Sets, Relations, And Functions JEE Main PYQ - 1

Total Time: 25 Minute

Total Marks: 40

Instructions

Instructions

1. Test will auto submit when the Time is up.
2. The Test comprises of multiple choice questions (MCQ) with one or more correct answers.
3. The clock in the top right corner will display the remaining time available for you to complete the examination.

Navigating & Answering a Question

1. The answer will be saved automatically upon clicking on an option amongst the given choices of answer.
2. To deselect your chosen answer, click on the clear response button.
3. The marking scheme will be displayed for each question on the top right corner of the test window.

1. If $X = \{4^n - 3n - 1 : n \in N\}$ and $Y = \{9(n-1) : n \in N\}$,where N is the set of natural numbers, then $X \cup Y$ is equal to   **(+4, −1)**

   [Online April 11, 2014]

   a. N

   b. Y−X

   c. X

   d. Y

2. In a certain town, 25% of the families own a phone and 15% own a car 65% families own neither a phone nor a car and $2,000$ families own both a car and a phone. Consider the following three statements : (a) 5% families own both a car and a phone. (b) 35% families own either a car or a phone. (c) $40,000$ families live in the town. Then,   **(+4, −1)**

   [April 8, 2019 (I)]

   a. Only (a) and (b) are correct

   b. Only (a) and (c) are correct

   c. Only (b) and (c) are correct

   d. All (a), (b) and (c) are correct

3. In a class of 140 students numbered 1 to 140, all even numbered students opted mathematics course, those whose number is divisible by 3 opted Physics course and those whose number is divisible by 5 opted Chemistry course. Then the number of students who did not opt for any of the three courses is :   **(+4, −1)**

   [Online April 11, 2014]

   a. 102

   b. 42

   c. 1

   d. 38

4. Let R be the set of real numbers A = {(x, y) ∈ R × R : y - x is an integer} is an equivalence relation on R. B = {(x, y) ∈ R × R : x = αy for some rational number α} is an equivalence relation on R.

(+4, −1)

a. Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1

b. Statement-1 is true, Statement-2 is false

c. Statement-1 is false, Statement-2 is true

d. Statement-1 is true, Statement-2 is true, Statement-2 is a correct explanation for statement -1

5. Let $x_1, x_2, ..., x_n$ be n observations, and let $\bar{x}$ be their arithmetic mean and σ^2 be the variance. Variance of $2x_1, 2x_2, ..., 2x_n$ is $4\sigma^2$. Arithmetic mean $2x_1, 2x_2, ..., 2x_n$ is $4\bar{x}$.

(+4, −1)

a. Statement-1 is false, Statement-2 is true

b. Statement-1 is true, statement-2 is true; statement-2 is a correct explanation for Statement-1

c. Statement-1 is true, statement-2 is true; statement-2 is not a correct explanation for Statement-1

d. Statement-1 is true, statement-2 is false

6. Let Z be the set of integers. If $A = \{x \in Z : 2^{(x+2)(x^2-5x+6)}\} = 1$ and $B = \{x \in Z : -3 < 2x - 1 < 9\}$, then the number of subsets of the set $A \times B$, is :

(+4, −1)

a. 2^{18}

b. 2^{10}

c. 2^{15}

d. 2^{12}

7. The variance of first n even natural numbers is $\frac{n^2-1}{4}$. The sum of first n natural numbers is $\frac{n(n+1)}{2}$ and the sum of squares of first n natural numbers is $\frac{n(n+1)(2n+1)}{6}$. **(+4, -1)**

   [2003]

   a. Statement-1 is true, Statement-2 is true Statement-2 is not a correct explanation for Statement-1

   b. Statement-1 is true, Statement-2 is false

   c. Statement-1 is false, Statement-2 is true

   d. Statement-1 is true, Statement-2 is true, Statement-2 is a correct explanation for statement -1

8. The set of all $\alpha \epsilon R$, for which $w = \frac{1+(1-8\alpha)z}{1-z}$ is a purely imaginary number, for all $z \neq 1$, is : **(+4, -1)**

   [Jan. 11, 2019 (I)]

   a. an empty set

   b. $\{0\}$

   c. $\left\{0, \frac{1}{4}, -\frac{1}{4}\right\}$

   d. equal to R

9. Let for $x \in R$ $f(x) = \frac{x+|x|}{2}$ and $g(x) = \begin{cases} x, & x < 0 \\ x^2, & x \geq 0 \end{cases}$ **(+4, -1)**

   Then area bounded by the curve $y = (f \circ g)(x)$ and the lines $y = 0, 2y - x = 15$ is equal to

   [2003]

10. Let $f^1(x) = \frac{3x+2}{2x+3}, x \in R - \left\{\frac{-3}{2}\right\}$ For $n \geq 2$, define $f^n(x) = f^1 o f^{n-1}(x)$ if $f^5(x) = \frac{ax+b}{bx+a}, \gcd(a,b) = 1$, then $a + b$ is equal to _____. **(+4, -1)**

    [Jan. 11, 2019 (I)]

Answers

1. **Answer: d**

Explanation:

$$X = \{(1+3)^n - 3n - 1, n \in N\}$$
$$= 3^2(^nC_2 + {}^nC_3 \cdot 3 + \ldots + 3^{n-2}), n \in N\}$$
$$= \{ \text{ Divisible by 9} \}$$
$$Y = \{9(n-1), n \in N\}$$
$$= (\text{ All multiples of 9} \}$$

So, $X \subseteq Y$

i.e., $X \cup Y = Y$

Concepts:

1. Sets:

Set is the collection of well defined objects. Sets are represented by capital letters, eg. A={}. Sets are composed of elements which could be numbers, letters, shapes, etc.

Example of set: Set of vowels A={a,e,i,o,u}

Representation of Sets

There are three basic notation or representation of sets are as follows:

Statement Form: The statement representation describes a statement to show what are the elements of a set.

- For example, Set A is the list of the first five odd numbers.

Roster Form: The form in which elements are listed in set. Elements in the set is seperatrd by comma and enclosed within the curly braces.

- For example represent the set of vowels in roster form.

$$A=\{a,e,i,o,u\}$$

Set Builder Form:

1. The set builder representation has a certain rule or a statement that specifically describes the common feature of all the elements of a set.
2. The set builder form uses a vertical bar in its representation, with a text describing the character of the elements of the set.
3. For example, A = { k | k is an even number, k ≤ 20}. The statement says, all the elements of set A are even numbers that are less than or equal to 20.
4. Sometimes a ":" is used in the place of the "|".

---

2. **Answer: d**

Explanation:
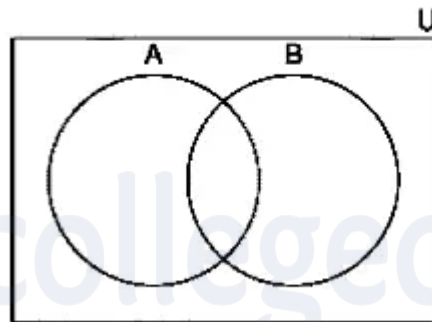
$n(P) = 25$

$n(C) = 15$

$n\left(P' \cup C'\right) = 65\%$

$\Rightarrow n\left(P \cup C\right)' = 65\%$

$n\left(P \cup C\right) = 35\%$

$n\left(P \cap C\right) = n\left(P\right) + n\left(C\right) - n\left(P \cup C\right)$

$25 + 15 - 35 = 5\%$

$x \times 5\% = 2000$

$x = 40,000$

Concepts:

1. Sets:

Set is the collection of well defined objects. Sets are represented by capital letters, eg. A={}. Sets are composed of elements which could be numbers, letters, shapes, etc.

Example of set: Set of vowels A={a,e,i,o,u}

Representation of Sets

There are three basic notation or representation of sets are as follows:

Statement Form: The statement representation describes a statement to show what are the elements of a set.

- For example, Set A is the list of the first five odd numbers.

Roster Form: The form in which elements are listed in set. Elements in the set is seperatrd by comma and enclosed within the curly braces.

- For example represent the set of vowels in roster form.

$$A=\{a,e,i,o,u\}$$

Set Builder Form:

1. The set builder representation has a certain rule or a statement that specifically describes the common feature of all the elements of a set.
2. The set builder form uses a vertical bar in its representation, with a text describing the character of the elements of the set.
3. For example, A = { k | k is an even number, k ≤ 20}. The statement says, all the elements of set A are even numbers that are less than or equal to 20.
4. Sometimes a ":" is used in the place of the "|".

---

3. **Answer: d**

Explanation:

Let $n(A)$ = number of students opted
Mathematics = 70,
$n(B)$ = number of students opted Physics = 46,
$n(C)$ = number of students opted Chemistry
= 28,
$n(A \cap B) = 23$,
$n(B \cap C) = 9$,
$n(A \cap C) = 14$,

$n(A \cap B \cap C) = 4,$

Now $n(A \cup B \cup C)$

$= n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C)$

$-n(A \cap C) + n(A \cap B \cap C)$

$= 70 + 46 + 28 - 23 - 9 - 14 + 4 = 102$

So number of students not opted for any course

= Total - $n(A \cup B \cup C)$

$= 140 - 102 = 38$

Concepts:

1. Sets:

Set is the collection of well defined objects. Sets are represented by capital letters, eg. A={}. Sets are composed of elements which could be numbers, letters, shapes, etc.

Example of set: Set of vowels A={a,e,i,o,u}

Representation of Sets

There are three basic notation or representation of sets are as follows:

Statement Form: The statement representation describes a statement to show what are the elements of a set.

- For example, Set A is the list of the first five odd numbers.

Roster Form: The form in which elements are listed in set. Elements in the set is seperatrd by comma and enclosed within the curly braces.

- For example represent the set of vowels in roster form.

<p align="center">A={a,e,i,o,u}</p>

Set Builder Form:

1. The set builder representation has a certain rule or a statement that specifically describes the common feature of all the elements of a set.
2. The set builder form uses a vertical bar in its representation, with a text describing the character of the elements of the set.

3. For example, A = { k | k is an even number, k ≤ 20}. The statement says, all the elements of set A are even numbers that are less than or equal to 20.

4. Sometimes a ":" is used in the place of the "|".

---

4. Answer: b

Explanation:

x − y is an integer.

∵ x − x = 0 is an integer ⇒ A is reflexive.

Let x − y is an integer

⇒ y − x is an integer

⇒ A is symmetric

Let x − y, y − z are integers

⇒ x − y + y − z is also an integer

⇒ x − z is an integer

⇒ A is transitive

∴ A is an equivalence relation.

Hence statement 1 is true.

Also B can be considered as

xBy if $\frac{x}{y} = \alpha$, a rational number

∵ $\frac{x}{x} = 1$ is a rational number

⇒ B is reflexive

But $\frac{x}{y} = \alpha$, a rational number need not imply $\frac{y}{x} = \frac{1}{\alpha}$, a rational number because $\frac{0}{1}$ is rational ⇒ $\frac{1}{0}$ is not rational

∴ B is not an equivalence relation.

Concepts:

1. Sets:

Set is the collection of well defined objects. Sets are represented by capital letters, eg. A={}. Sets are composed of elements which could be numbers, letters, shapes, etc.

Example of set: Set of vowels A={a,e,i,o,u}

Representation of Sets

There are three basic notation or representation of sets are as follows:

Statement Form: The statement representation describes a statement to show what are the elements of a set.

- For example, Set A is the list of the first five odd numbers.

Roster Form: The form in which elements are listed in set. Elements in the set is seperatrd by comma and enclosed within the curly braces.

- For example represent the set of vowels in roster form.

$$A=\{a,e,i,o,u\}$$

Set Builder Form:

1. The set builder representation has a certain rule or a statement that specifically describes the common feature of all the elements of a set.
2. The set builder form uses a vertical bar in its representation, with a text describing the character of the elements of the set.
3. For example, A = { k | k is an even number, k ≤ 20}. The statement says, all the elements of set A are even numbers that are less than or equal to 20.
4. Sometimes a ":" is used in the place of the "|".

---

5. **Answer: d**

Explanation:

If each observation is multiplied by k, mean gets multiplied by k and variance gets multiplied by k^2. Hence the new mean should be $2\bar{x}$ and new variance should be $k^2\sigma^2$. So statement-1 is true and statement-2 is false.

Concepts:

1. Sets:

Set is the collection of well defined objects. Sets are represented by capital letters, eg. A={}. Sets are composed of elements which could be numbers, letters, shapes, etc.

Example of set: Set of vowels A={a,e,i,o,u}

Representation of Sets

There are three basic notation or representation of sets are as follows:

Statement Form: The statement representation describes a statement to show what are the elements of a set.

- For example, Set A is the list of the first five odd numbers.

Roster Form: The form in which elements are listed in set. Elements in the set is seperatrd by comma and enclosed within the curly braces.

- For example represent the set of vowels in roster form.

$$A=\{a,e,i,o,u\}$$

Set Builder Form:

1. The set builder representation has a certain rule or a statement that specifically describes the common feature of all the elements of a set.
2. The set builder form uses a vertical bar in its representation, with a text describing the character of the elements of the set.
3. For example, A = { k | k is an even number, k ≤ 20}. The statement says, all the elements of set A are even numbers that are less than or equal to 20.
4. Sometimes a ":" is used in the place of the "|".

---

6. **Answer: c**

Explanation:

$A = \left\{ x \in z : 2^{(x+2)(x^2-5x+6)} = 1 \right\}$

$2^{(x+2)(x^2-5x+6)} = 2^0 \Rightarrow x = -2, 2, 3$

$A = \{-2, 2, 3\}$

$B = x \in Z : -3 < 2x - 1 < 9$

$A \times B = 3 \times 15 = 15$ has 15 elements so number of subsets

of $A \times B$ is 2^{15}

Q40

Sol : Given that :

$$A = \{ x \in z : 2^{(x+2)(x^2-5x+6)} = 1 \}$$

$$\therefore \quad 2^{(x+2)(x^2-5x+6)} = 2^0$$

$$\therefore (x+2)(x^2-5x+6) = 0$$

$$\Rightarrow \boxed{x = -2} \quad \text{and} \quad \text{for} \quad x^2-5x+6 = 0$$

$$\Rightarrow (x-2)(x-3) = 0$$

$$\Rightarrow x = 2 \quad \& \quad x = 3$$

$$\therefore \boxed{x = -2, 2, 3}$$

$$A = \{-2, 2, 3\}$$

$$B = \{ x \in z : -3 < 2x-1 < 9 \}$$

$$B = \{0, 1, 2, 3, 4\}$$

$$\therefore A \times B = 3 \times 5 = 15 \text{ elements so no}$$

$$\text{subsets of } A \times B = 2^{15} \quad (Ans)$$

Concepts:

1. Sets:

Set is the collection of well defined objects. Sets are represented by capital letters, eg. A={}. Sets are composed of elements which could be numbers, letters, shapes, etc.

Example of set: Set of vowels A={a,e,i,o,u}

Representation of Sets

There are three basic notation or representation of sets are as follows:

Statement Form: The statement representation describes a statement to show what are the elements of a set.

- For example, Set A is the list of the first five odd numbers.

Roster Form: The form in which elements are listed in set. Elements in the set is seperatrd by comma and enclosed within the curly braces.

- For example represent the set of vowels in roster form.

$$A=\{a,e,i,o,u\}$$

Set Builder Form:

1. The set builder representation has a certain rule or a statement that specifically describes the common feature of all the elements of a set.
2. The set builder form uses a vertical bar in its representation, with a text describing the character of the elements of the set.
3. For example, A = { k | k is an even number, k ≤ 20}. The statement says, all the elements of set A are even numbers that are less than or equal to 20.
4. Sometimes a ":" is used in the place of the "|".

---

7. **Answer: c**

Explanation:

The correct answer is C:Statement-1 is false, Statement-2 is true
Given that;
Sum of first 'n' even natural numbers
$= 2 + 4 + 6 + ... + 2n$
$= 2(1 + 2 + ...n)$
$\frac{2(n+1)n}{2} = n(n + 1)$
For the numbers 2, 4, 6, 8, ..., 2n
$\bar{x} = \frac{2[n(n+1)]}{2n} = (n + 1)$
And $Var = \frac{\sum(x-\bar{x})^2}{2n} = \frac{\sum x^2}{n} - (\bar{x})^2$
$= \frac{4\sum n^2}{n} - (n + 1)^2 = \frac{4n(n+1)(2n+1)}{6n} - (n + 1)^2$
$= \frac{2(2n+1)(n+1)}{3} - (n + 1)^2 = (n + 1)\left[\frac{4n+2-3n-3}{3}\right]$

$$= \frac{(n+1)(n-1)}{3} = \frac{n^2-1}{3}$$

$\therefore$ Statement-1 is false. Clearly, statement - 2 is true .

$$= \frac{(n+1)(n-1)}{3} = \frac{n^2-1}{3}$$

$\therefore$ Statement-1 is false. Clearly, statement - 2 is true .

243) **8th**

Given that

Sum of first 'n' even natural numbers

$$= 2+4+6+\cdots+2n$$

$$= 2(1+2+\cdots+n)$$

$$= \frac{2(n+1)n}{2} = n(n+1)$$

Mean $(\bar{x}) = \frac{(n+1)n}{n} = n+1$

∴ variance

$$\frac{1}{n}(\Sigma x_i)^2 - (\bar{x})^2$$

$$= \frac{1}{n}(2^2+4^2+\cdots(2n)^2) - (n+1)^2$$

$$= \frac{4}{n} \times \frac{n(n+1)(2n+1)}{6} - (n+1)^2$$

$$= \frac{2}{3}(n+1)(2n+1) - (n+1)^2$$

$$= \frac{(n+1)}{3}[2(2n+1) - 3(n+1)]$$

$$= \frac{n+1}{3}(n-1)$$

$$= \frac{n^2-1}{3} \quad (Ans)$$

Concepts:

1. Sets:

Set is the collection of well defined objects. Sets are represented by capital letters, eg. A={}. Sets are composed of elements which could be numbers, letters, shapes, etc.

Example of set: Set of vowels A={a,e,i,o,u}

Representation of Sets

There are three basic notation or representation of sets are as follows:

Statement Form: The statement representation describes a statement to show what are the elements of a set.

- For example, Set A is the list of the first five odd numbers.

Roster Form: The form in which elements are listed in set. Elements in the set is seperatrd by comma and enclosed within the curly braces.

- For example represent the set of vowels in roster form.

$$A=\{a,e,i,o,u\}$$

Set Builder Form:

1. The set builder representation has a certain rule or a statement that specifically describes the common feature of all the elements of a set.
2. The set builder form uses a vertical bar in its representation, with a text describing the character of the elements of the set.
3. For example, A = { k | k is an even number, k ≤ 20}. The statement says, all the elements of set A are even numbers that are less than or equal to 20.
4. Sometimes a ":" is used in the place of the "|".

---

8. Answer: b

Explanation:

As ω is purely imaginary $\omega + \bar{\omega} = 0$ $\frac{1+(1-8\alpha)z}{1-z} + \frac{1+(1-8a)\bar{z}}{1-z} = 0$ $\frac{1-\bar{z}+(1-8a)(z-1)+a-z+(1-8\alpha)(\bar{z}-1)}{(1-z)(1-\bar{z})} = 0$ $1 - \bar{z} + z - 1 - 8az + 8a + 1 - z + \bar{z} - 1 - 8\bar{z} - 1 - 8\bar{z}\alpha + 8\alpha = 0$ $-8a(z + \bar{z}) + 16\alpha = 0$ $8a[2 - (2 + 2)] = 0$ if $Re(z) \neq 1$

Concepts:

1. Sets:

Set is the collection of well defined objects. <u>Sets</u> are represented by capital letters, eg. A={}. Sets are composed of elements which could be numbers, letters, shapes, etc.

Example of set: Set of vowels A={a,e,i,o,u}

Representation of Sets

There are three basic notation or representation of sets are as follows:

Statement Form: The statement representation describes a statement to show what are the elements of a set.

- For example, Set A is the list of the first five odd numbers.

Roster Form: The form in which elements are listed in set. Elements in the set is seperatrd by comma and enclosed within the curly braces.

- For example represent the set of vowels in roster form.

$$A=\{a,e,i,o,u\}$$

Set Builder Form:

1. The set builder representation has a certain rule or a statement that specifically describes the common feature of all the elements of a set.
2. The set builder form uses a vertical bar in its representation, with a text describing the character of the elements of the set.
3. For example, A = { k | k is an even number, k ≤ 20}. The statement says, all the elements of set A are even numbers that are less than or equal to 20.
4. Sometimes a ":" is used in the place of the "|".

---

9. **Answer: 72 - 72**

Explanation:

The correct answer is 72.

$$f(x) = \frac{x + |x|}{2} = \begin{cases} x & x \geq 0 \\ 0 & x < 0 \end{cases}$$

$$g(x) = \begin{cases} x^2 & x \geq 0 \\ x & x < 0 \end{cases}$$

$$fog(x) = f[g(x)] = \begin{cases} g(x) & g(x) \geq 0 \\ 0 & g(x) < 0 \end{cases}$$

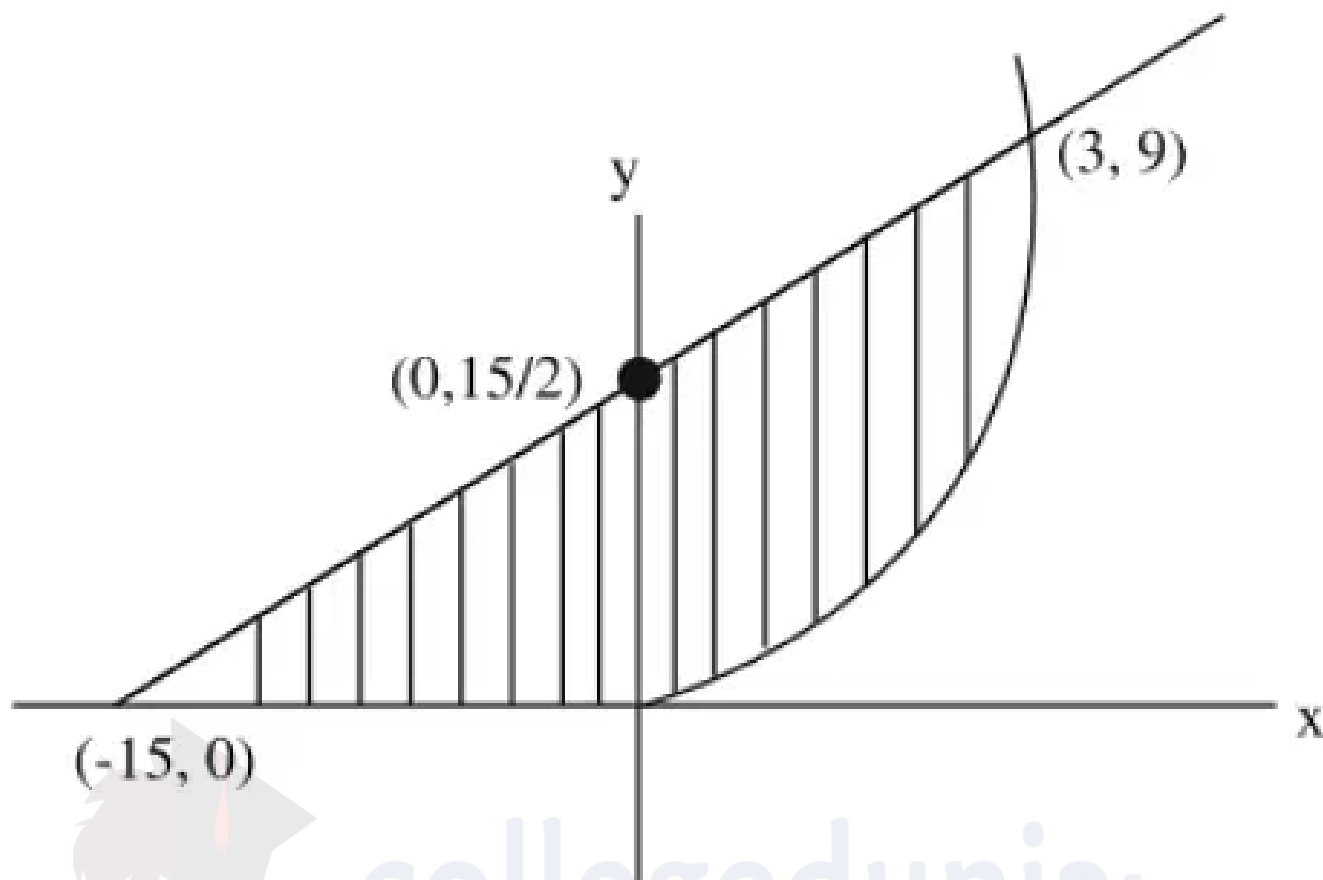$$fog(x) = \begin{cases} x^2 & x \geq 0 \\ 0 & x < 0 \end{cases}$$

$$2y - x = 15$$

2y−x=15

A=0∫3(2x+15−x2)dx+21×215×15

4x2+215x−3x3‖03+4225

=49+245−9+4225=499−36+225

=4288=72

Concepts:

1. Relations and functions:

A **relation** R from a non-empty set B is a subset of the cartesian product A × B. The subset is derived by describing a relationship between the first element and the second element of the ordered pairs in A × B.

A relation f from a set A to a set B is said to be a **function** if every element of set A has one and only one image in set B. In other words, no two distinct elements of B have the same pre-image.
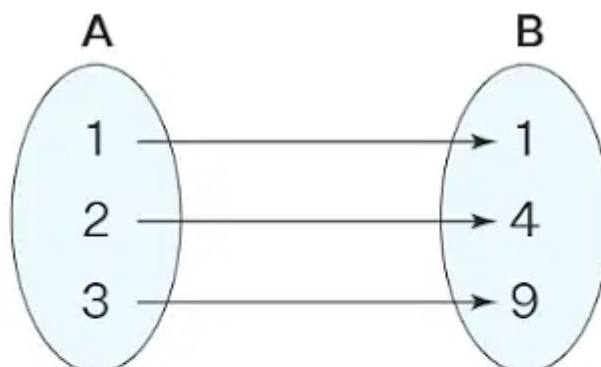
Representation of Relation and Function

Relations and functions can be represented in different forms such as arrow representation, algebraic form, set-builder form, graphically, roster form, and tabular form. Define a function f: A = {1, 2, 3} → B = {1, 4, 9} such that f(1) = 1, f(2) = 4, f(3) = 9. Now, represent this function in different forms.

1. Set-builder form - {(x, y): f(x) = y², x ∈ A, y ∈ B}

2. Roster form – {(1, 1), (2, 4), (3, 9)}

3. Arrow Representation

- Table Representation -

x	y
1	1
2	4
3	9

10. **Answer: 3125 – 3125**

Explanation:

The correct answer is 3125.

$f^1(x) = \frac{3x+2}{2x+3}$

$\Rightarrow f^2(x) = \frac{13x+12}{12x+13}$

$\Rightarrow f^3(x) = \frac{63x+62}{62x+63}$

$\therefore f^5(x) = \frac{1563x+1562}{1562x+1563}$

$a + b = 3125$

Concepts:

1. Relations and functions:
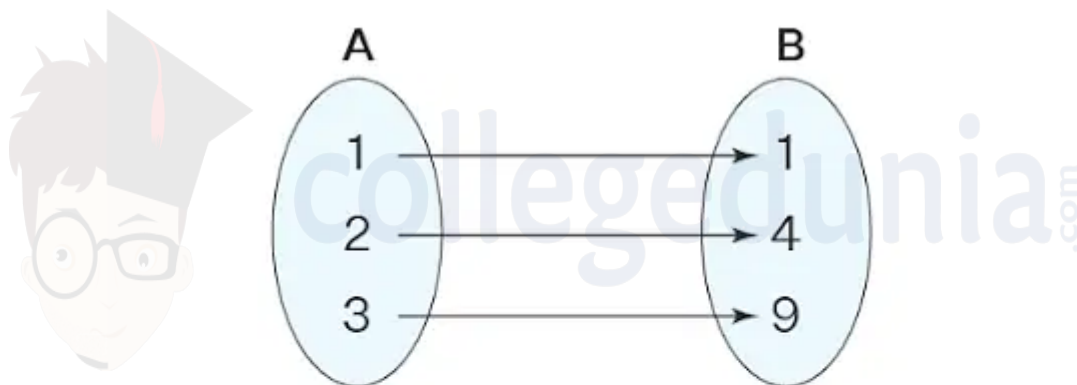
A **relation** R from a non-empty set B is a subset of the cartesian product A × B. The subset is derived by describing a relationship between the first element and the second element of the ordered pairs in A × B.

A relation f from a set A to a set B is said to be a **function** if every element of set A has one and only one image in set B. In other words, no two distinct elements of B have the same pre-image.

Representation of Relation and Function

Relations and functions can be represented in different forms such as arrow representation, algebraic form, set-builder form, graphically, roster form, and tabular form. Define a function f: A = {1, 2, 3} → B = {1, 4, 9} such that f(1) = 1, f(2) = 4, f(3) = 9. Now, represent this function in different forms.

1. Set-builder form - {(x, y): f(x) = y^2, x ∈ A, y ∈ B}
2. Roster form - {(1, 1), (2, 4), (3, 9)}
3. Arrow Representation

- Table Representation -

x	y
1	1
2	4
3	9