Sets, Relations, And Functions JEE Main PYQ - 2

Total Time: 25 Minute **Total Marks:** 40

Instructions

Instructions

1. Test will auto submit when the Time is up.
2. The Test comprises of multiple choice questions (MCQ) with one or more correct answers.
3. The clock in the top right corner will display the remaining time available for you to complete the examination.

Navigating & Answering a Question

1. The answer will be saved automatically upon clicking on an option amongst the given choices of answer.
2. To deselect your chosen answer, click on the clear response button.
3. The marking scheme will be displayed for each question on the top right corner of the test window.

Sets, Relations, And Functions

1. For any two statements and , the negation of the expression ( ) is: **(+4, -1)**

a.

b.

c.

d.

---

2. Let a, b, c be such that b (+) ≠ 0 . If **(+4, -1)**

+ 1 − 1 + 1 + 1 − 1

| − + 1 − 1 | + | − 1 − 1 + 1 | = 0Then the value of is?

− 1 + 1 (− 1) + 2 (− 1) + 1 (− 1)

a. (A) Zero

b. (B) Any even integer

c. (C) Any odd integer

d. (D) Any integer

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3. Let $S = x : x \in R$ and $\left(\sqrt{3}+\sqrt{2})^{x^2-4} + (\sqrt{3}-\sqrt{2})^{x^2-4} = 10\right\}$ Then $n(S)$ is equal to **(+4, -1)**

a. 2

b. 4

c. 0

d. 6

---

4. The number of ways of selecting two numbers a and $b, a \in \{2, 4, 6, \ldots, 100\}$ and $b \in \{1, 3, 5, \ldots, 99\}$ such that 2 is the remainder when $a + b$ is divided by 23 is **(+4, -1)**

a. 54

b. 108

c. 268

d. 186

---

5. Let $P(S)$ denote the power set of $S = \{1, 2, 3, \ldots, 10\}$. Define the relations R_1 and R_2 on $P(S)$ as A_1B if $(A \cap B^c) \cup (B \cap A^c) = \emptyset$ and A_2B if $A \cup B^c = B \cup A^c, \forall A, B \in P(S)$. Then : **(+4, -1)**

a. both R_1 and R_2 are not equivalence relations

b. only R_2 is an equivalence relation

c. only R_1 is an equivalence relation

d. both R_1 and R_2 are equivalence relations

---

6. Let awards in event A is 48 and awards in event B is 25 and awards in event C is 18 and also n(A ∪ B ∪ C) = 60, n(A ⋂ B ⋂ C) = 5, then how many got exactly two awards is? **(+4, -1)**

a. 21

b. 25

c. 24

d. 23

---

7. Let awards in event A is 48 and awards in event B is 25 and awards in event C is 18 and also n(A ∪ B ∪ C) = 60, n(A ⋂ B ⋂ C) = 5, then how many got exactly two awards is? **(+4, -1)**

a. 21

b. 25

c. 24

d. 23

---

8. Two sets A and B are as under: $A = \{(a, b) \in R \times R : |a - 5| < 1$ and $|b - 5| < 1\}$; $B = \{(a, b) \in R \times R : 4(a - 6)2 + 9(b - 5)^2 \leq 36\}$. Then (+4, -1)

a. $B \subset A$

b. $A \subset B$

c. $A \cap B = \phi$ (an empty set)

d. neither $A \subset B$ nor $B \subset A$

---

9. Let $S = \{1, 2, 3, 4, 5, 6\}$ Then the number of one-one functions $f : S \to P(S)$, where $P(S)$ denote the power set of S, such that $f(m) \subset f(m)$ where $n < m$ is _______ (+4, -1)

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10. Let $A : \{1, 2, 3, 4, 5, 6, 7\}$. Define $B = \{T \subseteq A :$ either $1 \notin T$ or $2 \in T\}$ and $C = \{T \subseteq A : T$ the sum of all the elements of T is a prime number $\}$. Then the number of elements in the set $B \cup C$ is _______ . (+4, -1)

Answers

1. Answer: d

Explanation:

Explanation:
The given expression is ( ).
Thus, the negation of this expression can be written as: (( ))
= ( )[( )]
= ( )[( )]
= ( ) ( )[( ) ( ) ( )]
= ( )[ ]
= ( )[ ]
Hence, the correct option is (D).

Concepts:

1. Sets:

In mathematics, a set is a well-defined collection of objects. Sets are named and demonstrated using capital letter. In the set theory, the elements that a set comprises can be any sort of thing: people, numbers, letters of the alphabet, shapes, variables, etc.

Read More: Set Theory

Elements of a Set:

The items existing in a set are commonly known to be either elements or members of a set. The elements of a set are bounded in curly brackets separated by commas.

Read Also: Set Operation

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Read More: Types of Sets

2. **Answer: c**

Explanation:

Explanation:

$$| - \begin{matrix} +1 & -1 \\ +1 & -1 \\ -1 & +1 \end{matrix} | + | \begin{matrix} (-1)^{+2} & +1 & -1 \\ (-1)^{+1} & +1 & -1 \\ (-1) & -1 & +1 \end{matrix} | = | - \begin{matrix} +(-1)^{+2} & +1 & -1 \\ +(-1)^{+1} & +1 & -1 \\ +(-1) & -1 & +1 \end{matrix} | = 0 \text{ if } \quad \text{is}$$

an odd integer.Hence, the correct option is (C).

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3. **Answer: b**

Explanation:

The correct answer is (B) : 4

$\text{Let } (\sqrt{3}+\sqrt{2})^{x^2-4} = t$

$t + \frac{1}{t} = 10$

$\Rightarrow t = 5 + 2\sqrt{6}, 5 - 2\sqrt{6}$

$\Rightarrow (\sqrt{3}+\sqrt{2})^{x^2-4} = 5 + 2\sqrt{6}, 5 - 2\sqrt{6}$

$\Rightarrow x^2 - 4 = 2, -2$ or $x^2 = 6, 2$

$\Rightarrow x = \pm\sqrt{2}, \pm\sqrt{6}$

Concepts:

1. **Sets:**

Set is the collection of well defined objects. Sets are represented by capital letters, eg. A={}. Sets are composed of elements which could be numbers, letters, shapes, etc.

Example of set: Set of vowels A={a,e,i,o,u}

Representation of Sets

There are three basic notation or representation of sets are as follows:

Statement Form: The statement representation describes a statement to show what are the elements of a set.

- For example, Set A is the list of the first five odd numbers.

Roster Form: The form in which elements are listed in set. Elements in the set is seperatrd by comma and enclosed within the curly braces.

- For example represent the set of vowels in roster form.

A={a,e,i,o,u}

Set Builder Form:

1. The set builder representation has a certain rule or a statement that specifically describes the common feature of all the elements of a set.
2. The set builder form uses a vertical bar in its representation, with a text describing the character of the elements of the set.
3. For example, A = { k | k is an even number, k ≤ 20}. The statement says, all the elements of set A are even numbers that are less than or equal to 20.
4. Sometimes a ":" is used in the place of the "|".

---

4. Answer: b

Explanation:

$a \in 2, 4, 6, 8, 10, \ldots., 100$
$b \in 1, 3, 5, 7, 9, \ldots\ldots, 99$
Now, $a + b \in 25, 71, 117, 163$

(i) $a + b = 25$, no. of ordered pairs (a, b) is 12
(ii) $a + b = 71$, no. of ordered pairs (a, b) is 35
(iii) $a + b = 117$, no. of ordered pairs (a, b) is 42
(iv) $a + b = 163$, no. of ordered pairs (a, b) is 19

$\therefore total = 108\ pairs$

Therefore, the correct option is (B):108

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---

5. Answer: a

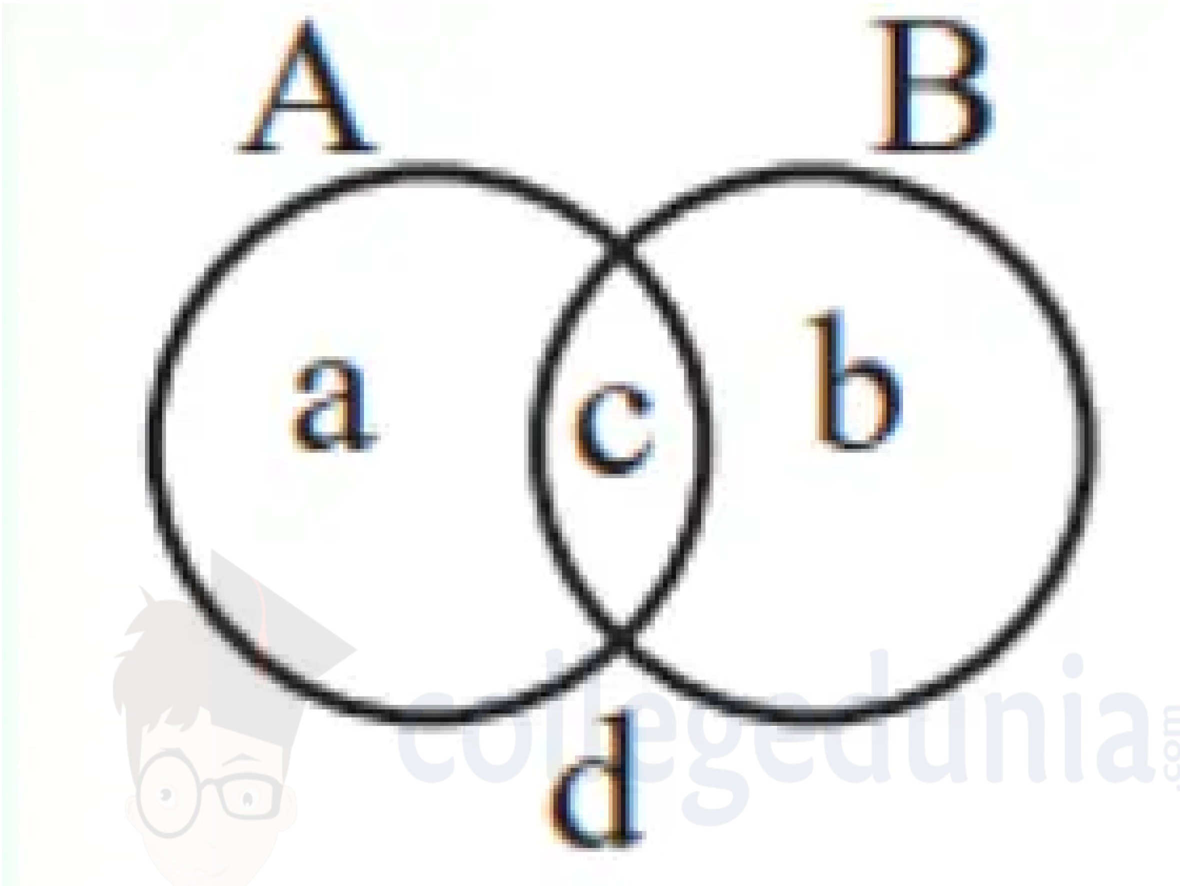
Explanation:

$$S=\{1,2,3,\ldots\ldots 10\}$$
$$P(S)=\text{ power set of } S$$
$$AR,B \Rightarrow (A\cap \overrightarrow{B})\cup(\overrightarrow{A}\cap B)=\phi$$
$$R1 \text{ is reflexive, symmetric}$$
$$\text{ For transitive}$$

$(A \cap \overrightarrow{B}) \cup (\overrightarrow{A} \cap B) = \phi; \{a\} = \phi = \{b\} A = B$

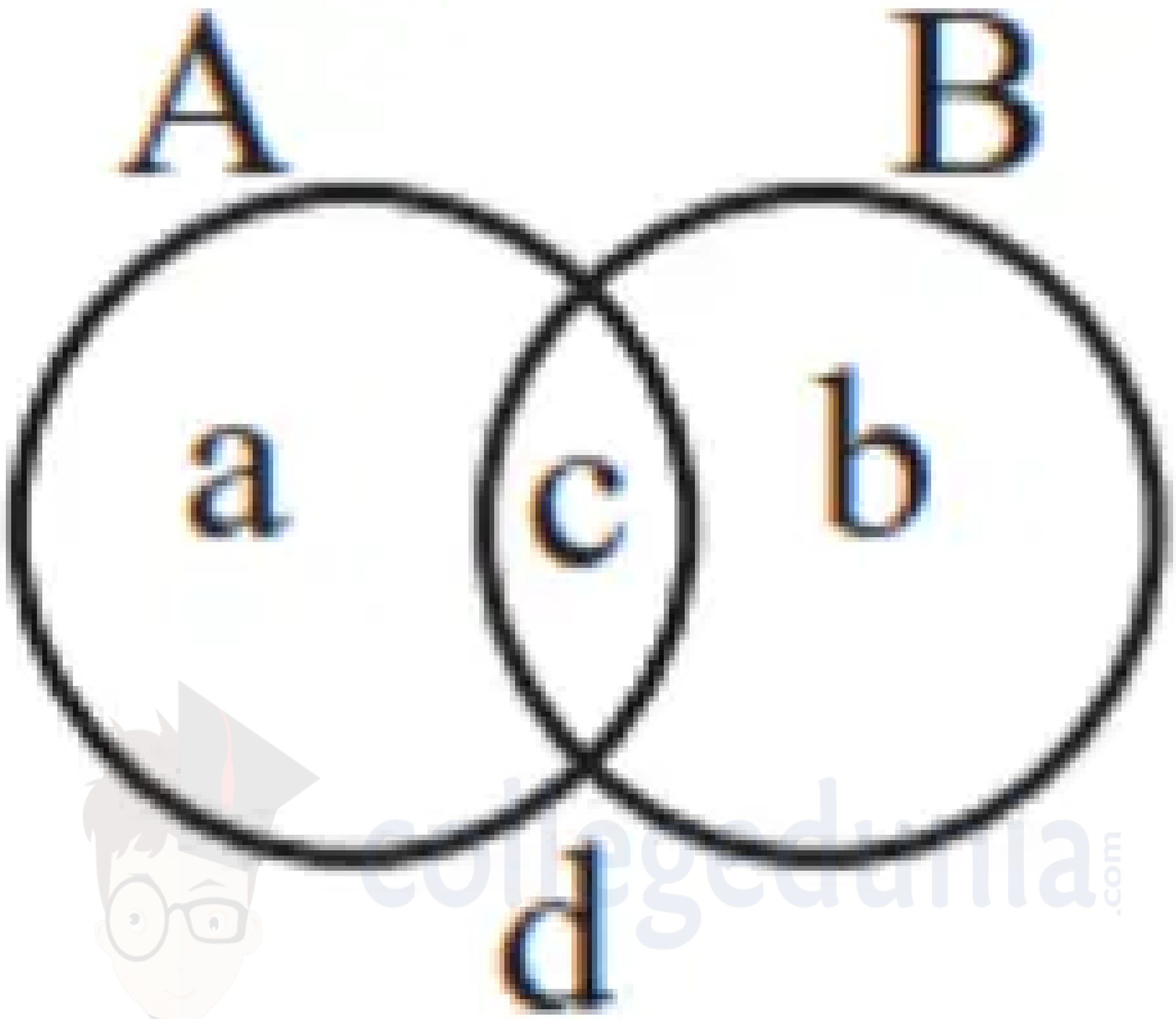
$(B \cap \overrightarrow{C}) \cup (\overrightarrow{B} \cap C) = \phi \therefore B = C$

$\therefore A = C$ equivalence.



$R_2 \equiv A \cup \overrightarrow{B} = \overrightarrow{A} \cup B$

$R_2 \rightarrow$ Reflexive, symmetric for transitive



$$A \cup \overrightarrow{B} = \overrightarrow{A} \cup B \Rightarrow \{a, c, d\} = \{b, c, d\}$$
$$\{a\} = \{b\}$$
$$\therefore A = B$$
$$B \cup \overrightarrow{C} = \overrightarrow{B} \cup C \Rightarrow B = C$$
$$\therefore A = C$$
$$\therefore A \cup \overrightarrow{C} = \overrightarrow{A} \cup C$$
$$\therefore \text{ Equivalence}$$

Concepts:

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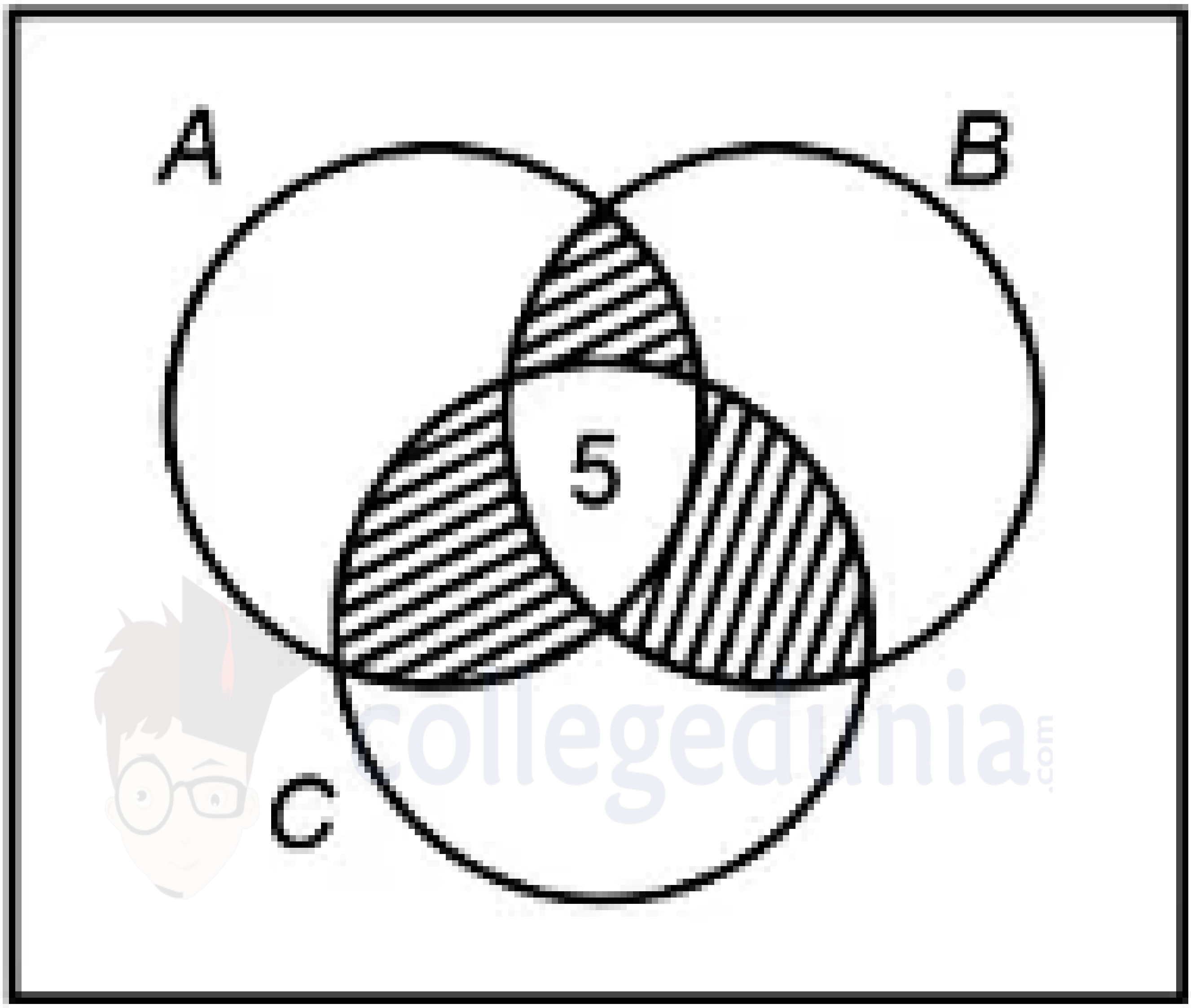
6. Answer: a

Explanation:

The correct answer is option (A): 21



∴ Number of persons who get exactly two awards

$= n(A) + n(B) + n(C) - n(A \cup B \cup C) - 2n(A \cap B \cap C)$

$= 48 + 25 + 18 - 60 - 10$

$= 21$

Concepts:

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Elements of a Set:

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Read Also: Set Operation

Cardinal Number of a Set:

The **cardinal number**, cardinality, or order of a set indicates the total number of elements in the set.

Read More: Types of Sets

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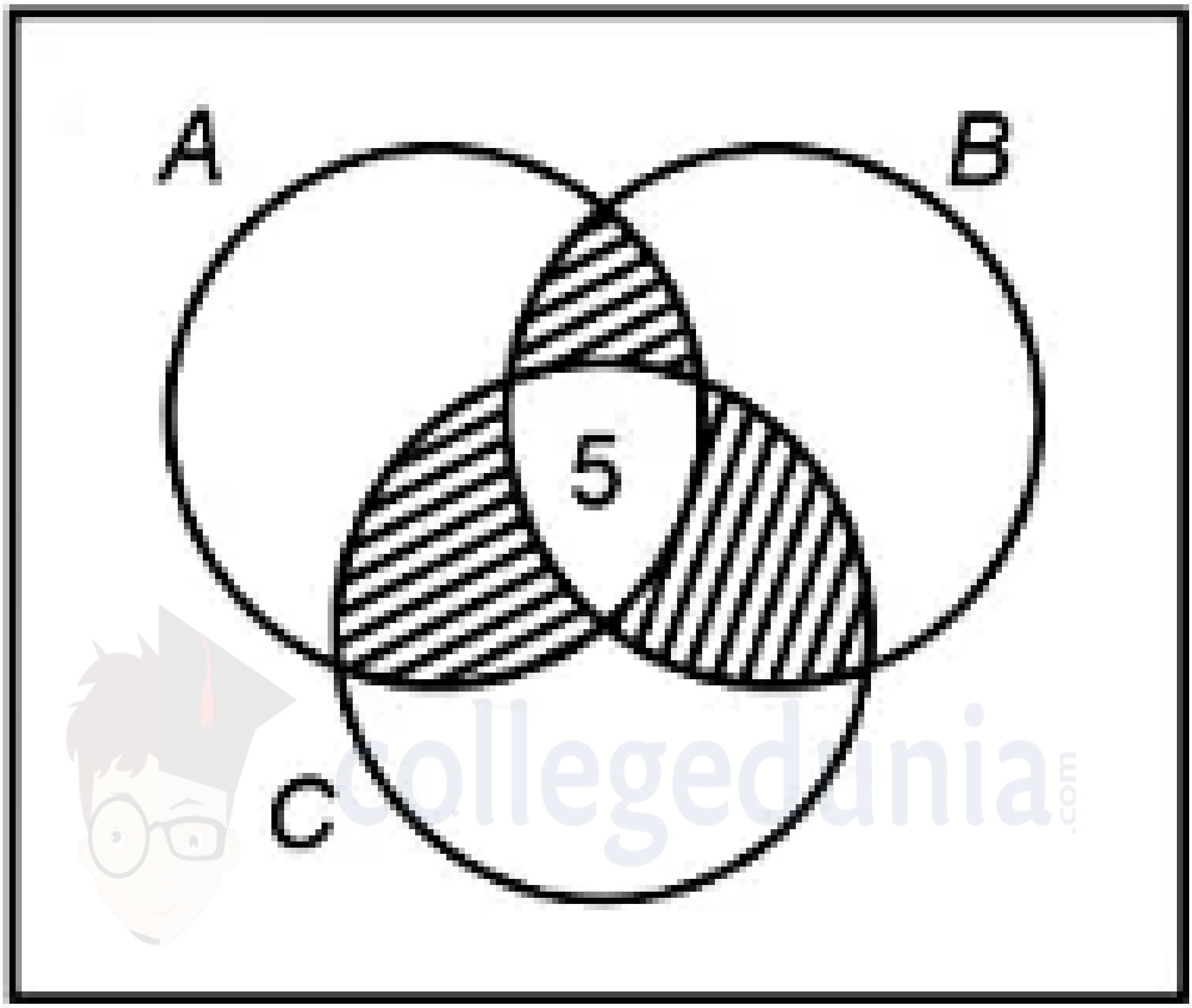
7. Answer: a

Explanation:

The correct answer is option (A) : 21



Number of persons who get exactly 2 awards : $n(A)+n(B)+n(C)-n(A \cup B \cup C)-2n(A \cap B \cap C)$

$= 48+25+18-60-10$

$= 21$

Concepts:

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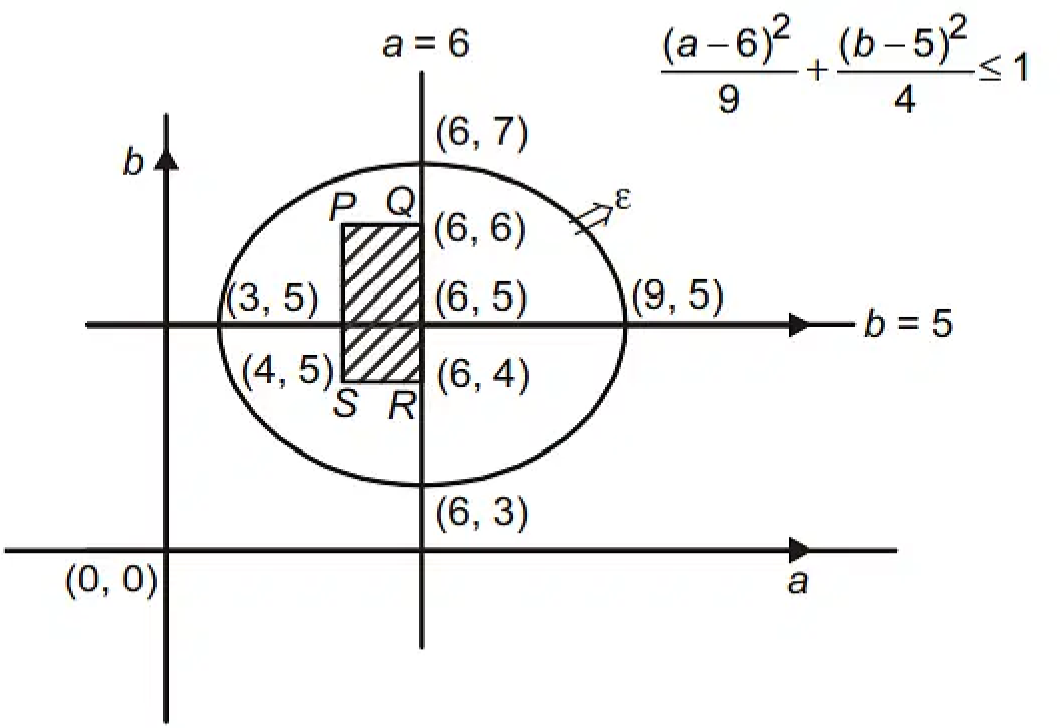
Read More: Types of Sets

---

8. **Answer: b**

Explanation:

As, $|a-5|<1$ and $|b-5|<1 \Rightarrow 4<a, b<6$ and $\frac{(a-6)^2}{9}+\frac{(b-5)^2}{4} \leq 1$ Taking axes as a-axis and b-axis



The set A represents square $PQRS$ inside set B representing ellipse and hence $A \subset B$
.

Concepts:

1. Types of Sets:

Sets are of various types depending on their features. They are as follows:

- **Empty Set** - It is a set that has no element in it. It is also called a null or void set and is denoted by Φ or $\{\}$.
- **Singleton Set** - It is a set that contains only one element.
- **Finite Set** - A set that has a finite number of elements in it.
- **Infinite Set** - A set that has an infinite number of elements in it.
- **Equal Set** - Sets in which elements of one set are similar to elements of another set. The sequence of elements can be any but the same elements exist in both sets.
- **Sub Set** - Set X will be a subset of Y if all the elements of set X are the same as the element of set Y.
- **Power Set** - It is the collection of all subsets of a set X.
- **Universal Set** - A basic set that has all the elements of other sets and forms the base for all other sets.
- **Disjoint Set** - If there is no common element between two sets, i.e if there is no element of Set A present in Set B and vice versa, then they are called disjoint sets.
- **Overlapping Set** - It is the set of two sets that have at least one common element, called overlapping sets.

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9. Answer: 3240 - 3240

Explanation:

The correct answer is 3240.
Let $S = \{1, 2, 3, 4, 5, 6\}$, then the number of one-one functions, $f : S \cdot P(S)$, where $P(S)$ denotes the power set of S, such that $f(n) < f(m)$ where $n < m$ is
$n(S) = 6$
$$P(S) = \begin{Bmatrix} \phi, \{1\}, \ldots \{6\}, \{1, 2\}, \ldots, \\ \{5, 6\}, \ldots, \{1, 2, 3, 4, 5, 6\} \end{Bmatrix}$$

-64 elements
case -1
$f(6)=S$ i.e. 1 option,
$f(5)=$ any 5 element subset A of S i.e. 6 options,
$f(4)=$ any 4 element subset B of A i.e. 5 options,
$f(3)=$ any 3 element subset C of B i.e. 4 options,
$f(2)=$ any 2 element subset D of C i.e. 3 options,
$f(1)=$ any 1 element subset E of D or empty subset i.e. 3 options,
Total functions $=1080$
Case -2
$f(6)=$ any 5 element subset A of S i.e. 6 options,
$f(5)=$ any 4 element subset B of A i.e. 5 options,
$f'(4)=$ any 3 element subset C of B i.e. 4 options,
$f(3)=$ any 2 element subset D of C i.e. 3 options,
$f'(2)=$ any 1 element subset E of D i.e. 2 options,
$f(1)=$ empty subset i.e. 1 option
Total functions $=720$
Case -3
$f(6)=S$
$f(5)=$ any 4 element subset A of ' S i.e. 15 options,
$f(4)=$ any 3 element subset B of A i.e. 4 options,
$f(3)=$ any 2 element subset C of B i.e. 3 options,
$f(2)=$ any 1 element subset D of C i.e. 2 options,
$f(1)=$ empty subset i.e. 1 option
Total functions $=360$
Case -4
$f(6)=S$
$f(5)=$ any 5 element subset A of S i.e. 6 options,
$f(4)=$ any 3 element subset B of A i.e. 10 options,
$f(3)=$ any 2 element subset C of B i.e. 3 options,
$f(2)=$ any 1 element subset D of C i.e. 2 options,
$f(1)=$ empty subset i.e. 1 option
Total functions $=360$
Case -5
$f(6)=S$
$f(5)=$ any 5 element subset A of S i.e. 6 options,
$f(4)=$ any 4 element subset B of A i.e. 5 options,

$f(3) =$ any 2 element subset C of B i.e. 6 options,
$f(2) =$ any 1 element subset D of C i.e. 2 options,
$f(1) =$ empty subset i.e. 1 option
Total functions $= 360$
Case -6
$f(6) = S$
$f(5) =$ any 5 element suhset A of S i e 6 options
$f(4) =$ any 4 element subset B of A i.e. 5 options,
$f(3) =$ any 3 element subset C of B i.e. 4 options,
$f(2) =$ any 1 element subset D of C i.e. 3 options,
$f(1) =$ empty subset i.e. 1 option
Total functions $= 360$
$\therefore$ Number of surch funstions $= 3240$

Concepts:

1. Relations and functions:

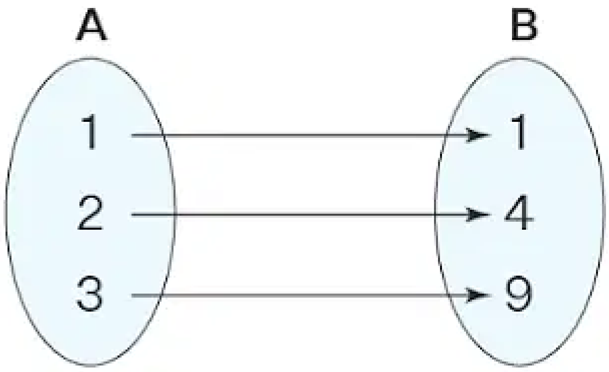
A **relation** R from a non-empty set B is a subset of the cartesian product A × B. The subset is derived by describing a relationship between the first element and the second element of the ordered pairs in A × B.

A relation f from a set A to a set B is said to be a **function** if every element of set A has one and only one image in set B. In other words, no two distinct elements of B have the same pre-image.

Representation of Relation and Function

Relations and functions can be represented in different forms such as arrow representation, algebraic form, set-builder form, graphically, roster form, and tabular form. Define a function f: A = {1, 2, 3} → B = {1, 4, 9} such that f(1) = 1, f(2) = 4, f(3) = 9. Now, represent this function in different forms.

1. Set-builder form - {(x, y): f(x) = y^2, x ∈ A, y ∈ B}
2. Roster form - {(1, 1), (2, 4), (3, 9)}
3. Arrow Representation



- Table Representation -

x	y
1	1
2	4
3	9

---

10. **Answer: 107 - 107**

Explanation:

$A : \{1,2,3,4,5,6,7\}$

Number of elements in set B

$= n(1 \notin T) + n(2 \in T) - n[(1 \notin T) \cap (2 \in T)]$

$= 2^6 + 2^6 - 2^5 = 96$

Number of elements in set C

$= \{\{2\},\{3\},\{5\},\{7\},\{1,2\},\{1,4\},\{1,6\},$
$\{2,3\},\{2,5\},\{3,4\},\{4,7\},\{5,6\},\{6,7\}$
$\{1,2,4\},\{1,3,7\},\{1,4,6\},\{1,5,7\},\{2,3,$
$6\},\{2,4,5\},\{2,4,7\},\{2,5,6\},\{3,4,6\},$
$\{4,6,7\},\{1,2,4,6\},\{2,4,6,7\},\{2,4,6,$
$5\},\{3,5,7,4\},\{1,3,5,4\},\{1,5,7,4\},\{1,$
$2,3,5\},\{1,2,3,7\},\{1,3,6,7\},\{1,5,6,7\},$
$\{2,3,5,7\},\{1,5,7,2,4\},\{3,5,7,2,6\},\{1,$
$3,7,2,4\},\{1,4,5,6,7\},$
$\{1,3,4,5,6\},\{1,2,3,6,7\},\{1,2,3,5,6\},$
$\{1,2,3,4,6,7\}$

Number of elementrrs in $C = 42$

$\Rightarrow n(B \cup C) = n(B) + n(C) - n(B \cap C)$

$= 96 + 42 - 31 = 107$

The correct answer is 107.

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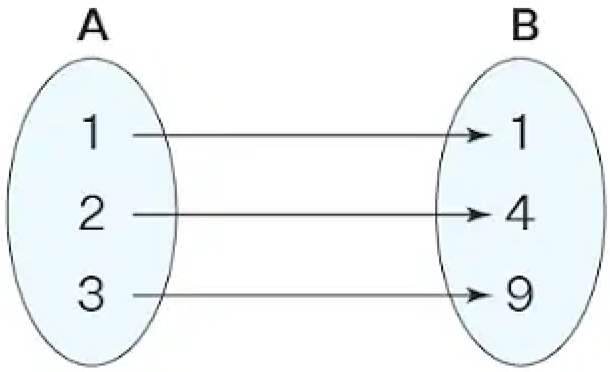

- Table Representation -

x	y
1	1
2	4
3	9