Straight lines JEE Main PYQ -3

Total Time: 20 Minute **Total Marks:** 40

Instructions

Instructions

1. Test will auto submit when the Time is up.
2. The Test comprises of multiple choice questions (MCQ) with one or more correct answers.
3. The clock in the top right corner will display the remaining time available for you to complete the examination.

Navigating & Answering a Question

1. The answer will be saved automatically upon clicking on an option amongst the given choices of answer.
2. To deselect your chosen answer, click on the clear response button.
3. The marking scheme will be displayed for each question on the top right corner of the test window.

Straight lines

1. Let S_1 and S_2 be respectively the sets of all $a \in R - \{0\}$ for which the system of linear equations $ax + 2ay - 3az = 1$ $(2a+1)x + (2a+3)y + (a+1)z = 2$ $(3a+5)x + (a+5)y + (a+2)z = 3$ has unique solution and infinitely many solutions Then **(+4)**

   a. $S_1 = \Phi$ and $S_2 = R - \{0\}$

   b. S_1 is an infinite set and $n(S_2) = 2$

   c. $S_1 = R - \{0\}$ and $S_2 = \Phi$

   d. $n(S_1) = 2$ and S_2 is an infinite set

---

2. Let S_1 and S_2 be respectively the sets of all $a \in R - \{0\}$ for which the system of linear equations **(+4)**
   $ax + 2ay - 3az = 1$
   $(2a+1)x + (2a+3)y + (a+1)z = 2$
   $(3a+5)x + (a+5)y + (a+2)z = 3$
   has unique solution and infinitely many solutions. Then

   a. $S_1 = \Phi$ and $S_2 = R - \{0\}$

   b. S_1 is an infinite set and $n(S_2) = 2$

   c. $S_1 = R - \{0\}$ and $S_2 = \Phi$

   d. $n(S_1) = 2$ and S_2 is an infinite set

---

3. Let the six numbers $a_1, a_2, a_3, a_4, a_5, a_6$, be in AP and $a_1 + a_3 = 10$ If the mean of these six numbers is $\frac{19}{2}$ and their variance is σ^2, then $8\sigma^2$ is equal to : **(+4)**

   a. 105

   b. 210

   c. 200

   d. 220

4. Orthocentre of triangle having vertices as A (1,2), B(3,-4), C(0,6) is **(+4)**

a. (-129, -37)

b. (9, -1)

c. (7, -3)

d. (28, -16)

5. A rectangle is drawn by lines x=0, x=2, y=0 and y=5. Points A and B lie on coordinate axes. If line AB divides the area of rectangle in 4:1, then the locus of mid-point of AB is? **(+4)**

a. Circle

b. Hyperbola

c. Ellipse

d. Straight line

6. Two sides of a rhombus are along the lines, $x - y + 1 = 0$ and $7x - y - 5 = 0$. If its diagonals intersect at $(-1, -2)$, then which one of the following is a vertex of this rhombus? **(+4)**

a. (-3 , -9)

b. (-3 , -8)

c. $\left(\frac{1}{3}, -\frac{8}{3}\right)$

d. $\left(-\frac{10}{3}, -\frac{7}{3}\right)$

7. The line passing through the points (5, 1, a) and (3, b, 1) crosses the yz- plane at the point 0, $\frac{17}{2}$, $\frac{-13}{2}$. Then; **(+4)**

a. (A) a = 2, b = 8

b. (B) a = 4, b = 6

c. (C) a = 6, b = 4

d. (D) a = 8, b = 6

---

8. For $\alpha, \beta \in R$, suppose the system of linear equations $x - y + z = 5$ $2x + 2y + \alpha z = 8$ $3x - y + 4z = \beta$ has infinitely many solutions Then α and β are the roots of **(+4)**

a. $x^2 - 18x + 56 = 0$

b. $x^2 + 14x + 24 = 0$

c. $x^2 - 10x + 16 = 0$

d. $x^2 + 18x + 56 = 0$

---

9. If the x-intercept of a focal chord of the parabola $y^2 = 8x + 4y + 4$ is 3 , then the length of this chord is equal to ___ **(+4)**

---

10. Let S be the set of all $a \in N$ such that the area of the triangle formed by the tangent at the point $P(b, c), b, c \in N$, on the parabola $y^2 = 2$ ax and the lines $x = b, y = 0$ is 16 unit ^{2}, then $\sum_{a \in S} a$ is equal to _____ **(+4)**

Answers

1. Answer: c

Explanation:

$$? = \begin{vmatrix} a & 2a & -3a \\ 2a+1 & 2a+3 & a+1 \\ 3a+5 & a+5 & a+2 \end{vmatrix}$$

$= a\left(15a^2 + 31a + 36\right) = 0 \; ? \; a = 0$

$? \neq 0$�for�all�$a \; ? \; R$ - $\{0\}$

Hence $S_1 = R$ - $\{0\} S_2 = \text{F}$

Concepts:

1. Straight lines:

A **straight line** is a line having the shortest distance between two points.

A straight line can be represented as an equation in various forms, as show in the image below:

Standard Form : $ax + by = c$

Slope-Intercept Form : $y = mx + c$

Point-Slope Form : $y - y_1 = m(x - x_1)$

The following are the many forms of the equation of the line that are presented in straight line-

1. Slope – Point Form

Assume P0(x0, y0) is a fixed point on a non-vertical line L with m as its slope. If P (x, y) is an arbitrary point on L, then the point (x, y) lies on the line with slope m through the fixed point (x0, y0) if and only if its coordinates fulfil the equation below.

$y - y_0 = m(x - x_0)$

2. Two – Point Form

Let's look at the line. L crosses between two places. $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ are general points on L, while P (x, y) is a general point on L. As a result, the three points P_1, P_2, and P are collinear, and it becomes

The slope of P_2P = The slope of P_1P_2, i.e.

$$\frac{y-y_1}{x-x_1} = \frac{y_2-y_1}{x_2-x_1}$$

Hence, the equation becomes:

$$y - y_1 = \frac{y_2-y_1}{x_2-x_1}(x - x1)$$

3. Slope-Intercept Form

Assume that a line L with slope m intersects the y-axis at a distance c from the origin, and that the distance c is referred to as the line L's y-intercept. As a result, the coordinates of the spot on the y-axis where the line intersects are (0, c). As a result, the slope of the line L is m, and it passes through a fixed point (0, c). The equation of the line L thus obtained from the slope – point form is given by

$y - c = m(x - 0)$

As a result, the point (x, y) on the line with slope m and y-intercept c lies on the line, if and only if

$$y = mx + c$$

2. **Answer: c**

Explanation:

The correct answer is (C) : $S_1 = R - \{0\}$ and $S_2 = \Phi$

$$\Delta = \begin{vmatrix} a & 2a & -3a \\ 2a+1 & 2a+3 & a+1 \\ 3a+5 & a+5 & a+2 \end{vmatrix}$$

$= a\left(15a^2 + 31a + 36\right) = 0 \Rightarrow a = 0$

$\Delta \neq 0$ for all $a \in R - \{0\}$

Hence $S_1 = R - \{0\} S_2 = \Phi$

Concepts:

1. **Solution of System of Linear Inequalities in Two Variables:**

A System of Linear Inequalities is a set of 2 or more linear inequalities which have the same variables.

Example

$x + y \geq 5$

$x - y \leq 3$

Here are two inequalities having two same variables that are, x and y.

Solution of System of Linear Inequalities in Two Variables

The solution of a system of a linear inequality is the ordered pair which is the solution of all inequalities in the studied system and the graph of the system of a linear inequality is the graph of the common solution of the system.

Therefore, the Solution of the System of Linear Inequalities could be:

Graphical Method:

For the Solution of the System of Linear Inequalities, the Graphical Method is the easiest method. In this method, the process of making a graph is entirely similar to the graph of linear inequalities in two variables.

Non-Graphical Method:

In the Non-Graphical Method, there is no need to make a graph but we can find the solution to the system of inequalities by finding the interval at which the system persuades all the inequalities.

In this method, we have to find the point of intersection of the two inequalities by resolving them. It could be feasible that there is no intersection point between them.

---

3. **Answer: b**

Explanation:

The correct answer is (C) : 210

$a_1 + a_3 = 10 = a_1 + d \Rightarrow 5$

$a_1 + a_2 + a_3 + a_4 + a_5 + a_6 = 57$

$\Rightarrow \frac{6}{2}[a_1 + a_6] = 57$

$\Rightarrow a_1 + a_6 = 19$

$\Rightarrow 2a_1 + 5d = 19 \text{ and } a_1 + d = 5$

$\Rightarrow a_1 = 2, d = 3$

Numbers : $2, 5, 8, 11, 14, 17$

Variance $= \sigma^2 =$ mean of squares-square of mean

$= \frac{2^2+5^2+8^2+(11)^2+(14)^2+(17)^2}{6} - \left(\frac{19}{2}\right)^2$

$= \frac{699}{6} - \frac{361}{4} = \frac{105}{4}$

$8\sigma^2 = 210$

Concepts:

1. Straight lines:

A **straight line** is a line having the shortest distance between two points.

A straight line can be represented as an equation in various forms, as show in the image below:

Standard Form : $ax + by = c$

Slope-Intercept Form : $y = mx + c$

Point-Slope Form : $y - y_1 = m(x - x_1)$

The following are the many forms of the equation of the line that are presented in straight line-

1. Slope – Point Form

Assume P0(x0, y0) is a fixed point on a non-vertical line L with m as its slope. If P (x, y) is an arbitrary point on L, then the point (x, y) lies on the line with slope m through the fixed point (x0, y0) if and only if its coordinates fulfil the equation below.

$$y - y_0 = m(x - x_0)$$

2. Two – Point Form

Let's look at the line. L crosses between two places. $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ are general points on L, while P (x, y) is a general point on L. As a result, the three points P_1, P_2, and P are collinear, and it becomes

The slope of P_2P = The slope of P_1P_2, i.e.

$$\frac{y-y_1}{x-x_1} = \frac{y_2-y_1}{x_2-x_1}$$

Hence, the equation becomes:

$$y - y_1 = \frac{y_2-y_1}{x_2-x_1}(x - x1)$$

3. Slope-Intercept Form

Assume that a line L with slope m intersects the y-axis at a distance c from the origin, and that the distance c is referred to as the line L's y-intercept. As a result, the coordinates of the spot on the y-axis where the line intersects are (0, c). As a result, the slope of the line L is m, and it passes through a fixed point (0, c). The equation of the line L thus obtained from the slope – point form is given by

$y - c = m(x - 0)$

As a result, the point (x, y) on the line with slope m and y-intercept c lies on the line, if and only if

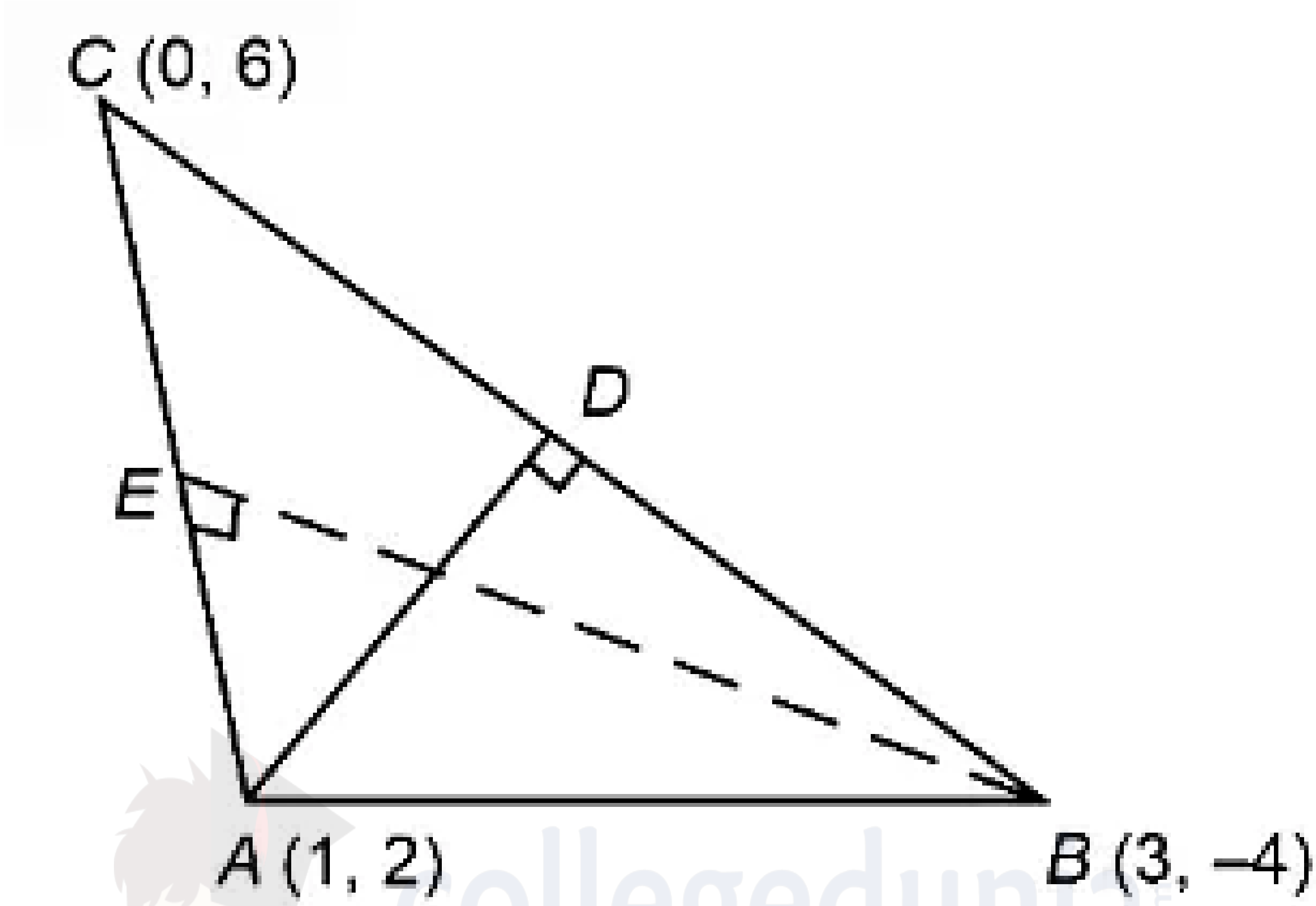
$y = mx + c$

---

4. **Answer: a**

   Explanation:

   Correct answer is (A). (-129, -37)



AD = $(y-2)=\frac{3}{10}(x-1)$

3x−10y+17=0....(i)

BE = $(y+4)= \frac{1}{4}(x-3)$

x−4y=19 .....(ii)

solving (i) and (ii),

(-129, -37) is Orthocentre

Concepts:

1. Straight lines:

A **straight line** is a line having the shortest distance between two points.

A straight line can be represented as an equation in various forms, as show in the image below:

Standard Form : $ax + by = c$

Slope-Intercept Form : $y = mx + c$

Point-Slope Form : $y - y_1 = m(x - x_1)$

The following are the many forms of the equation of the line that are presented in straight line-

1. Slope – Point Form

Assume P0(x0, y0) is a fixed point on a non-vertical line L with m as its slope. If P (x, y) is an arbitrary point on L, then the point (x, y) lies on the line with slope m through the fixed point (x0, y0) if and only if its coordinates fulfil the equation below.

$$y - y_0 = m(x - x_0)$$

2. Two – Point Form

Let's look at the line. L crosses between two places. $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ are general points on L, while P (x, y) is a general point on L. As a result, the three points P_1, P_2, and P are collinear, and it becomes

The slope of P_2P = The slope of P_1P_2, i.e.

$$\frac{y-y_1}{x-x_1} = \frac{y_2-y_1}{x_2-x_1}$$

Hence, the equation becomes:

$$y - y_1 = \frac{y_2-y_1}{x_2-x_1}(x - x1)$$

3. Slope-Intercept Form

Assume that a line L with slope m intersects the y-axis at a distance c from the origin, and that the distance c is referred to as the line L's y-intercept. As a result, the coordinates of the spot on the y-axis where the line intersects are (0, c). As a result, the slope of the line L is m, and it passes through a fixed point (0, c). The equation of the line L thus obtained from the slope – point form is given by

$y - c = m(x - 0)$

As a result, the point (x, y) on the line with slope m and y-intercept c lies on the line, if and only if

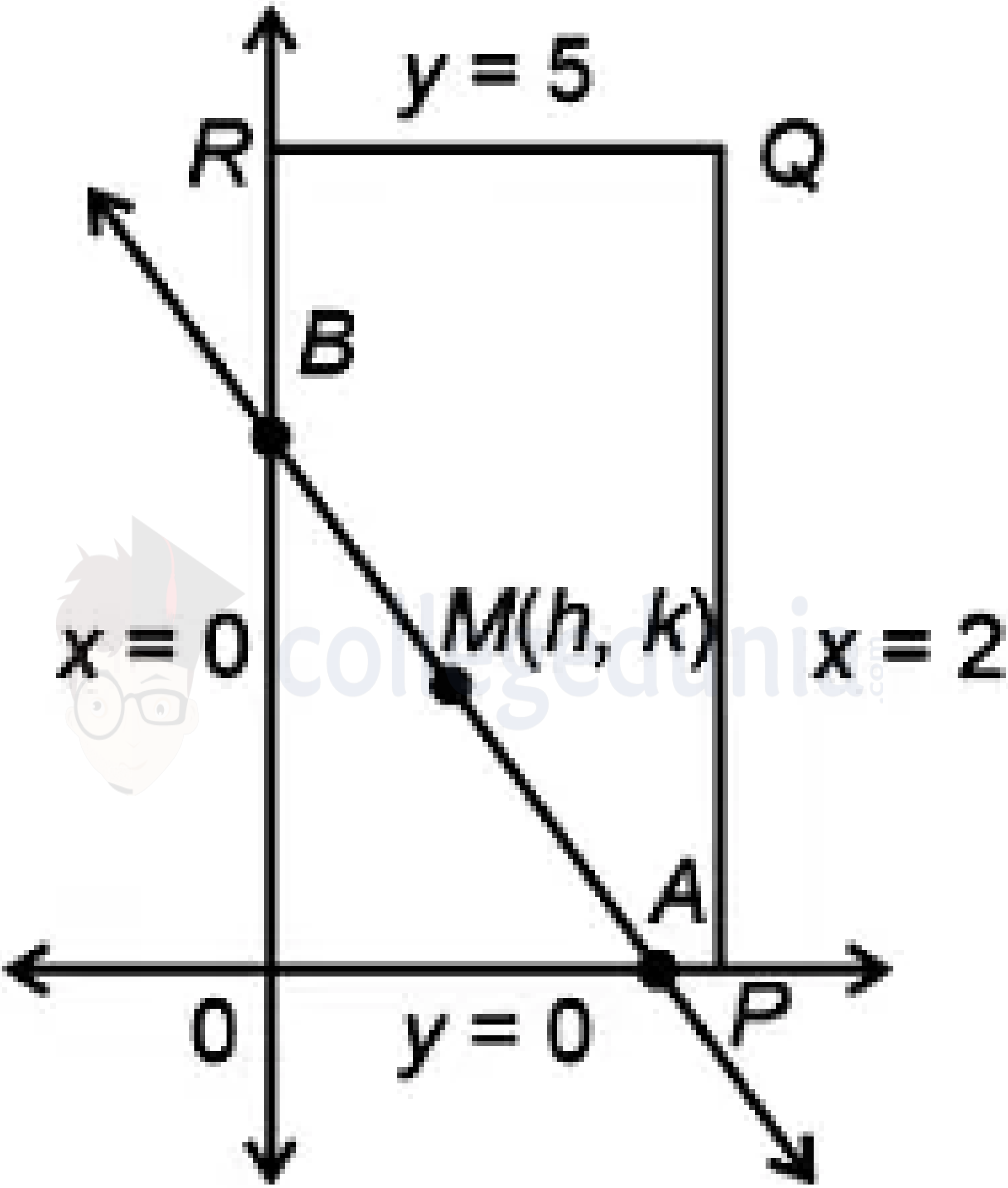
$y = mx + c$

---

5. **Answer: b**

Explanation:

The correct answer is option (B): Hyperbola



A(2h,0), B(0,2k)

Area of $\triangle$OAB=8

$\frac{1}{2} \times 2h \times 2k = 8$

$hk = 4$

Locus of XY = 4

Concepts:

1. Straight lines:

A **straight line** is a line having the shortest distance between two points.

A straight line can be represented as an equation in various forms, as show in the image below:

Standard Form : $ax + by = c$

Slope-Intercept Form : $y = mx + c$

Point-Slope Form : $y - y_1 = m(x - x_1)$

The following are the many forms of the equation of the line that are presented in straight line-

1. Slope – Point Form

Assume P0(x0, y0) is a fixed point on a non-vertical line L with m as its slope. If P (x, y) is an arbitrary point on L, then the point (x, y) lies on the line with slope m through the fixed point (x0, y0) if and only if its coordinates fulfil the equation below.

$$y - y_0 = m(x - x_0)$$

2. Two – Point Form

Let's look at the line. L crosses between two places. $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ are general points on L, while P (x, y) is a general point on L. As a result, the three points P_1, P_2, and P are collinear, and it becomes

The slope of P_2P = The slope of P_1P_2 , i.e.

$$\frac{y-y_1}{x-x_1} = \frac{y_2-y_1}{x_2-x_1}$$

Hence, the equation becomes:

$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x1)$

3. Slope-Intercept Form

Assume that a line L with slope m intersects the y-axis at a distance c from the origin, and that the distance c is referred to as the line L's y-intercept. As a result, the coordinates of the spot on the y-axis where the line intersects are (0, c). As a result, the slope of the line L is m, and it passes through a fixed point (0, c). The equation of the line L thus obtained from the slope – point form is given by

$y - c = m(x - 0)$

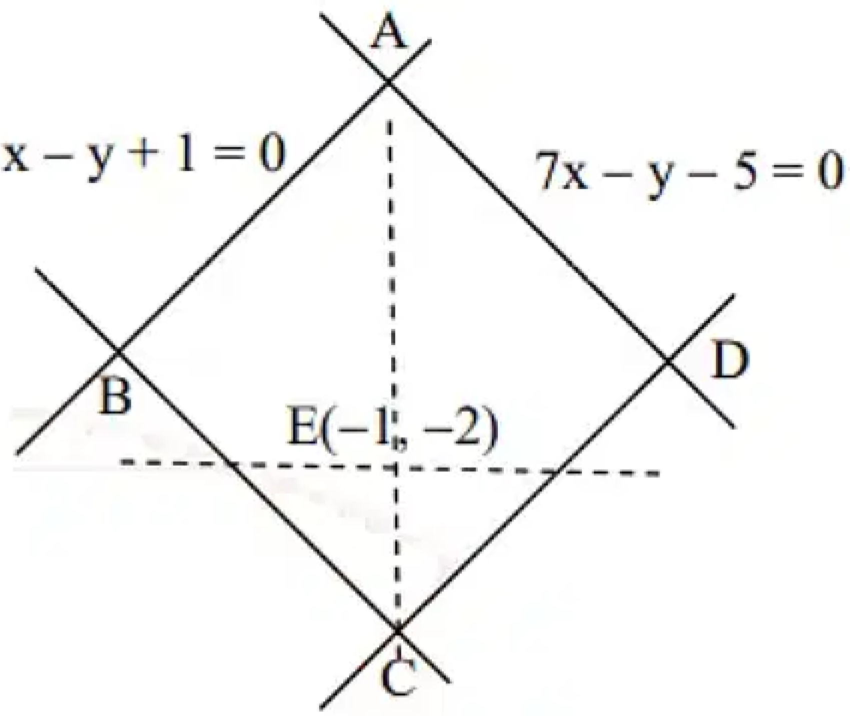
As a result, the point (x, y) on the line with slope m and y-intercept c lies on the line, if and only if

$y = mx + c$

---

6. **Answer: c**

Explanation:



Coordinates of $A \equiv (1, 2)$

$\therefore$ Slope of $AE = 2$

$\Rightarrow$ Slope of $BD = -\frac{1}{2} \Rightarrow$

E of BD is $\frac{y+2}{x+1} = -\frac{1}{2}$

$\Rightarrow x + 2y + 5 = 0$

$\therefore$ Co-ordinates of $D = \left(\frac{1}{3}, \frac{-8}{3}\right)$

So, the correct option is (C): $\left(\frac{1}{3}, -\frac{8}{3}\right)$

Concepts:

1. Straight lines:

A **straight line** is a line having the shortest distance between two points.

A straight line can be represented as an equation in various forms, as show in the image below:

Standard Form : $ax + by = c$

Slope-Intercept Form : $y = mx + c$

Point-Slope Form : $y - y_1 = m(x - x_1)$

The following are the many forms of the equation of the line that are presented in straight line-

1. Slope – Point Form

Assume P0(x0, y0) is a fixed point on a non-vertical line L with m as its slope. If P (x, y) is an arbitrary point on L, then the point (x, y) lies on the line with slope m through the fixed point (x0, y0) if and only if its coordinates fulfil the equation below.

$$y - y_0 = m (x - x_0)$$

2. Two – Point Form

Let's look at the line. L crosses between two places. $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ are general points on L, while P (x, y) is a general point on L. As a result, the three points P_1, P_2, and P are collinear, and it becomes

The slope of P_2P = The slope of P_1P_2 , i.e.

$$\frac{y-y_1}{x-x_1} = \frac{y_2-y_1}{x_2-x_1}$$

Hence, the equation becomes:

$$y - y_1 = \frac{y_2-y_1}{x_2-x_1}(x - x1)$$

3. Slope-Intercept Form

Assume that a line L with slope m intersects the y-axis at a distance c from the origin, and that the distance c is referred to as the line L's y-intercept. As a result, the coordinates of the spot on the y-axis where the line intersects are (0, c). As a result, the slope of the line L is m, and it passes through a fixed point (0, c). The equation of the line L thus obtained from the slope – point form is given by

$$y - c = m(x - 0)$$

As a result, the point (x, y) on the line with slope m and y-intercept c lies on the line, if and only if

$$y = m x + c$$

7. **Answer: c**

Explanation:

Explanation:
Given:The line passing through the points (5, 1, a) and (3, b, 1) crosses the yz- plane at the point 0, $\frac{17}{2}$, $\frac{-13}{2}$We have to find values of a, b.We know, Equation of Line in Symmetrical Form passing through points (x1, y1, z1) and (x, y, z) is given by $\frac{-\ \ 2}{1\ -\ \ 2} = \frac{-\ \ 2}{1\ -\ \ 2} = \frac{-\ \ 2}{1\ -\ \ 2}$ ...(i)Thus, the equation of line passing through (5, 1,) and (3, , 1) is$\frac{-3}{5-3} = \frac{-}{1-} = \frac{-1}{-1}$since the line passes through the point (0, $\frac{17}{2}$, $\frac{-13}{2}$). It satisfies equation (i), $\frac{0-3}{2} = \frac{\frac{17}{2}-}{1-} = \frac{\frac{-13}{2}-1}{-1}$ $\frac{-3}{2} = \frac{17-2}{2-2} = \frac{-15}{2\ \ -2}$...(ii)For $\frac{-3}{2} = \frac{17-2}{2-2}$ [using (ii)] $-6+6 = 34-4$ $10 = 40$ $b = \frac{40}{10} = 4$For $\frac{-3}{2} = \frac{-15}{2\ \ -2}$ [using (ii)] $-30 = -6\ \ +6$ $6 = 36$ $= \frac{36}{6} = 6$ $= 6,$ $= 4$Hence, the correct option is (C).

8. **Answer: a**

Explanation:

$$\begin{vmatrix} 1 & -1 & 1 \\ 2 & 2 & \alpha \\ 3 & -1 & 4 \end{vmatrix}$$

$= 0; 8 + \alpha - 2(-4 + 1) + 3(-\alpha - 2) = 0$

$\Rightarrow 8 + \alpha + 6 - 3\alpha - 6 = 0$

$\alpha = 4$

So, the correct Option is (A): $x^2 - 18x + 56 = 0$

Concepts:

1. Straight lines:

A **straight line** is a line having the shortest distance between two points.

A straight line can be represented as an equation in various forms, as show in the image below:

Standard Form : $ax + by = c$

Slope-Intercept Form : $y = mx + c$

Point-Slope Form : $y - y_1 = m(x - x_1)$

The following are the many forms of the equation of the line that are presented in straight line-

1. Slope – Point Form

Assume P0(x0, y0) is a fixed point on a non-vertical line L with m as its slope. If P (x, y) is an arbitrary point on L, then the point (x, y) lies on the line with slope m through the fixed point (x0, y0) if and only if its coordinates fulfil the equation below.

$$y - y_0 = m(x - x_0)$$

2. Two – Point Form

Let's look at the line. L crosses between two places. $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ are general points on L, while P (x, y) is a general point on L. As a result, the three points P_1, P_2, and P are collinear, and it becomes

The slope of P_2P = The slope of P_1P_2, i.e.

$$\frac{y-y_1}{x-x_1} = \frac{y_2-y_1}{x_2-x_1}$$

Hence, the equation becomes:

$$y - y_1 = \frac{y_2-y_1}{x_2-x_1}(x - x1)$$

3. Slope-Intercept Form

Assume that a line L with slope m intersects the y-axis at a distance c from the origin, and that the distance c is referred to as the line L's y-intercept. As a result, the coordinates of the spot on the y-axis where the line intersects are (0, c). As a result, the slope of the line L is m, and it passes through a fixed point (0, c). The equation of the line L thus obtained from the slope – point form is given by

y – c =m(x - 0)

As a result, the point (x, y) on the line with slope m and y-intercept c lies on the line, if and only if

y = m x +c

---

9. Answer: 16 - 16

Explanation:

The correct answer is 16.

$y^2 = 8x + 4y + 4$

$(y-2)^2 = 8(x+1)$

$y^2 = 4ax$

$a = 2, X = x+1, Y = y-2$

focus $(1,2)$

$y - 2 = m(x-1)$

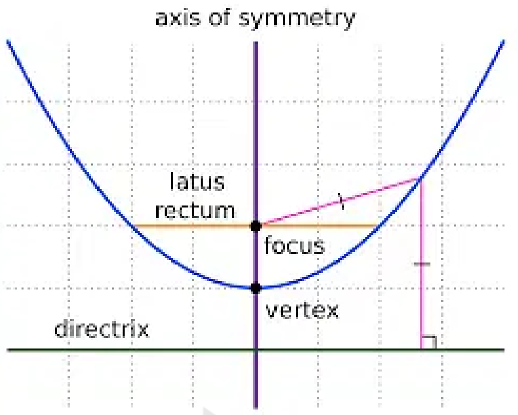
Put $(3,0)$ in the above line $m = -1$

Length of focal chord $= 16$

Concepts:

1. Parabola:

Parabola is defined as the locus of points equidistant from a fixed point (called focus) and a fixed-line (called directrix).



Parabola

Standard Equation of a Parabola

For horizontal parabola

- Let us consider
- Origin (0,0) as the parabola's vertex A,

1. Two equidistant points S(a,0) as focus, and Z(- a,0) as a directrix point,
2. P(x,y) as the moving point.

- Let us now draw SZ perpendicular from S to the directrix. Then, SZ will be the axis of the parabola.
- The centre point of SZ i.e. A will now lie on the locus of P, i.e. AS = AZ.
- The x-axis will be along the line AS, and the y-axis will be along the perpendicular to AS at A, as in the figure.
- By definition PM = PS

=> $MP^2 = PS^2$

- So, $(a + x)^2 = (x - a)^2 + y^2$.
- Hence, we can get the equation of horizontal parabola as $y^2 = 4ax$.

For vertical parabola

- Let us consider
- Origin (0,0) as the parabola's vertex A

1. Two equidistant points, S(0,b) as focus and Z(0, -b) as a directrix point
2. P(x,y) as any moving point

- Let us now draw a perpendicular SZ from S to the directrix.
- Then SZ will be the axis of the parabola. Now, the midpoint of SZ i.e. A, will lie on P's locus i.e. AS=AZ.
- The y-axis will be along the line AS, and the x-axis will be perpendicular to AS at A, as shown in the figure.
- By definition PM = PS

=> $MP^2 = PS^2$

So, $(b + y)^2 = (y - b)^2 + x^2$

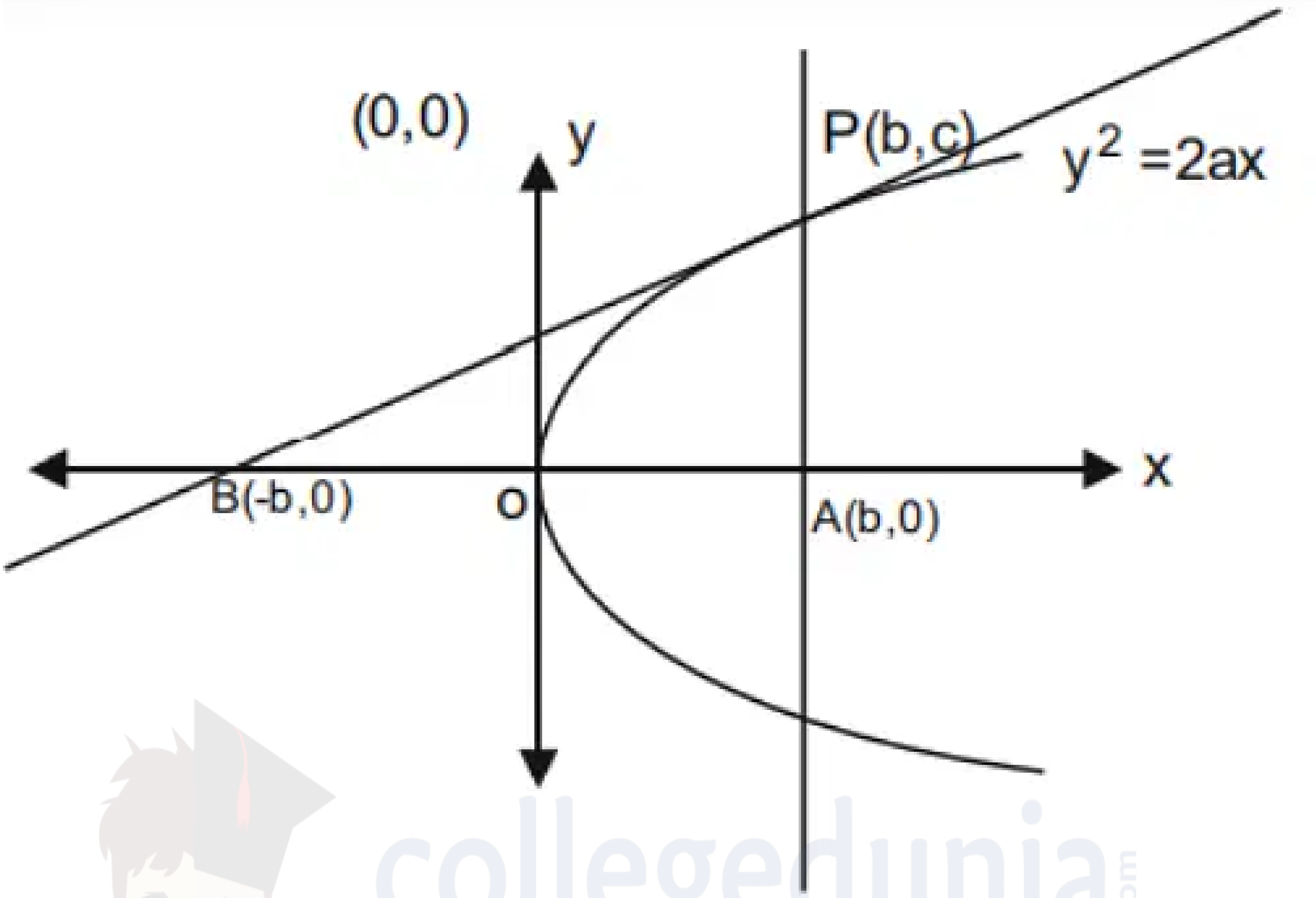
- As a result, the vertical parabola equation is $x^2 = 4by$.

---

10. **Answer: 146 - 146**

Explanation:

The correct answer is 146.



As $P(b, c)$ lies on parabola so $c^2 = 2ab$....(1)
Now equation of tangent to parabola $y^2 = 2ax$ in point
form is $yy_1 = 2a\frac{(x+x_1)}{2}, (x_1, y_1) = (b, c)$
$\Rightarrow yc = a(x + b)$
For point B, put $y = 0$, now $x = -b$
So, area of $\triangle PBA$, $\frac{1}{2} \times AB \times AP = 16$
$\Rightarrow \frac{1}{2} \times 2b \times c = 16$
$\Rightarrow bc = 16$
As b and c are natural number so possible values of (b, c) are $(1, 16), (2, 8), (4, 4), (8, 2)$ and $(16, 1)$
Now from equation (1) $a = \frac{c^2}{2b}$ and $a \in N$, so values of (b, c) are $(1, 16), (2, 8)$ and $(4, 4)$ now values of are $128, 16$ and 2 .
Hence sum of values of a is 146 .

Concepts:

1. Conjugate of a Complex Number:

A complex conjugate of a complex number is equivalent to the complex number whose real part is identical to the original complex number and the magnitude of the imaginary part is identical to the opposite sign.

A complex number is of the expression **a + ib**,

where,

a, b = real numbers, 'a' is named as the real part, 'b' is named as the imaginary part, and 'i' is an imaginary number equivalent to the root of negative 1.

The complex conjugate of a + ib with real part 'a' and imaginary part 'b' is stated by a - ib whose real part is 'a' and imaginary part is '-b'.

a - ib is the reflection of a + ib with reference to the real axis (X-axis) in the argand plane.