20, 30), C(30, 0).$$

Step 2: Substitute corner points into $Z = 4x + y$

Evaluate Z at each vertex:

$$Z_A = 4(0) + 50 = 50,$$

$$Z_B = 4(20) + 30 = 110,$$

$$Z_C = 4(30) + 0 = 120.$$

Step 3: Find the maximum value

The maximum value of Z occurs at $C(30, 0)$, where $Z = 120$.

Step 4: Verify the options

The maximum value is 120, which corresponds to option (C).

Quick Tip

For L.P.P., always evaluate the objective function at all vertices of the feasible region.

17. The probability distribution of a random variable X is:

X	0	1	2	3	4
$P(X)$	0.1	k	$2k$	k	0.1

where k is some unknown constant. The probability that the random variable X takes the value 2 is:

- (A) $\frac{1}{5}$
- (B) $\frac{2}{5}$
- (C) $\frac{4}{5}$
- (D) 1

Correct Answer: (B) $\frac{2}{5}$

Solution: Step 1: Write the given equation: The total probability is given as:

$$P(0) + P(1) + P(2) + P(3) + P(4) = 1$$

Substituting the known values:

$$0.1 + k + 2k + k + 0.1 = 1$$

where $P(1) = k$, $P(2) = 2k$, and $P(3) = k$.

Step 2: Simplify the equation: Combine the terms:

$$0.2 + 4k = 1$$

Subtract 0.2 from both sides:

$$4k = 0.8$$

Divide by 4 to find k :

$$k = 0.2 = \frac{1}{5}$$

Step 3: Find $P(2)$: Given $P(2) = 2k$, substitute the value of k :

$$P(2) = 2 \times \frac{1}{5} = \frac{2}{5}$$

Conclusion: The value of $P(2)$ is $\frac{2}{5}$.

Quick Tip

To solve probability equations, substitute the known values and simplify step by step. Ensure that the sum of all probabilities equals 1, as this is the fundamental property of probability distributions.

18. The function $f(x) = kx - \sin x$ is strictly increasing for:

- (A) $k > 1$
- (B) $k < 1$
- (C) $k > -1$

(D) $k < -1$

Correct Answer: (A) $k > 1$

Solution:

Step 1: Find the derivative

The derivative of $f(x)$ is:

$$f'(x) = k - \cos x.$$

Step 2: Condition for increasing function

For $f(x)$ to be strictly increasing:

$$f'(x) > 0 \implies k - \cos x > 0 \implies k > \cos x.$$

Step 3: Maximum value of $\cos x$

The maximum value of $\cos x$ is 1. Therefore:

$$k > 1.$$

Step 4: Verify the options

The function is strictly increasing for $k > 1$, which matches option (A).

Quick Tip

For strict monotonicity, check the sign of the derivative over the entire domain.

ASSERTION-REASON BASED QUESTIONS

Questions number 19 and 20 are Assertion and Reason based questions. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below.

19. Assertion (A): The relation $R = \{(x, y) : (x + y) \text{ is a prime number and } x, y \in \mathbb{N}\}$ is not a reflexive relation.

Reason (R): The number $2n$ is composite for all natural numbers n .

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

(B) Both Assertion (A) and Reason (R) are true and Reason (R) is not the correct explanation of Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

Correct Answer: (C) Assertion (A) is true, but Reason (R) is false.

Solution:

Step 1: Analyze Assertion (A)

For R to be reflexive, (x, x) must belong to R for all $x \in \mathbb{N}$. This means $x + x = 2x$ must be a prime number. However, for $x > 1$, $2x$ is not a prime number as it is divisible by 2. Therefore, R is not reflexive, and Assertion (A) is true.

Step 2: Analyze Reason (R)

The Reason states that $2n$ is composite for all n . This is false because when $n = 1$, $2n = 2$, which is a prime number. Therefore, Reason (R) is false.

Step 3: Conclusion

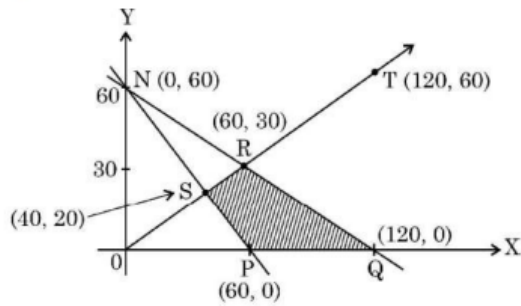
Since Assertion (A) is true and Reason (R) is false, the correct answer is option (C).

Quick Tip

A relation is reflexive if every element relates to itself; check this condition for all elements.

20. Assertion (A): The corner points of the bounded feasible region of a L.P.P. are shown below. The maximum value of $Z = x + 2y$ occurs at infinite points.

Reason (R): The optimal solution of a LPP having bounded feasible region must occur at corner points.



- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
- (B) Both Assertion (A) and Reason (R) are true and Reason (R) is not the correct explanation of Assertion (A).
- (C) Assertion (A) is true, but Reason (R) is false.
- (D) Assertion (A) is false, but Reason (R) is true.

Correct Answer: (B) Both Assertion (A) and Reason (R) are true and Reason (R) is not the correct explanation of Assertion (A).

Solution:

Step 1: Analyze Assertion (A)

From the graph, the line $Z = x + 2y$ passes through two corner points $(60, 0)$ and $(120, 60)$, providing the same maximum value. This indicates that the maximum value occurs at infinite points along this segment. Thus, Assertion (A) is true.

Step 2: Analyze Reason (R)

In general, the optimal solution of an LPP occurs at corner points of the feasible region. This is true; however, in this case, the solution lies along a line segment connecting two corner points. Thus, Reason (R) is not the correct explanation of Assertion (A).

Step 3: Conclusion

Both Assertion (A) and Reason (R) are true, but Reason (R) does not explain Assertion (A). Hence, the correct answer is option (B).

Quick Tip

In linear programming, always check if the objective function is constant along any edge of the feasible region.

SECTION B

This section comprises very short answer (VSA) type questions of 2 marks each.

21(a). Express $\tan^{-1}\left(\frac{\cos x}{1-\sin x}\right)$, where $-\frac{\pi}{2} < x < \frac{\pi}{2}$, in the simplest form.

Correct Answer: $\frac{\pi}{4} + \frac{x}{2}$

Solution:

Step 1: Simplify the expression inside \tan^{-1}

The given expression is:

$$\tan^{-1}\left(\frac{\cos x}{1-\sin x}\right).$$

Using trigonometric identities, rewrite:

$$1 - \sin x = \left(\cos^2 \frac{x}{2} + \sin^2 \frac{x}{2}\right) - 2 \sin \frac{x}{2} \cos \frac{x}{2} = \left(\cos \frac{x}{2} - \sin \frac{x}{2}\right)^2.$$

Step 2: Transform into a single tangent function

Substituting $1 - \sin x$ and $\cos x = \left(\cos^2 \frac{x}{2} - \sin^2 \frac{x}{2}\right)$:

$$\tan^{-1}\left(\frac{\cos x}{1-\sin x}\right) = \tan^{-1}\left[\tan\left(\frac{\pi}{4} + \frac{x}{2}\right)\right].$$

Step 3: Simplify using $\tan^{-1} \tan y = y$

Since $-\frac{\pi}{2} < x < \frac{\pi}{2}$, we simplify:

$$\tan^{-1}\left[\tan\left(\frac{\pi}{4} + \frac{x}{2}\right)\right] = \frac{\pi}{4} + \frac{x}{2}.$$

Conclusion: The simplest form is $\frac{\pi}{4} + \frac{x}{2}$.

Quick Tip

For expressions involving \tan^{-1} , rewrite in terms of trigonometric identities to simplify.

OR

21(b). Find the principal value of $\tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{\sqrt{2}}\right)$.

Correct Answer: $\frac{2\pi}{3}$

Solution:

Step 1: Evaluate each term

$$\tan^{-1}(1) = \frac{\pi}{4}, \quad \cos^{-1}\left(-\frac{1}{2}\right) = \pi - \frac{\pi}{3}, \quad \sin^{-1}\left(-\frac{1}{\sqrt{2}}\right) = -\frac{\pi}{4}.$$

Step 2: Add the terms

$$\frac{\pi}{4} + \left(\pi - \frac{\pi}{3}\right) - \frac{\pi}{4}.$$

Simplify:

$$\frac{\pi}{4} + \pi - \frac{\pi}{3} - \frac{\pi}{4} = \frac{2\pi}{3}.$$

Conclusion: The principal value is $\frac{2\pi}{3}$.

Quick Tip

Always compute inverse trigonometric values in their principal ranges.

22(a). If $y = \cos^3(\sec^2 2t)$, find $\frac{dy}{dt}$.

Correct Answer: $-12 \cos^2(\sec^2 2t) \sin(\sec^2 2t) \sec^2 2t \tan 2t$

Solution:

Step 1: Differentiate using the chain rule

We have:

$$\frac{dy}{dt} = 3 \cos^2(\sec^2 2t) \cdot [-\sin(\sec^2 2t)] \cdot \frac{d}{dt}(\sec^2 2t).$$

Step 2: Simplify derivatives

$$\frac{d}{dt}(\sec^2 2t) = 2 \sec^2 2t \tan 2t \cdot 2.$$

Substitute back:

$$\frac{dy}{dt} = -12 \cos^2(\sec^2 2t) \sin(\sec^2 2t) \sec^2 2t \tan 2t.$$

Conclusion: The derivative is $-12 \cos^2(\sec^2 2t) \sin(\sec^2 2t) \sec^2 2t \tan 2t$.

Quick Tip

When differentiating trigonometric functions, apply the chain rule carefully.

OR

22(b). If $x^y = e^{x-y}$, prove that $\frac{dy}{dx} = \frac{\log x}{(1+\log x)^2}$.

Correct Answer: $\frac{\log x}{(1+\log x)^2}$

Solution:

Step 1: Take logarithms of both sides

The given equation is:

$$x^y = e^{x-y}.$$

Taking the natural logarithm on both sides, we get:

$$\log(x^y) = \log(e^{x-y}).$$

Step 2: Simplify using logarithmic properties

Using the properties of logarithms:

$$y \log x = x - y.$$

Rearranging the terms to express y :

$$y(1 + \log x) = x.$$

Thus, we have:

$$y = \frac{x}{1 + \log x}.$$

Step 3: Differentiate y with respect to x

Differentiate both sides of $y = \frac{x}{1 + \log x}$ with respect to x using the quotient rule:

$$\frac{dy}{dx} = \frac{(1 + \log x) \cdot 1 - x \cdot \frac{1}{x}}{(1 + \log x)^2}.$$

Step 4: Simplify the derivative

Simplify the numerator:

$$\frac{dy}{dx} = \frac{(1 + \log x) - 1}{(1 + \log x)^2}.$$

This reduces to:

$$\frac{dy}{dx} = \frac{\log x}{(1 + \log x)^2}.$$

Conclusion: The derivative is proved to be:

$$\frac{dy}{dx} = \frac{\log x}{(1 + \log x)^2}.$$

Quick Tip

For functions involving both x and y , logarithmic differentiation often simplifies the process.

23. Find the interval in which the function $f(x) = x^4 - 4x^3 + 10$ is strictly decreasing.

Correct Answer: $(-\infty, 0) \cup (0, 3)$

Solution:

Step 1: Find the derivative of $f(x)$

$$f'(x) = 4x^3 - 12x^2 = 4x^2(x - 3).$$

Step 2: Solve $f'(x) < 0$

Factorize:

$$f'(x) < 0 \implies 4x^2(x - 3) < 0.$$

The critical points are $x = 0$ and $x = 3$. Analyze the sign of $f'(x)$ in intervals:

$$(-\infty, 0), (0, 3), \text{ and } (3, \infty).$$

Step 3: Determine intervals

$f'(x) < 0$ on $(-\infty, 0) \cup (0, 3)$.

Conclusion: The function is strictly decreasing on $(-\infty, 0) \cup (0, 3)$.

Quick Tip

Use the derivative sign test to find intervals of increase or decrease.

24. The volume of a cube is increasing at the rate of $6 \text{ cm}^3/\text{s}$. How fast is the surface area of the cube increasing, when the length of an edge is 8 cm ?

Correct Answer: $3 \text{ cm}^2/\text{s}$

Solution:

Step 1: Relate volume and surface area of the cube

The volume of the cube is:

$$V = x^3,$$

where x is the length of an edge. Differentiating with respect to t , we get:

$$\frac{dV}{dt} = 3x^2 \frac{dx}{dt}.$$

Substitute $\frac{dV}{dt} = 6 \text{ cm}^3/\text{s}$:

$$6 = 3(8)^2 \frac{dx}{dt}.$$

Solve for $\frac{dx}{dt}$:

$$\frac{dx}{dt} = \frac{6}{192} = \frac{1}{32} \text{ cm/s.}$$

Step 2: Find the rate of change of surface area

The surface area of the cube is:

$$S = 6x^2.$$

Differentiating with respect to t , we get:

$$\frac{dS}{dt} = 12x \frac{dx}{dt}.$$

Substitute $x = 8 \text{ cm}$ and $\frac{dx}{dt} = \frac{1}{32}$:

$$\frac{dS}{dt} = 12(8) \left(\frac{1}{32} \right) = 3 \text{ cm}^2/\text{s}.$$

Conclusion: The surface area of the cube is increasing at $3 \text{ cm}^2/\text{s}$.

Quick Tip

For related rates problems, identify the variables, write their relationships, and differentiate with respect to time.

25. Evaluate $\int \frac{dx}{x(x^2-1)}$.

Correct Answer: $\frac{1}{2} \log \left| \frac{x^2-1}{x^2} \right| + C$

Solution:

Step 1: Decompose the integral

The given integral is:

$$I = \int \frac{dx}{x(x^2-1)}.$$

Decompose into partial fractions:

$$\frac{1}{x(x^2-1)} = \frac{1}{x} - \frac{1}{x^2-1}.$$

Step 2: Simplify and integrate

The integral becomes:

$$I = \int \frac{1}{x} dx - \int \frac{1}{x^2 - 1} dx.$$

For $\int \frac{1}{x^2 - 1} dx$, use partial fractions:

$$\frac{1}{x^2 - 1} = \frac{1}{2} \left(\frac{1}{x - 1} - \frac{1}{x + 1} \right).$$

Thus:

$$I = \log |x| - \frac{1}{2} \log |x - 1| + \frac{1}{2} \log |x + 1| + C.$$

Step 3: Combine terms

Combine the logarithmic terms:

$$I = \frac{1}{2} \log \left| \frac{x^2 - 1}{x^2} \right| + C.$$

Conclusion: The integral evaluates to:

$$\int \frac{dx}{x(x^2 - 1)} = \frac{1}{2} \log \left| \frac{x^2 - 1}{x^2} \right| + C.$$

Quick Tip

For rational functions, decompose into partial fractions for simpler integration.

SECTION C

This section comprises short answer (SA) type questions of 3 marks each

26. Given that $y = (\sin x)^x \cdot x^{\sin x} + a^x$, **find** $\frac{dy}{dx}$.

Correct Answer: $(\sin x)^x x^{\sin x} [\log(\sin x) + x \cot x + \sin x \log x + \cos x \log x] + a^x \log a$

Solution:

Step 1: Express y in terms of simpler functions

We have:

$$y = (\sin x)^x \cdot x^{\sin x} + a^x.$$

Let $u = (\sin x)^x \cdot x^{\sin x}$. Thus:

$$\frac{dy}{dx} = \frac{du}{dx} + \frac{d(a^x)}{dx}.$$

Step 2: Differentiate u

Taking logarithm:

$$\log u = x \log(\sin x) + \sin x \log x.$$

Differentiating with respect to x :

$$\frac{1}{u} \frac{du}{dx} = \log(\sin x) + x \cot x + \log x \cdot \cos x + \sin x \cdot \frac{1}{x}.$$

Thus:

$$\frac{du}{dx} = u \cdot [\log(\sin x) + x \cot x + \sin x \log x + \cos x \log x].$$

Step 3: Add the derivative of a^x

The derivative of a^x is:

$$\frac{d}{dx}(a^x) = a^x \log a.$$

Conclusion: The derivative is:

$$\frac{dy}{dx} = (\sin x)^x x^{\sin x} [\log(\sin x) + x \cot x + \sin x \log x + \cos x \log x] + a^x \log a.$$

Quick Tip

For functions involving powers and products, logarithmic differentiation simplifies the process.

27(a). Evaluate $\int_0^{\pi/4} \frac{x}{1+\cos 2x+\sin 2x} dx$.

Correct Answer: $\frac{\pi}{16} \log 2$

Solution:

Step 1: Apply symmetry property of definite integrals

Let:

$$I = \int_0^{\pi/4} \frac{x}{1 + \cos 2x + \sin 2x} dx.$$

Using the property $\int_0^a f(x) dx = \int_0^a f(a-x) dx$, we get:

$$I = \int_0^{\pi/4} \frac{\pi/4 - x}{1 + \cos 2x + \sin 2x} dx.$$

Step 2: Combine integrals

Adding the two forms of I :

$$2I = \int_0^{\pi/4} \frac{\pi/4}{1 + \cos 2x + \sin 2x} dx.$$

Step 3: Simplify the integrand

Rewrite $1 + \cos 2x + \sin 2x$:

$$\cos 2x + \sin 2x = \sqrt{2} \sin(2x + \pi/4),$$

and simplify:

$$I = \frac{\pi}{16} \int_0^{\pi/4} \frac{1}{\cos^2 x + \sin x \cos x} dx.$$

Step 4: Integrate and simplify

The integral evaluates to:

$$I = \frac{\pi}{16} (\log |1 + \tan x|)_0^{\pi/4}.$$

Substitute limits:

$$I = \frac{\pi}{16} \log 2.$$

Conclusion: The integral evaluates to $\frac{\pi}{16} \log 2$.

Quick Tip

For definite integrals with symmetric limits, apply symmetry properties to simplify.

OR

27(b). Find: $\int e^x \left[\frac{1}{(1+x^2)^{3/2}} + \frac{x}{\sqrt{1+x^2}} \right] dx$.

Correct Answer: $e^x \frac{x}{\sqrt{1+x^2}} + C$

Solution:

Step 1: The given integral can be written as:

$$I = \int e^x \left(\frac{x}{\sqrt{1+x^2}} + \frac{1}{(1+x^2)^{\frac{3}{2}}} \right) dx$$

Let:

$$f(x) = \frac{x}{\sqrt{1+x^2}}$$

Step 2: Now, calculate the derivative of $f(x)$:

$$f'(x) = \frac{\sqrt{1+x^2} - \frac{x \cdot x}{\sqrt{1+x^2}}}{1+x^2} = \frac{\sqrt{1+x^2} - \frac{x^2}{\sqrt{1+x^2}}}{1+x^2}$$

Simplify the numerator:

$$f'(x) = \frac{\sqrt{1+x^2} - \frac{x^2}{\sqrt{1+x^2}}}{1+x^2} = \frac{1}{(1+x^2)^{\frac{3}{2}}}$$

Thus, the integral becomes:

$$I = \int e^x (f(x) + f'(x)) dx$$

Step 3: Using the standard result:

$$\int e^x (f(x) + f'(x)) dx = e^x f(x) + C$$

Substitute $f(x) = \frac{x}{\sqrt{1+x^2}}$:

$$I = e^x \frac{x}{\sqrt{1+x^2}} + C$$

Final Answer:

$$I = e^x \frac{x}{\sqrt{1+x^2}} + C$$

Explanation:

1. Splitting the Integral: The given integral is split into terms containing $\frac{x}{\sqrt{1+x^2}}$ and $\frac{1}{(1+x^2)^{\frac{3}{2}}}$.
2. Defining $f(x)$: The function $f(x)$ is chosen as $\frac{x}{\sqrt{1+x^2}}$ because its derivative results in the second term, $\frac{1}{(1+x^2)^{\frac{3}{2}}}$.
3. Applying the Formula: The integral formula for $\int e^x(f(x) + f'(x))dx = e^x f(x) + C$ is directly applied.
4. Substitution: Finally, substituting $f(x)$ into the formula gives the result.

Quick Tip

For integrals involving a combination of functions $f(x)$ and $f'(x)$, use the substitution $u = f(x)$ to simplify.

28. Find: $\int \frac{3x+5}{\sqrt{x^2+2x+4}} dx$.

Correct Answer: $3\sqrt{x^2 + 2x + 4} + 2 \log |x + 1 + \sqrt{x^2 + 2x + 4}| + C$

Solution:

Step 1: Simplify the denominator

The denominator $\sqrt{x^2 + 2x + 4}$ can be written as $\sqrt{(x + 1)^2 + 3}$.

Step 2: Split the integral

The integral becomes:

$$I = \int \frac{3x + 5}{\sqrt{x^2 + 2x + 4}} dx = \int \frac{2x + 2}{\sqrt{x^2 + 2x + 4}} dx + \int \frac{1}{\sqrt{x^2 + 2x + 4}} dx.$$

Step 3: Solve each part separately

For the first part:

$$\int \frac{2x + 2}{\sqrt{x^2 + 2x + 4}} dx = 2\sqrt{x^2 + 2x + 4}.$$

For the second part:

$$\int \frac{1}{\sqrt{x^2 + 2x + 4}} dx = \log |x + 1 + \sqrt{x^2 + 2x + 4}|.$$

Step 4: Combine results

Combine the results to get:

$$I = 3\sqrt{x^2 + 2x + 4} + 2 \log \left| x + 1 + \sqrt{x^2 + 2x + 4} \right| + C.$$

Conclusion: The integral evaluates to:

$$\int \frac{3x + 5}{\sqrt{x^2 + 2x + 4}} dx = 3\sqrt{x^2 + 2x + 4} + 2 \log \left| x + 1 + \sqrt{x^2 + 2x + 4} \right| + C.$$

Quick Tip

For integrals with quadratic expressions inside square roots, complete the square to simplify.

29(a). Find the particular solution of the differential equation $\frac{dy}{dx} = y \cot 2x$, given that $y\left(\frac{\pi}{4}\right) = 2$.

Correct Answer: $y = 2\sqrt{\sin 2x}$

Solution:

Step 1: Separating the Variables

Rewriting the equation:

$$\frac{1}{y} dy = \cot 2x dx$$

Step 2: Integrating Both Sides

Integrate both sides:

$$\int \frac{1}{y} dy = \int \cot 2x dx$$

The left-hand side becomes:

$$\log |y|$$

The right-hand side uses the integral of $\cot 2x$:

$$\int \cot 2x dx = \frac{1}{2} \log |\sin 2x|$$

So the equation becomes:

$$\log |y| = \frac{1}{2} \log |\sin 2x| + \log c$$

Here, $\log c$ is the constant of integration.

Step 3: Simplify the Expression

Combine the logarithms:

$$\log |y| = \log (c\sqrt{\sin 2x})$$

Exponentiate both sides to remove the logarithm:

$$y = c\sqrt{\sin 2x}$$

Step 4: Finding the Particular Solution

We are given the condition $y\left(\frac{\pi}{4}\right) = 2$. Substitute $x = \frac{\pi}{4}$ and $y = 2$ into the solution:

$$2 = c\sqrt{\sin\left(2 \cdot \frac{\pi}{4}\right)}$$

Simplify:

$$2 = c\sqrt{\sin\left(\frac{\pi}{2}\right)}$$

Since $\sin\left(\frac{\pi}{2}\right) = 1$, we have:

$$2 = c \cdot 1 \quad \Rightarrow \quad c = 2$$

Step 5: Final Solution

Substitute $c = 2$ back into the solution:

$$y = 2\sqrt{\sin 2x}$$

Final Answer:

$$\boxed{y = 2\sqrt{\sin 2x}}$$

This is the required particular solution to the given differential equation.

Quick Tip

For separable differential equations, isolate y and x , then integrate both sides.

OR

29(b). Find the particular solution of the differential equation $(x(e)^{y/x} + y) dx = x dy$, given that $y = 1$ when $x = 1$.

Solution:

Step 1: Rewrite the given differential equation

Rearrange the equation to separate variables:

$$\frac{dy}{dx} = x(e)^{y/x} + \frac{y}{x}$$

This is a homogeneous differential equation.

Step 2: Substitution

Let:

$$y = vx \quad \Rightarrow \quad \frac{dy}{dx} = v + x \frac{dv}{dx}$$

Substitute $y = vx$ into the equation:

$$v + x \frac{dv}{dx} = x(e)^{y/x} + v$$

Simplify:

$$x \frac{dv}{dx} = x(e)^{y/x}$$

Step 3: Solve for v

Rearrange the equation for integration:

$$\int e^{-v} dv = \int \frac{1}{x} dx$$

Evaluate the integrals:

$$-e^{-v} = \log |x| + c$$

Substitute $v = \frac{y}{x}$:

$$-e^{-\frac{y}{x}} = \log |x| + c \quad \dots (1)$$

Step 4: Apply the initial condition

We are given that $y = 1$ when $x = 1$. Substitute $x = 1$ and $y = 1$ into equation (1):

$$-e^{-\frac{1}{1}} = \log |1| + c$$

Since $\log |1| = 0$:

$$-e^{-1} = c$$

Thus:

$$c = -e^{-1}$$

Step 5: Substitute c back into the solution

Substitute $c = -e^{-1}$ into equation (1):

$$-e^{-\frac{y}{x}} = \log |x| - e^{-1}$$

Rearranging:

$$\log |x| + e^{-\frac{y}{x}} = e^{-1}$$

Final Answer:

$$\log |x| + e^{-\frac{y}{x}} = e^{-1}$$

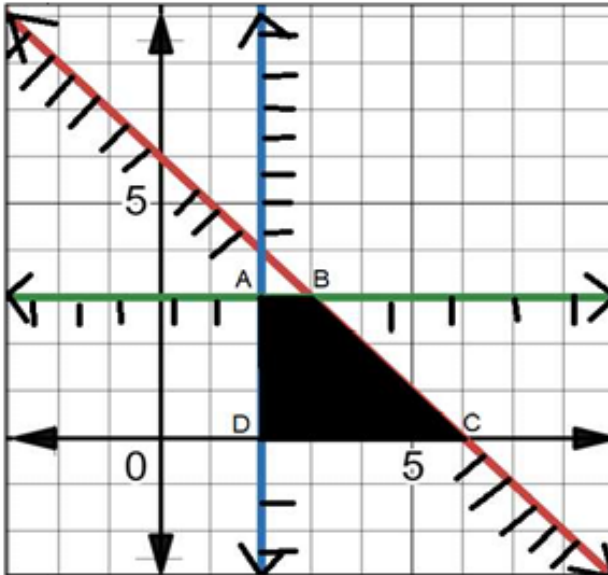
Quick Tip

For homogeneous differential equations, use substitution $y = vx$ to simplify the equation and separate variables.

30. Solve the following linear programming problem graphically: Maximise $Z = 2x + 3y$, subject to the constraints: $x + y \leq 6$, $x \geq 2$, $y \geq 3$, $x \geq 0$, $y \geq 0$.

Correct Answer: Maximum value is $Z = 15$ at $(3, 3)$.

Solution:



Step 1: Plot the constraints

Graph the constraints $x + y \leq 6$, $x \geq 2$, $y \geq 3$, $x \geq 0$, $y \geq 0$ on the Cartesian plane. The feasible region is the shaded region bounded by these lines.

Step 2: Find corner points

The corner points of the feasible region are:

$$A(2, 3), B(3, 3), C(6, 0), D(2, 0).$$

Step 3: Evaluate $Z = 2x + 3y$ at each point

$$Z(2, 3) = 13, \quad Z(3, 3) = 15, \quad Z(6, 0) = 12, \quad Z(2, 0) = 4.$$

Conclusion: The maximum value of Z is 15, which occurs at $(3, 3)$.

Quick Tip

In linear programming, always evaluate the objective function at the corner points of the feasible region.

31(a). A card from a well-shuffled deck of 52 playing cards is lost. From the remaining cards of the pack, a card is drawn at random and is found to be a King. Find the proba-

bility of the lost card being a King.

Solution:

Step 1: Define the events

Let E_1 be the event that the lost card is a King, and E_2 be the event that the lost card is not a King. Let A be the event of drawing a King from the remaining 51 cards.

Step 2: Assign probabilities to the events

$$P(E_1) = \frac{1}{13}, \quad P(E_2) = \frac{12}{13}, \quad P(A|E_1) = \frac{3}{51}, \quad P(A|E_2) = \frac{4}{51}$$

Step 3: Use Bayes' Theorem

The required probability is $P(E_1|A)$, which is given by:

$$P(E_1|A) = \frac{P(A|E_1) \cdot P(E_1)}{P(A|E_1) \cdot P(E_1) + P(A|E_2) \cdot P(E_2)}$$

Substituting the values:

$$P(E_1|A) = \frac{\frac{1}{13} \cdot \frac{3}{51}}{\frac{1}{13} \cdot \frac{3}{51} + \frac{12}{13} \cdot \frac{4}{51}} = \frac{\frac{3}{663}}{\frac{3}{663} + \frac{48}{663}} = \frac{3}{51} = \frac{1}{17}$$

Step 4: Final result

The probability that the lost card is a King is $\frac{1}{17}$.

Quick Tip

When solving problems involving missing or conditional probabilities, use Bayes' Theorem and clearly define all events and conditional probabilities.

OR

31(b). A biased die is twice as likely to show an even number as an odd number. If such a die is thrown twice, find the probability distribution of the number of sixes. Also, find the mean of the distribution.

Solution:

Step 1: Assign probabilities

Let $P(3) = P(5) = p$, so $P(2) = P(4) = P(6) = 2p$.

As the total probability is 1:

$$P(1) + P(2) + P(3) + P(4) + P(5) + P(6) = 1 \implies 9p = 1 \implies p = \frac{1}{9}$$

Thus, $P(6) = 2p = \frac{2}{9}$, and $P(\text{Not getting six}) = 1 - P(6) = \frac{7}{9}$.

Step 2: Define the random variable X

Let X represent the number of sixes. The possible values of X are 0, 1, 2.

Step 3: Compute probabilities for X

$$P(X = 0) = \left(\frac{7}{9}\right)^2 = \frac{49}{81}, \quad P(X = 1) = 2 \cdot \frac{2}{9} \cdot \frac{7}{9} = \frac{28}{81}, \quad P(X = 2) = \left(\frac{2}{9}\right)^2 = \frac{4}{81}$$

Step 4: Probability distribution of X

X	$P(X)$
0	$\frac{49}{81}$
1	$\frac{28}{81}$
2	$\frac{4}{81}$

Step 5: Compute the mean of X

The mean is given by:

$$\mu = \sum_{i=1}^3 X_i \cdot P(X_i) = 0 \cdot \frac{49}{81} + 1 \cdot \frac{28}{81} + 2 \cdot \frac{4}{81} = \frac{28}{81} + \frac{8}{81} = \frac{36}{81} = \frac{4}{9}$$

Step 6: Final result

The probability distribution of X is:

X	$P(X)$
0	$\frac{49}{81}$
1	$\frac{28}{81}$
2	$\frac{4}{81}$

The mean of the distribution is $\frac{4}{9}$.

Quick Tip

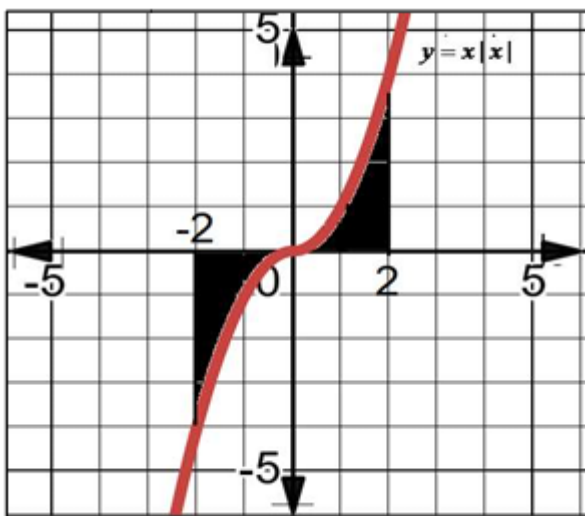
When working with biased probability distributions, ensure the total probability sums to 1 and carefully calculate probabilities for each outcome.

SECTION D

This section comprises long answer (LA) type questions of 5 marks each

32(a). Sketch the graph of $y = x|x|$ and hence find the area bounded by this curve, the X-axis, and the ordinates $x = -2$ and $x = 2$, using integration.

Solution:



Step 1: Rewrite the function $y = x|x|$

The function $y = x|x|$ can be expressed as:

$$y = \begin{cases} -x^2, & x < 0 \\ x^2, & x \geq 0 \end{cases}$$

Step 2: Graph the function

The graph of $y = x|x|$ is a parabola, concave downwards for $x < 0$ and concave upwards for

$x \geq 0$. (Refer to the attached graph.)

Step 3: Area computation using integration

The area of the shaded region between $x = -2$ and $x = 2$ is given by:

$$\text{Area} = \int_{-2}^2 |y| dx = 2 \int_0^2 x^2 dx$$

Step 4: Evaluate the integral

$$\int_0^2 x^2 dx = \left[\frac{x^3}{3} \right]_0^2 = \frac{2^3}{3} - \frac{0^3}{3} = \frac{8}{3}$$

Thus, the total area is:

$$\text{Area} = 2 \cdot \frac{8}{3} = \frac{16}{3}$$

Step 5: Final result

The area bounded by the curve $y = x|x|$, the X-axis, and the ordinates $x = -2$ and $x = 2$ is $\frac{16}{3}$.

Quick Tip

When finding the area bounded by curves, split the integral into regions where the function behaves differently (e.g., absolute values or piecewise functions).

OR

32(b). Using integration, find the area bounded by the ellipse $9x^2 + 25y^2 = 225$, the lines $x = -2, x = 2$, and the X-axis.

Solution:

Step 1: Rewrite the equation of the ellipse

The equation of the ellipse is:

$$9x^2 + 25y^2 = 225 \implies y = \pm \frac{3}{5} \sqrt{25 - x^2}$$

Step 2: Set up the integral for the area

The area of the region bounded by the ellipse, the X-axis, and the lines $x = -2$ and $x = 2$ is given by:

$$\text{Area} = 2 \int_0^2 \frac{3}{5} \sqrt{25 - x^2} dx$$

Step 3: Simplify the integral

Let $I = \int \sqrt{a^2 - x^2} dx$, where $a = 5$. Using the standard formula:

$$\int \sqrt{a^2 - x^2} dx = \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \left(\frac{x}{a} \right) + C$$

Step 4: Evaluate the integral

Substitute $a = 5$ and evaluate $\int_0^2 \sqrt{25 - x^2} dx$:

$$\int_0^2 \sqrt{25 - x^2} dx = \left[\frac{x}{2} \sqrt{25 - x^2} + \frac{25}{2} \sin^{-1} \left(\frac{x}{5} \right) \right]_0^2$$

At $x = 2$:

$$\frac{2}{2} \sqrt{25 - 2^2} + \frac{25}{2} \sin^{-1} \left(\frac{2}{5} \right) = \sqrt{21} + \frac{25}{2} \sin^{-1} \left(\frac{2}{5} \right)$$

At $x = 0$:

$$\frac{0}{2} \sqrt{25 - 0^2} + \frac{25}{2} \sin^{-1}(0) = 0$$

Thus:

$$\int_0^2 \sqrt{25 - x^2} dx = \sqrt{21} + \frac{25}{2} \sin^{-1} \left(\frac{2}{5} \right)$$

Step 5: Final area calculation

Multiply by $\frac{6}{5}$ to account for $\frac{3}{5}$ and the factor of 2:

$$\text{Area} = \frac{6}{5} \left(\sqrt{21} + \frac{25}{2} \sin^{-1} \left(\frac{2}{5} \right) \right)$$

Step 6: Final result

The area bounded by the ellipse, the X-axis, and the lines $x = -2$ and $x = 2$ is:

$$\frac{6\sqrt{21}}{5} + 15 \sin^{-1} \left(\frac{2}{5} \right)$$

Quick Tip

When integrating to find areas involving ellipses or circles, use symmetry and standard integral formulas for $\sqrt{a^2 - x^2}$.

33(a). Let $A = \mathbb{R} - \{5\}$ and $B = \mathbb{R} - \{1\}$. Consider the function $f : A \rightarrow B$, defined by $f(x) = \frac{x-3}{x-5}$. Show that f is one-one and onto.

Solution:

Step 1: Prove that f is one-one

Let $f(x_1) = f(x_2)$, where $x_1, x_2 \in A$:

$$\frac{x_1 - 3}{x_1 - 5} = \frac{x_2 - 3}{x_2 - 5}$$

Cross-multiply:

$$(x_1 - 3)(x_2 - 5) = (x_2 - 3)(x_1 - 5)$$

Simplify:

$$\begin{aligned}x_1x_2 - 5x_1 - 3x_2 + 15 &= x_1x_2 - 5x_2 - 3x_1 + 15 \\-5x_1 - 3x_2 &= -5x_2 - 3x_1 \implies 5(x_2 - x_1) = 3(x_2 - x_1)\end{aligned}$$

If $x_1 \neq x_2$, this leads to a contradiction. Hence, $x_1 = x_2$, proving f is one-one.

Step 2: Prove that f is onto

Let $y \in B$. Solve $f(x) = y$:

$$\begin{aligned}\frac{x - 3}{x - 5} = y &\implies x - 3 = y(x - 5) \\x - 3 &= yx - 5y \implies x - yx = -5y + 3 \\x(1 - y) &= -5y + 3 \implies x = \frac{-5y + 3}{1 - y}\end{aligned}$$

For $y \neq 1$ (since $y \in B$), x exists in A . Thus, f is onto.

Step 3: Conclude the function properties

Since f is both one-one and onto, f is a bijection.

Quick Tip

To prove a function is one-one, show that $f(x_1) = f(x_2)$ implies $x_1 = x_2$. To prove onto, solve $f(x) = y$ and ensure x lies in the domain.

OR

33(b). Check whether the relation S in the set of real numbers \mathbb{R} , defined by $S = \{(a, b) : a - b + \sqrt{2} \text{ is an irrational number}\}$, is reflexive, symmetric, or transitive.

Solution:

Step 1: Reflexivity

For $a \in \mathbb{R}$:

$$a - a + \sqrt{2} = \sqrt{2} \text{ is irrational.}$$

Thus, $(a, a) \in S$, and S is reflexive.

Step 2: Symmetry

Let $(a, b) \in S$, so:

$$a - b + \sqrt{2} \text{ is irrational.}$$

Now, check if $(b, a) \in S$:

$$b - a + \sqrt{2} \text{ may or may not be irrational.}$$

For example:

$$a = \sqrt{2}, b = 1 \implies a - b + \sqrt{2} = \sqrt{2} - 1 + \sqrt{2} = 2\sqrt{2} - 1 \text{ (irrational), but}$$

$$b - a + \sqrt{2} = 1 - \sqrt{2} + \sqrt{2} = 1 \text{ (rational).}$$

Thus, S is not symmetric.

Step 3: Transitivity

Let $(a, b) \in S$ and $(b, c) \in S$, so:

$$a - b + \sqrt{2} \text{ is irrational, and } b - c + \sqrt{2} \text{ is irrational.}$$

Check if $(a, c) \in S$:

$$a - c + \sqrt{2} = (a - b + \sqrt{2}) + (b - c + \sqrt{2}) - \sqrt{2} \text{ may or may not be irrational.}$$

For example:

$$a = 1, b = \sqrt{3}, c = \sqrt{3} - \sqrt{2} \implies a - c + \sqrt{2} = 1 - (\sqrt{3} - \sqrt{2}) + \sqrt{2} = 1 - \sqrt{3} + 2\sqrt{2}.$$

This is irrational, but a counterexample exists for other values. Thus, S is not transitive.

Step 4: Final conclusion

The relation S is reflexive but neither symmetric nor transitive.

Quick Tip

To test reflexivity, verify if $(a, a) \in S$ for all a . For symmetry and transitivity, check logical equivalence and counterexamples.

34. If $A = \begin{bmatrix} 2 & 1 & -3 \\ 3 & 2 & 1 \\ 1 & 2 & -1 \end{bmatrix}$, find A^{-1} and hence solve the following system of equations:

$$2x + y - 3z = 13, \quad 3x + 2y + z = 4, \quad x + 2y - z = 8$$

Solution:

Step 1: Compute A^{-1}

The given matrix A is:

$$A = \begin{bmatrix} 2 & 1 & -3 \\ 3 & 2 & 1 \\ 1 & 2 & -1 \end{bmatrix}$$

The determinant of A is:

$$|A| = 2(2 \cdot -1 - 1 \cdot 2) - 1(3 \cdot -1 - 1 \cdot 1) + (-3)(3 \cdot 2 - 2 \cdot 1) = -16$$

Since $|A| \neq 0$, A^{-1} exists. The adjoint of A is:

$$\text{adj}(A) = \begin{bmatrix} -4 & -5 & 7 \\ 4 & 1 & -11 \\ 4 & -3 & 1 \end{bmatrix}$$

Thus:

$$A^{-1} = \frac{1}{|A|} \text{adj}(A) = -\frac{1}{16} \begin{bmatrix} -4 & -5 & 7 \\ 4 & 1 & -11 \\ 4 & -3 & 1 \end{bmatrix}$$

Step 2: Represent the system in matrix form

The system of equations can be written as:

$$AX = B, \quad \text{where } A = \begin{bmatrix} 2 & 1 & -3 \\ 3 & 2 & 1 \\ 1 & 2 & -1 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad B = \begin{bmatrix} 13 \\ 4 \\ 8 \end{bmatrix}$$

To solve for X :

$$X = A^{-1}B$$

Step 3: Solve for X

$$X = -\frac{1}{16} \begin{bmatrix} -4 & -5 & 7 \\ 4 & 1 & -11 \\ 4 & -3 & 1 \end{bmatrix} \begin{bmatrix} 13 \\ 4 \\ 8 \end{bmatrix}$$
$$X = -\frac{1}{16} \begin{bmatrix} -16 \\ -32 \\ 48 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}$$

Step 4: Final result

The solution to the system of equations is:

$$x = 1, \quad y = 2, \quad z = -3$$

Quick Tip

Always verify the determinant before finding the inverse of a matrix. If $|A| = 0$, the inverse does not exist.

35(a). Find the distance between the line $\frac{x}{2} = \frac{2y-6}{4} = \frac{1-z}{-1}$ and another line parallel to it passing through the point $(4, 0, -5)$.

Solution:

Step 1: Standardize the equations of the lines

The given line L_1 is:

$$\frac{x}{2} = \frac{2y-6}{4} = \frac{1-z}{-1} \implies \vec{r}_1 = \vec{0} + \lambda(2\hat{i} + \hat{j} + \hat{k})$$

The line L_2 parallel to L_1 and passing through $(4, 0, -5)$ is:

$$\vec{r}_2 = (4\hat{i} - 5\hat{k}) + \mu(2\hat{i} + \hat{j} + \hat{k})$$

Step 2: Vector between the lines

Let $\vec{a}_2 - \vec{a}_1 = (4\hat{i} - 5\hat{k}) - (0) = 4\hat{i} - 5\hat{k}$. The direction vector $\vec{b} = 2\hat{i} + \hat{j} + \hat{k}$.

Step 3: Find the shortest distance

The shortest distance $S.D.$ is given by:

$$S.D. = \frac{|(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b})|}{|\vec{b}|}$$

Compute $\vec{b} \times (\vec{a}_2 - \vec{a}_1)$:

$$\vec{b} \times (\vec{a}_2 - \vec{a}_1) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & 1 \\ 4 & 0 & -5 \end{vmatrix} = 9\hat{i} - 16\hat{j} + 14\hat{k}$$

The magnitude:

$$|\vec{b}| = \sqrt{2^2 + 1^2 + 1^2} = 3$$
$$S.D. = \frac{\sqrt{81 + 256 + 196}}{3} = \frac{\sqrt{533}}{3} \text{ units.}$$

Quick Tip

For the shortest distance between skew or parallel lines, use the cross-product approach for accuracy.

OR

35(b). If the lines $\frac{x-1}{-3} = \frac{y-2}{2k} = \frac{z-3}{2}$ and $\frac{x-1}{3k} = \frac{y-1}{1} = \frac{z-6}{-7}$ are perpendicular to each other, find the value of k and hence write the vector equation of a line perpendicular to these two lines passing through the point $(3, -4, 7)$.

Solution:

Step 1: Find k

The direction ratios of the first line are $\langle -3, 2k, 2 \rangle$ and for the second line are $\langle 3k, 1, -7 \rangle$. Since the lines are perpendicular:

$$(-3)(3k) + (2k)(1) + (2)(-7) = 0$$

$$-9k + 2k - 14 = 0 \implies -7k = 14 \implies k = -2$$

Step 2: Find the vector equation of the perpendicular line

The direction vectors are:

$$\vec{b}_1 = \langle -3, -4, 2 \rangle, \quad \vec{b}_2 = \langle -6, 1, -7 \rangle$$

The perpendicular vector is:

$$\vec{b} = \vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -3 & -4 & 2 \\ -6 & 1 & -7 \end{vmatrix} = 26\hat{i} - 33\hat{j} - 27\hat{k}$$

The required equation is:

$$\vec{r} = \langle 3, -4, 7 \rangle + \lambda(26\hat{i} - 33\hat{j} - 27\hat{k})$$

Quick Tip

For perpendicular lines, use the dot product of their direction vectors to solve for unknown parameters.

Case Study - 1

36. A store has been selling calculators at Rs. 350 each. A market survey indicates that a reduction in price (p) of calculators increases the number of units (x) sold. The relation between the price and quantity sold is given by the demand function:

$$p = 450 - \frac{x}{2}.$$

Based on the above information, answer the following questions:



36.(i). Determine the number of units (x) that should be sold to maximise the revenue $R(x) = x \cdot p(x)$. Also verify the result.

Solution:

Step 1: Express revenue as a function of x

Revenue is given by:

$$R(x) = x \cdot p(x) = x \cdot \left(450 - \frac{x}{2}\right) = 450x - \frac{x^2}{2}.$$

Step 2: Differentiate to find critical points

The first derivative of $R(x)$ is:

$$\frac{dR}{dx} = 450 - x.$$

For maximum or minimum, set $\frac{dR}{dx} = 0$:

$$450 - x = 0 \implies x = 450.$$

Step 3: Verify using the second derivative

The second derivative of $R(x)$ is:

$$\frac{d^2R}{dx^2} = -1 < 0.$$

Since $\frac{d^2R}{dx^2} < 0$, $R(x)$ is maximum when $x = 450$.

Step 4: Final result

The number of units that should be sold to maximise revenue is $x = 450$.

Quick Tip

To maximise revenue or profit, always verify the nature of the critical point using the second derivative test.

36.(ii). What rebate in price of the calculator should the store give to maximise the revenue?

Solution:

Step 1: Calculate the price at $x = 450$

The price is given by:

$$p = 450 - \frac{x}{2} = 450 - \frac{450}{2} = 225.$$

Step 2: Compute the rebate

The original price is Rs. 350. The rebate is:

$$\text{Rebate} = 350 - 225 = 125 \text{ (Rs. per calculator).}$$

Step 3: Final result

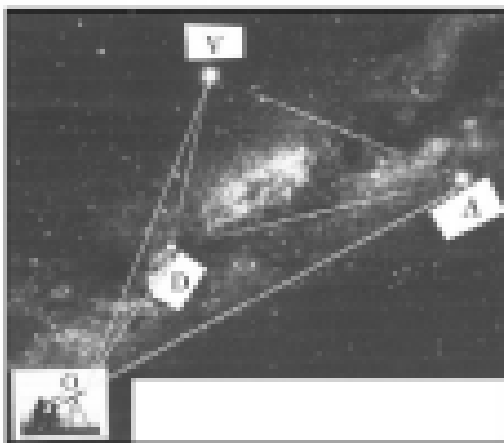
The rebate required to maximise the revenue is Rs. 125 per calculator.

Quick Tip

For pricing and revenue problems, calculate the optimal price after determining the maximum quantity sold.

Case Study - 2

37. An instructor at the astronomical centre shows three among the brightest stars in a particular constellation. Assume that the telescope is located at $O(0, 0, 0)$ and the three stars have their locations at points $D, A,$ and $V,$ having position vectors $2\hat{i} + 3\hat{j} + 4\hat{k}, 7\hat{i} + 5\hat{j} + 8\hat{k},$ and $-3\hat{i} + 7\hat{j} + 11\hat{k},$ respectively. Based on the above information, answer the following questions:



37.(i). How far is the star V from star A ?

Solution:

Step 1: Compute the position vector of \overrightarrow{AV}

$$\overrightarrow{AV} = \text{Position vector of } V - \text{Position vector of } A$$

$$\overrightarrow{AV} = (-3\hat{i} + 7\hat{j} + 11\hat{k}) - (7\hat{i} + 5\hat{j} + 8\hat{k}) = -10\hat{i} + 2\hat{j} + 3\hat{k}.$$

Step 2: Compute the magnitude of \overrightarrow{AV}

$$|\overrightarrow{AV}| = \sqrt{(-10)^2 + 2^2 + 3^2} = \sqrt{100 + 4 + 9} = \sqrt{113}.$$

Step 3: Final result

The distance between star V and star A is $\sqrt{113}$ units.

Quick Tip

To find the distance between two points, use the magnitude of the difference of their position vectors.

37.(ii). Find a unit vector in the direction of \overrightarrow{DA} .

Solution:

Step 1: Compute \overrightarrow{DA}

$$\overrightarrow{DA} = \text{Position vector of } A - \text{Position vector of } D$$

$$\overrightarrow{DA} = (7\hat{i} + 5\hat{j} + 8\hat{k}) - (2\hat{i} + 3\hat{j} + 4\hat{k}) = 5\hat{i} + 2\hat{j} + 4\hat{k}.$$

Step 2: Find the magnitude of \overrightarrow{DA}

$$|\overrightarrow{DA}| = \sqrt{(5)^2 + (2)^2 + (4)^2} = \sqrt{25 + 4 + 16} = \sqrt{45} = 3\sqrt{5}.$$

Step 3: Compute the unit vector

The unit vector is:

$$\hat{u} = \frac{\overrightarrow{D\hat{A}}}{|\overrightarrow{D\hat{A}}|} = \frac{5\hat{i} + 2\hat{j} + 4\hat{k}}{3\sqrt{5}} = \frac{5}{3\sqrt{5}}\hat{i} + \frac{2}{3\sqrt{5}}\hat{j} + \frac{4}{3\sqrt{5}}\hat{k}.$$

Step 4: Final result

The unit vector in the direction of $\overrightarrow{D\hat{A}}$ is:

$$\frac{5}{3\sqrt{5}}\hat{i} + \frac{2}{3\sqrt{5}}\hat{j} + \frac{4}{3\sqrt{5}}\hat{k}.$$

Quick Tip

To find a unit vector, divide the vector by its magnitude.

37.(iii)(a). Find the measure of $\angle VDA$.

Solution:

Step 1: Recall the formula for the angle between vectors

The angle θ between two vectors \overrightarrow{VD} and $\overrightarrow{D\hat{A}}$ is given by:

$$\cos \theta = \frac{\overrightarrow{VD} \cdot \overrightarrow{D\hat{A}}}{|\overrightarrow{VD}| \cdot |\overrightarrow{D\hat{A}}|}.$$

Step 2: Compute \overrightarrow{VD} and $\overrightarrow{D\hat{A}}$

From previous calculations:

$$\overrightarrow{VD} = \overrightarrow{V} - \overrightarrow{D} = (-3\hat{i} + 7\hat{j} + 11\hat{k}) - (2\hat{i} + 3\hat{j} + 4\hat{k}) = -5\hat{i} + 4\hat{j} + 7\hat{k}.$$

$$\overrightarrow{D\hat{A}} = 5\hat{i} + 2\hat{j} + 4\hat{k}.$$

Step 3: Compute $\overrightarrow{VD} \cdot \overrightarrow{D\hat{A}}$

$$\overrightarrow{VD} \cdot \overrightarrow{D\hat{A}} = (-5)(5) + (4)(2) + (7)(4) = -25 + 8 + 28 = 11.$$

Step 4: Compute magnitudes of \overrightarrow{VD} and $\overrightarrow{D\hat{A}}$

$$|\overrightarrow{VD}| = \sqrt{(-5)^2 + 4^2 + 7^2} = \sqrt{25 + 16 + 49} = \sqrt{90}.$$

$$|\overrightarrow{D\hat{A}}| = \sqrt{(5)^2 + (2)^2 + (4)^2} = \sqrt{25 + 4 + 16} = \sqrt{45}.$$

Step 5: Compute $\cos \theta$

$$\cos \theta = \frac{11\sqrt{2}}{\sqrt{90} \cdot \sqrt{45}} = \frac{11\sqrt{2}}{\sqrt{4050}} = \frac{11\sqrt{2}}{90}.$$

Step 6: Final result

The measure of $\angle VDA$ is:

$$\theta = \cos^{-1} \left(\frac{11\sqrt{2}}{90} \right).$$

Quick Tip

For angles between vectors, always use the dot product formula and ensure the magnitude is correctly computed.

37.(iii)(b). What is the projection of vector \overrightarrow{DV} on vector \overrightarrow{DA} ?

Solution:

Step 1: Recall the formula for projection

The projection of \overrightarrow{DV} on \overrightarrow{DA} is given by:

$$\text{Projection} = \frac{\overrightarrow{DV} \cdot \overrightarrow{DA}}{|\overrightarrow{DA}|}.$$

Step 2: Compute \overrightarrow{DV}

$$\overrightarrow{DV} = \overrightarrow{V} - \overrightarrow{D} = (-5\hat{i} + 4\hat{j} + 7\hat{k}).$$

Step 3: Compute $\overrightarrow{DV} \cdot \overrightarrow{DA}$

From the previous calculations:

$$\overrightarrow{DV} \cdot \overrightarrow{DA} = (-5)(5) + (4)(2) + (7)(4) = -25 + 8 + 28 = 11.$$

Step 4: Compute $|\overrightarrow{DA}|$

$$|\overrightarrow{DA}| = \sqrt{(5)^2 + (2)^2 + (4)^2} = \sqrt{45} = 3\sqrt{5}.$$

Step 5: Compute the projection

$$\text{Projection} = \frac{\overrightarrow{DV} \cdot \overrightarrow{DA}}{|\overrightarrow{DA}|} = \frac{11}{3\sqrt{5}}$$

Step 6: Final result

The projection of \overrightarrow{DV} on \overrightarrow{DA} is:

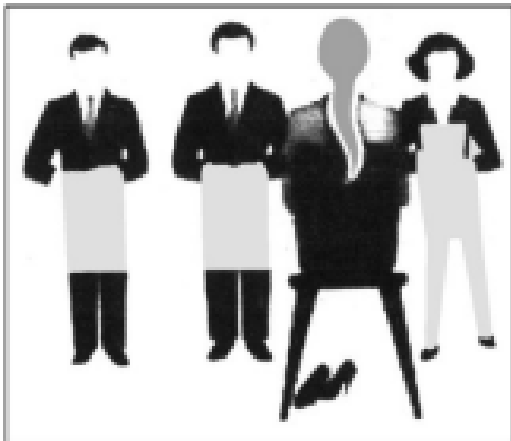
$$\frac{11\sqrt{5}}{15}$$

Quick Tip

The projection of one vector onto another gives the component of the first vector along the direction of the second.

Case Study - 3

38 Rohit, Jaspreet, and Alia appeared for an interview for three vacancies in the same post. The probability of Rohit's selection is $\frac{1}{5}$, Jaspreet's selection is $\frac{1}{3}$, and Alia's selection is $\frac{1}{4}$. The events of selection are independent of each other.



Based on the above information, answer the following questions:

38.(i). What is the probability that at least one of them is selected?

Solution:

Step 1: Probability of no one being selected

The probability that none of them are selected is:

$$P(\text{No one selected}) = \left(1 - \frac{1}{5}\right) \cdot \left(1 - \frac{1}{3}\right) \cdot \left(1 - \frac{1}{4}\right) = \frac{4}{5} \cdot \frac{2}{3} \cdot \frac{3}{4} = \frac{2}{5}.$$

Step 2: Probability of at least one being selected

The probability that at least one of them is selected is:

$$P(\text{At least one selected}) = 1 - P(\text{No one selected}) = 1 - \frac{2}{5} = \frac{3}{5}.$$

Final Result: The probability that at least one of them is selected is $\frac{3}{5}$.

Quick Tip

To find the probability of "at least one" event happening, use the complement rule:

$$P(\text{At least one}) = 1 - P(\text{None}).$$

38.(ii). Find $P(G \cap \bar{H})$, where G is the event of Jaspreet's selection and \bar{H} denotes the event that Rohit is not selected.

Solution:

- $P(G|\bar{H})$ is the conditional probability of G given \bar{H} .
- $P(G \cap \bar{H})$ is the probability that both G and \bar{H} occur.
- $P(\bar{H})$ is the probability that H does not occur.

$$P(G|\bar{H}) = \frac{P(G \cap \bar{H})}{P(\bar{H})} = \frac{1}{3}$$

Quick Tip

For independent events, the probability of their intersection is the product of their individual probabilities.

38.(iii)(a). Find the probability that exactly one of them is selected.

Solution:

Step 1: Compute the probability of exactly one being selected

The probability of exactly one being selected is:

$$P(\text{Exactly one selected}) = P(R) \cdot P(\bar{J}) \cdot P(\bar{A}) + P(\bar{R}) \cdot P(J) \cdot P(\bar{A}) + P(\bar{R}) \cdot P(\bar{J}) \cdot P(A),$$

where:

$$P(\bar{J}) = 1 - P(J) = \frac{2}{3}, \quad P(\bar{A}) = 1 - P(A) = \frac{3}{4}, \quad P(\bar{R}) = 1 - P(R) = \frac{4}{5}.$$

Substitute the values:

$$P(\text{Exactly one selected}) = \frac{1}{5} \cdot \frac{2}{3} \cdot \frac{3}{4} + \frac{4}{5} \cdot \frac{1}{3} \cdot \frac{3}{4} + \frac{4}{5} \cdot \frac{2}{3} \cdot \frac{1}{4}.$$

Simplify each term:

$$P(\text{Exactly one selected}) = \frac{6}{60} + \frac{12}{60} + \frac{8}{60} = \frac{26}{60} = \frac{13}{30}.$$

Final Result: The probability that exactly one of them is selected is $\frac{13}{30}$.

Quick Tip

To calculate "exactly one" selection, consider all cases where one succeeds, and the others fail, then sum the probabilities.

38.(iii)(b). Find the probability that exactly two of them are selected.

Solution:

Step 1: Compute the probability of exactly two being selected

The probability of exactly two being selected is:

$$P(\text{Exactly two selected}) = P(R) \cdot P(J) \cdot P(\bar{A}) + P(R) \cdot P(\bar{J}) \cdot P(A) + P(\bar{R}) \cdot P(J) \cdot P(A).$$

Substitute the values:

$$P(\text{Exactly two selected}) = \frac{1}{5} \cdot \frac{1}{3} \cdot \frac{3}{4} + \frac{1}{5} \cdot \frac{2}{3} \cdot \frac{1}{4} + \frac{4}{5} \cdot \frac{1}{3} \cdot \frac{1}{4}.$$

Simplify each term:

$$P(\text{Exactly two selected}) = \frac{3}{60} + \frac{2}{60} + \frac{4}{60} = \frac{9}{60} = \frac{3}{20}.$$

Final Result: The probability that exactly two of them are selected is $\frac{3}{20}$.

Quick Tip

For "exactly two" events, consider all pairs of selections and one failure, and sum their probabilities.
