Sets, Relations, And Functions JEE Main PYQ - 3

Total Time: 25 Minute **Total Marks:** 40

Instructions

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1. Test will auto submit when the Time is up.
2. The Test comprises of multiple choice questions (MCQ) with one or more correct answers.
3. The clock in the top right corner will display the remaining time available for you to complete the examination.

Navigating & Answering a Question

1. The answer will be saved automatically upon clicking on an option amongst the given choices of answer.
2. To deselect your chosen answer, click on the clear response button.
3. The marking scheme will be displayed for each question on the top right corner of the test window.

Sets, Relations, And Functions

1. Let R be a relation on R, given by $R = \{(a, b) : 3a - 3b + \sqrt{7}$ is an irrational number $\}$Then R is **(+4, -1)**

   a. reflexive and symmetric but not transitive

   b. reflexive and transitive but not symmetric

   c. reflexive but neither symmetric nor transitive

   d. an equivalence relation

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2. Let $f(x) = 2x + \tan^{-1} x$ and $g(x) = \log_e \left(\sqrt{1 + x^2} + x\right), x \in [0, 3]$ Then **(+4, -1)**

   a. $\min f'(x) = 1 + \max g'(x)$

   b. there exist $0 < x_1 < x_2 < 3$ such that $f(x) < g(x), \forall x \in (x_1, x_2)$

   c. $\max f(x) > \max g(x)$

   d. there exists $\hat{x} \in [0, 3]$ such that $f'(\hat{x}) < g'(\hat{x})$

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3. Let $f(n) = \left[\frac{1}{3} + \frac{3n}{100}\right] n$, where $[n]$ denotes the greatest integer less than or equal to n. Then $\sum\limits_{n=1}^{56} \Delta_r f(n)$ is equal to : **(+4, -1)**

   a. 56

   b. 689

   c. 1287

   d. 1399

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4. If A = {1, 2, 3, 4, 6} and R is a relation on A such that R = {(a, b) : a, b ∈ A and b is exactly divisible by a} then find the number of elements present in the range of R? **(+4, -1)**

   a. 2

b. 4

c. 6

d. 5

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5. If A = {1, 2, 3, 4, 6} and R is a relation on A such that R = {(a, b) : a, b ∈ A and b is exactly divisible by a} then find the number of elements present in the range of R? **(+4, -1)**

a. (A) 2

b. (B) 4

c. (C) 6

d. (D) 5

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6. Let , : [− 1, 2] be continuous functions which are twice differentiable on the interval (-1,2). Let the values of and at the points -1,0 and 2 be as given in the following table: **(+4, -1)**

	= − 1	= 0	= 2
( )	3	6	0
( )	0	1	−1

In each of the intervals (-1,0) and (0,2) the function (− 3) never vanishes. Then the correct statement(s) is (are):

a. (A) ′( ) − 3 ′( ) = 0 has exactly three solutions in (-1,0) (0, 2)

b. (B) ′( ) − 3 ′( ) = 0 has exactly one solution in (-1,0) and exactly one solutions in (0,2)

c. (C) ′( ) − 3 ′( ) = 0 has exactly two solutions in (-1,0) and exactly two solutions in (0,2)

d. (D) None of the above

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7. Let R be a relation on $N \times N$ defined by $(a, b)R(c, d)$ if and only if $ad(b-c) = bc(a-d)$ Then R is **(+4, -1)**

a. transitive but neither reflexive nor symmetric

b. symmetric but neither reflexive nor transitive

c. symmetric and transitive but not reflexive

d. reflexive and symmetric but not transitive

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8. Let $f : (0, 1) \to R$ be a function defined by $f(x) = \frac{1}{1-e^{-x}}$, and $g(x) = (f(-x) - f(x))$ Consider two statements (I) g is an increasing function in $(0, 1)$ (II) g is one-one in $(0, 1)$Then, **(+4, -1)**

a. Only (I) is true

b. Both (I) and (II) are true

c. Neither (I) nor (II) is true

d. Only (II) is true

---

9. Let $f : (0, 1) \to R$ be a function defined by **(+4, -1)**

$f(x) = \frac{1}{1-e^{-x}}$, and $g(x) = (f(-x) - f(x))$ Consider two statements

(I) g is an increasing function in $(0, 1)$

(II) g is one-one in $(0, 1)$Then,

a. Only (I) is true

b. Both (I) and (II) are true

c. Neither (I) nor (II) is true

d. Only (II) is true

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10. If the function ( ) = ^{3} + $^{/2}$ and ( ) = $^{-1}$, then the value of $'$(1) is **(+4, -1)**

Answers

1. **Answer: c**

Explanation:

Check for reflexivity:
As $3(a-a)+\sqrt{7}=\sqrt{7}$
which belongs to relation so relation is reflexive
Check for symmetric:
Take $a=\frac{\sqrt{7}}{3}, b=0$
Now (a, b) $\in R$ but $(b, a) \notin R$
As $3(b-a)+\sqrt{7}=0$
which is rational so relation is not symmetric.
Check for Transitivity:
Take (a, b) as $\left(\frac{\sqrt{7}}{3}, 1\right)$
$\&(b, c)$ as $\left(1, \frac{2\sqrt{7}}{3}\right)$
So now (a, b) $\in R \&(b, c) \in R$ but $(a, c) \notin R$ which means relation is not transitive

Concepts:

1. Operations on Sets:

Some important operations on sets include union, intersection, difference, and the complement of a set, a brief explanation of operations on sets is as follows:

1. Union of Sets:

- The union of sets lists the elements in set A and set B or the elements in both set A and set B.
- For example, {3,4} ∪ {1, 4} = {1, 3, 4}
- It is denoted as "A U B"

2. Intersection of Sets:

- Intersection of sets lists the common elements in set A and B.
- For example, {3,4} ∪ {1, 4} = {4}
- It is denoted as "A ∩ B"

3.Set Difference:

- Set difference is the list of elements in set A which is not present in set B
- For example, {3,4} - {1, 4} = {3}
- It is denoted as “A - B”

4.Set Complement:

- The set complement is the list of all elements present in the Universal set except the elements present in set A
- It is denoted as “U-A”

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2. Answer: c

Explanation:

The correct answer is (C) : $\max f(x) > \max g(x)$
$f'(x) = 2 + \frac{1}{1+x^2},\ g'(x) = \frac{1}{\sqrt{1+x^2}}$
Both does not have critical values
$f(0) = 0, f(3) = 6 + \tan^{-1}(3)$
$g(0) = 0, g(3) = \log(3 + \sqrt{10})$
Let h(x) = f(x) - g(x)
$h'(x) > 0 \forall x \in (0,3)$
∴ h(x) is increasing function

Concepts:

1. Relations and functions:

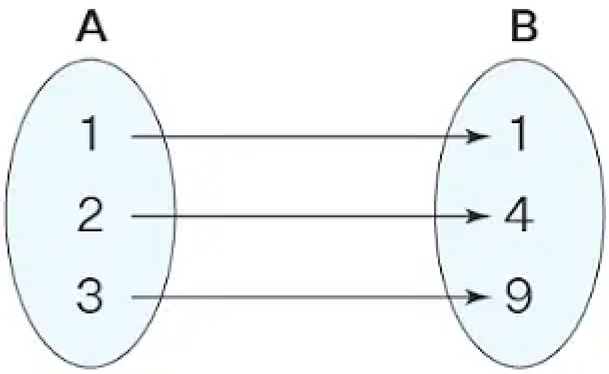
A **relation** R from a non-empty set B is a subset of the cartesian product A × B. The subset is derived by describing a relationship between the first element and the second element of the ordered pairs in A × B.

A relation f from a set A to a set B is said to be a **function** if every element of set A has one and only one image in set B. In other words, no two distinct elements of B have the same pre-image.

Representation of Relation and Function

Relations and functions can be represented in different forms such as arrow representation, algebraic form, set-builder form, graphically, roster form, and tabular form. Define a function f: A = {1, 2, 3} → B = {1, 4, 9} such that f(1) = 1, f(2) = 4, f(3) = 9. Now, represent this function in different forms.

1. Set-builder form - $\{(x, y): f(x) = y^2, x \in A, y \in B\}$
2. Roster form - {(1, 1), (2, 4), (3, 9)}
3. Arrow Representation



- Table Representation -

x	y
1	1
2	4
3	9

---

3. Answer: d

Explanation:

$$\sum_{n=1}^{56} f(x) = \left[\frac{1}{3} + \frac{3 \times 1}{100}\right] \times 1 + \ldots.. + \left[\frac{1}{3} + \frac{3 \times 22}{100}\right] \times 22$$
$$+ \left[\tfrac{1}{3} + \tfrac{3 \times 23}{100}\right] \times 23 \text{ .... + ...}$$
$$\left[\tfrac{1}{3} + \tfrac{3 \times 55}{100}\right] \times 55 + \left[\tfrac{1}{3} + \tfrac{3 \times 56}{100}\right] \times 56$$
$$= 0 + \ldots\ldots.. + 0 + 23 + 24 + \ldots\ldots. + 55 + 2 \times 56$$
$$= \tfrac{55(56)}{2} - \tfrac{22(23)}{2} + 112$$
$$= 11(5 \times 28 - 23) + 112$$
$$= 11 \times 117 + 112$$
$$= 1287 + 112$$
$$= 1399$$

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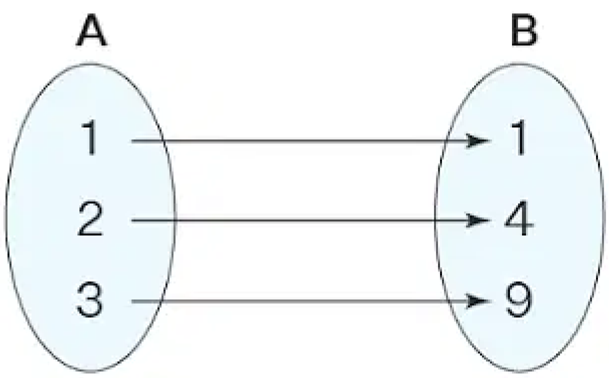

- Table Representation -

x	y
1	1
2	4
3	9

4. **Answer: d**

Explanation:

Explanation:

It is given that A = {1, 2, 3, 4, 6} and R is a relation on A such that R = {(a, b) : a, b ∈ A and b is exactly divisible by a}

The given R can be re-written in roaster form as R = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 6), (2, 2), (2, 4), (2, 6), (3, 3), (3, 6), (4, 4), (6, 6)}.

As we know, Range (R) = {b: (a, b) ∈ R}

Therefore, range (R) = {1, 2, 3, 4, 6} = A∈ n(A) = 5

Hence, the correct option is (D).

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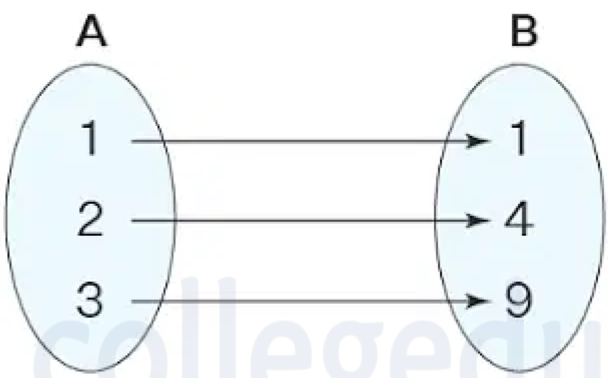

- Table Representation -

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5. **Answer: d**

Explanation:

Explanation:

It is given that, A = {1, 2, 3, 4, 6} and R is a relation on A such that R = {(a, b) : a, b ∈ A and b is exactly divisible by a}The given R can be re-written in roaster form as R = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 6), (2, 2), (2, 4), (2, 6), (3, 3), (3, 6), (4, 4), (6, 6)}.As we know that, Range (R) = {b: (a, b) ∈ R}Therefore, range (R) = {1, 2, 3, 4, 6} = A⇒ n(A) = 5Hence, the correct option is (D).

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6. Answer: b

Explanation:

Explanation:
, : [− 1, 2] ( ) is twice differentiable on (-1,2) (− 1) = 3, (− 1) = 0 (0) = 6, (0) = 1 (2) = 0, (2) = − 1(− 3)″ ≠ 0 on (-1,0) and (0,2)Number of solutions of ′( ) − 3 ′( ) = 0 in (− 1, 0) (0, 2) = ? Let ( ) = ( ) − 3 ( ). Then (− 1) = (− 1) − 3 (− 1) = 3 (0) = (0) − 3 (0) = 6 − 3(1) = 3Therefore, by Rolle's theorem, ′( ) that is, ′( ) − 3 ′( ) = 0 has at least one root in (-1,0).Also (2) = (2) − 3 (2) = 0 − 3(− 1) = 3 Hence, again by Rolle's theorem, ′( ) − 3 ′( ) = 0 has at least one root in (0,2).That is, ′( ) − 3 ′( ) = 0 has at least 2 roots in (-1,2).Since (− 3)″ ≠ 0 for (-1,0) and (0,2)So, ( ) has no point of inflexion in (-1,0) and (0,2) .Therefore, (′ − 3 ′)( ) ≠ 0 in (-1,0) and (0,2) , that is, (′ − 3 ′)( ) ≠ 0 exactly once in (-1,0) and exactly once in (0,2).Hence, the correct option is (B).

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7. Answer: b

Explanation:

(a, b) $R(c, d) \Rightarrow ad(b-c) = bc(a-d)$
Symmetric:
$(c, d)R(a, b) \Rightarrow cb(d-a) = da(c-b) \Rightarrow$
Symmetric Reflexive:
(a, b) $R(a, b) \Rightarrow ab(b-a) \neq ba(a-b) \Rightarrow$
Not reflexive
Transitive: $(2,3)R(3,2)$ and $(3,2)R(5,30)$ but
$((2,3),(5,30)) \notin R \Rightarrow$ Not transitive

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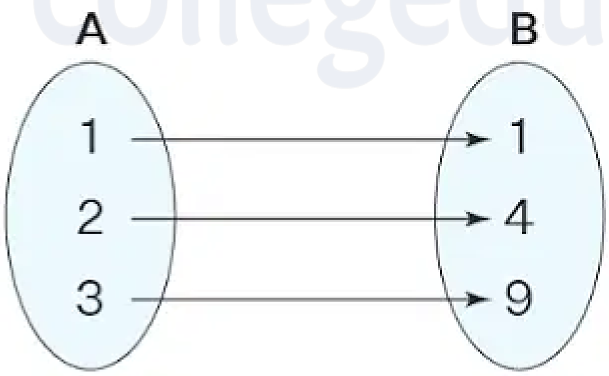

- Table Representation -

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---

8. **Answer: b**

Explanation:

$g(x) = f(\text{-}x)\text{ - }f(x) = \frac{1+e^x}{1\text{-}e^x}$
$?\ g^{'}(x) = \frac{2e^x}{(1\text{-}e^x)^2} > 0$
$?\ g$�is�increasing�in�$(0, 1)$
$?\ g$�is�one-one�in�$(0, 1)$

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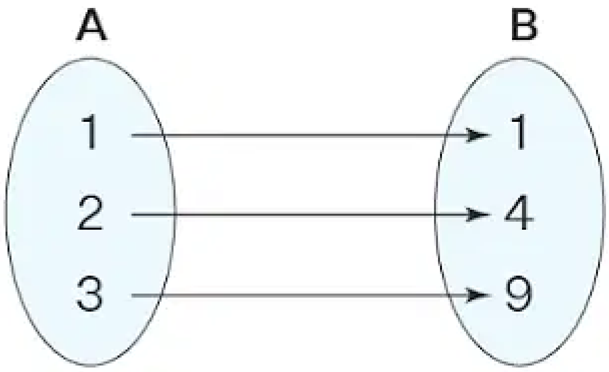

- Table Representation -

x	y
1	1
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9. **Answer: b**

Explanation:

$g(x) = f(-x) - f(x) = \frac{1+e^x}{1-e^x}$

$\Rightarrow g'(x) = \frac{2e^x}{(1-e^x)^2} > 0$

$\Rightarrow g$ is increasing in $(0, 1)$

$\Rightarrow g$ is one-one in $(0, 1)$

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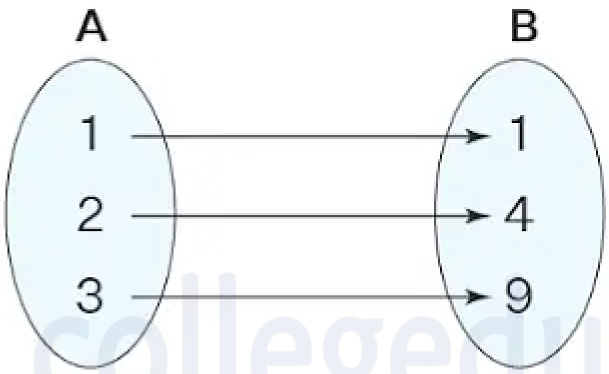

- Table Representation -

x	y
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---

10. **Answer: 2 - 2**

Explanation:

Explanation:

( ) is the inverse of ( )

So, (( )) =

' '(( ))' '( ) = 1

' '(( )) = 1/( )

We need to find out when $(\ ) = 1$

Clearly this happens for $= 0$

So $g'(1) = \frac{1}{'(0)}$

$'(\ ) = 3\ ^2 + \frac{1}{2}\ ^{\bar{z}}$

$'(0) = 0 + \frac{1}{2} = \frac{1}{2}$

So $'(1) = 2$

Hence, the correct answer is (2).

Concepts:

1. Types of Functions:

Types of Functions

One to One Function

A function is said to be one to one function when f: A → B is One to One if for each element of A there is a distinct element of B.

Many to One Function

A function which maps two or more elements of A to the same element of set B is said to be many to one function. Two or more elements of A have the same image in B.

Onto Function

If there exists a function for which every element of set B there is (are) pre-image(s) in set A, it is Onto Function.

One – One and Onto Function

A function, f is One – One and Onto or Bijective if the function f is both One to One and Onto function.

Read More: **Types of Functions**