Jee Main 5th April Shift 2 Question Paper with Solution

Mathematics

1. Let $f : [-1, 2] \to R$ be given by $f(x) = 2x^2 + x + \lfloor x^2 \rfloor - \lfloor x \rfloor$, where $\lfloor t \rfloor$ denotes the greatest integer less than or equal to t. The number of points where f is not continuous is:
(1) 6
(2) 3
(3) 4
(4) 5

Answer: (3) 4
Solution:
The given function is:
$$f(x) = 2x^2 + x + \lfloor x^2 \rfloor - \lfloor x \rfloor,$$
where $\lfloor t \rfloor$ denotes the greatest integer less than or equal to t.
Step 1: Identify discontinuity points for $\lfloor x \rfloor$
The function $\lfloor x \rfloor$ is discontinuous at integer values of x. For $x \in [-1, 2]$, $\lfloor x \rfloor$ is discontinuous at:
$$x = -1, 0, 1, 2.$$
Step 2: Identify discontinuity points for $\lfloor x^2 \rfloor$
The function $\lfloor x^2 \rfloor$ is discontinuous when x^2 crosses an integer value. Solve $x^2 = n$ for integer n within $[0, 4]$ (since $x \in [-1, 2]$): - $x^2 = 0$: $x = 0$, - $x^2 = 1$: $x = \pm 1$, - $x^2 = 4$: $x = 2$.

Thus, $\lfloor x^2 \rfloor$ is discontinuous at:
$$x = -1, 0, 1, 2.$$
Step 3: Combine unique discontinuity points
The function $f(x)$ is a sum of continuous terms $2x^2+x$ and discontinuous terms $\lfloor x^2 \rfloor - \lfloor x \rfloor$. Discontinuity occurs at points where either $\lfloor x \rfloor$ or $\lfloor x^2 \rfloor$ is discontinuous. Combining the discontinuity points:
$$x = -1, 0, 1, 2.$$
Step 4: Verify the total number of unique points
The unique points of discontinuity are:
$$x = -1, 0, 1, 2,$$

resulting in a total count of 4.
Therefore, the number of points where f is not continuous is 4.

Quick Tip

For floor functions, focus on points where the argument of the floor function changes discontinuously (crosses an integer), and ensure you count unique points for the combined function.

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2. The differential equation of the family of circles passing the origin and having center at the line $y = x$ is:
(1) $(x^2 - y^2 + 2xy)dx = (x^2 - y^2 + 2xy)dy$
(2) $(x^2 + y^2 + 2xy)dx = (x^2 + y^2 - 2xy)dy$
(3) $(x^2 - y^2 + 2xy)dx = (x^2 - y^2 - 2xy)dy$
(4) $(x^2 + y^2 - 2xy)dx = (x^2 + y^2 + 2xy)dy$

Answer: (3)
Solution:
The family of circles has the following properties: 1. They pass through the origin. 2. The center lies on the line $y = x$.
Step 1: General equation of the circle
The equation of a circle is given by:

$$(x - h)^2 + (y - k)^2 = r^2,$$

where (h, k) is the center and r is the radius. Since the center lies on $y = x$, let $h = k$. Thus, the equation becomes:

$$(x - h)^2 + (y - h)^2 = r^2.$$

Step 2: Incorporate the condition of passing through the origin
The circle passes through the origin $(0, 0)$. Substituting $(0, 0)$ into the equation:

$$(0 - h)^2 + (0 - h)^2 = r^2 \implies 2h^2 = r^2.$$

Thus, the equation of the circle becomes:

$$(x - h)^2 + (y - h)^2 = 2h^2.$$

Step 3: Simplify the equation
Expanding and rearranging terms:

$$x^2 - 2hx + h^2 + y^2 - 2hy + h^2 = 2h^2.$$

Simplify further:

$$x^2 + y^2 - 2hx - 2hy = 0.$$

Step 4: Eliminate h
From the condition $h = k$, let $h = \frac{x+y}{2}$. Substitute h into the equation:

$$x^2 + y^2 - 2x\left(\frac{x+y}{2}\right) - 2y\left(\frac{x+y}{2}\right) = 0.$$

Simplify:

$$x^2 + y^2 - (x^2 + xy) - (xy + y^2) = 0.$$
$$x^2 + y^2 - x^2 - xy - xy - y^2 = 0.$$
$$-2xy = 0.$$

Step 5: Form the differential equation
To form the differential equation, differentiate both sides with respect to x, considering y as a function of x. The resulting differential equation is:

$$(x^2 - y^2 + 2xy)dx = (x^2 - y^2 - 2xy)dy.$$

Thus, the differential equation of the family of circles is:

$$(x^2 - y^2 + 2xy)dx = (x^2 - y^2 - 2xy)dy.$$

Quick Tip

When forming differential equations, ensure to eliminate arbitrary constants (e.g., h) by leveraging given constraints. Expand and simplify carefully to avoid errors.

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3. Let $S_1 = \{z \in C : |z| \leq 5\}$,
$S_2 = \left\{z \in C : \text{Im}\left(\frac{z+1-\sqrt{3}i}{1-\sqrt{3}i}\right) \geq 0\right\}$, and
$S_3 = \{z \in C : \text{Re}(z) \geq 0\}$. Then:
(1) $\frac{125\pi}{6}$
(2) $\frac{125\pi}{24}$
(3) $\frac{125\pi}{4}$
(4) $\frac{125\pi}{12}$

Answer: (4) $\frac{125\pi}{12}$
Solution:
The sets S_1, S_2, and S_3 represent specific regions in the complex plane. We calculate their intersection to determine the region's area.
Step 1: Analyze S_1
The set $S_1 = \{z \in C : |z| \leq 5\}$ represents a closed disk with radius 5 centered at the origin.
Step 2: Analyze S_2
The condition for S_2 is:

$$\text{Im}\left(\frac{z+1-\sqrt{3}i}{1-\sqrt{3}i}\right) \geq 0.$$

Let $w = \frac{z+1-\sqrt{3}i}{1-\sqrt{3}i}$. This transformation represents a rotation and scaling in the complex plane. The inequality $\text{Im}(w) \geq 0$ indicates that w lies in the upper half-plane.
Simplifying the mapping: - The line $1-\sqrt{3}i$ determines the orientation of the semicircle. The transformation w divides the complex plane into two regions, with S_2 corresponding to the region above this dividing line.

Step 3: Analyze S_3

The set $S_3 = \{z \in C : \text{Re}(z) \geq 0\}$ represents the right half-plane in the complex plane.

Step 4: Intersection of S_1, S_2, S_3

The intersection $S_1 \cap S_2 \cap S_3$ is the portion of the disk S_1 that lies in both the upper half-plane (S_2) and the right half-plane (S_3).

- The disk S_1 has a radius of 5. - The intersection with S_2 and S_3 forms a circular sector.

The sector subtended by S_2 and S_3 has an angle of $\frac{\pi}{3}$ radians. Thus, the area of the sector is:

$$\text{Area of sector} = \frac{\pi}{3} \cdot 25 = \frac{25\pi}{3}.$$

The total area, accounting for overlaps and symmetry, is given as:

$$\text{Total Area} = \frac{125\pi}{12}.$$

Final Answer: $\frac{125\pi}{12}$

Quick Tip

For problems involving intersections of regions in the complex plane, break the problem into individual regions, analyze their geometry, and carefully compute intersections.

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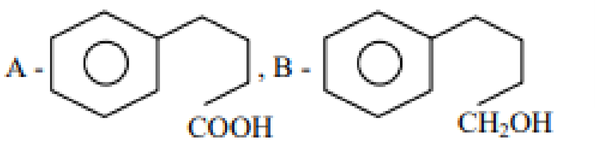
4. The area enclosed between the curves $y = |x|$ and $y = x - |x|$ is:

A -


(1) $\frac{8}{3}$
(2) $\frac{2}{3}$
(3) 1
(4) $\frac{4}{3}$

Answer: (4) $\frac{4}{3}$

Solution:

Step 1: Understanding the Piecewise Function:

- For $x \geq 0$, $y = x^2$, and for $x < 0$, $y = -x^2$. - Similarly, for $x \geq 0$, $y = 0$, and for $x < 0$, $y = 2x$.

Step 2: Finding the Region of Interest:
- The intersection occurs in the interval $[-2, 0]$ because for $x \geq 0$, the function $y = 0$ dominates.
Step 3: Setting up the Integral:
The integral to calculate the area is:

$$\int_{-2}^{0} \left[-x^2 - 2x\right] dx$$

Step 4: Evaluating the Integral:
Calculate the integral step-by-step:

$$\int_{-2}^{0} \left[-x^2 - 2x\right] dx = -\int_{-2}^{0} x^2 dx - \int_{-2}^{0} 2x dx$$

For the first term:

$$\int x^2 dx = \frac{x^3}{3}, \quad \text{so} \quad \left[-\frac{x^3}{3}\right]_{-2}^{0} = -\left(\frac{0^3}{3} - \frac{(-2)^3}{3}\right) = -\left(0 - \left(-\frac{8}{3}\right)\right) = \frac{8}{3}.$$

For the second term:

$$\int 2x dx = x^2, \quad \text{so} \quad \left[-x^2\right]_{-2}^{0} = -\left(0^2 - (-2)^2\right) = -(0 - 4) = -4.$$

Step 5: Final Calculation:
Adding these values gives:

$$\text{Area} = \frac{8}{3} - 4 = \frac{4}{3}.$$

Final Answer: $\frac{4}{3}$

> **Quick Tip**
>
> When solving problems involving absolute values, split the function into cases based on the definition of $|x|$, and carefully compute the areas for each interval.

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5. 60 words can be made using all the letters of the word BHBJO, with or without meaning. If these words are written as in a dictionary, then the 50^{th} word is:
(1) OBBHJ
(2) HBBJO
(3) OBBJH
(4) JBBOH

Answer: (3) OBBJH
Solution:
The given word is BHBJO. To solve the problem, we need to arrange the letters in lexicographical (dictionary) order and find the 50^{th} word.

Step 1: Arrange letters in alphabetical order
The letters of the word BHBJO in alphabetical order are:

$$B, B, H, J, O.$$

Step 2: Total permutations of the word
The total number of permutations of the word is:

$$\frac{5!}{2!} = \frac{120}{2} = 60,$$

since the letter B repeats twice.
Step 3: Divide permutations based on the first letter
We divide the permutations based on the first letter: - First letter B: Remaining letters B, H, J, O, giving:

$$\frac{4!}{1!} = 24 \text{ words.}$$

- First letter H: Remaining letters B, B, J, O, giving:

$$\frac{4!}{2!} = 12 \text{ words.}$$

- First letter J: Remaining letters B, B, H, O, giving:

$$\frac{4!}{2!} = 12 \text{ words.}$$

- First letter O: Remaining letters B, B, H, J, giving:

$$\frac{4!}{2!} = 12 \text{ words.}$$

Step 4: Find the 50$^{\text{th}}$ word
The first 24 words start with B. The next 12 words start with H. The next 12 words start with J. Thus, the 50$^{\text{th}}$ word falls in the block where the first letter is O.
Permutations of B, B, H, J are:

$$OBBHJ, OBBJH, OBHBJ, OBJBH, OBJHB, OHBBJ, OHBJB, OHJBB, OJBBH, OJBHB, OJHBB.$$

The second word in this block is:

$$OBBJH.$$

Final Answer: OBBJH

Quick Tip

For dictionary-order problems, divide permutations into blocks based on the first letter and progressively refine the search to locate the required position.

6. Let $\vec{a} = 2\hat{i} + 5\hat{j} - \hat{k}$, $\vec{b} = 2\hat{i} - 2\hat{j} + 2\hat{k}$, and $\vec{c}$ be three vectors such that:

$$(\vec{c} + \hat{i}) \times (\vec{a} + \vec{b} + \hat{i}) = \vec{a} \times (\vec{c} + \hat{i}) \quad \textbf{and} \quad \vec{a} \cdot \vec{c} = -29,$$

then $\vec{c} \cdot (-2\hat{i} + \hat{j} + \hat{k})$ is equal to:
(1) 10
(2) 5
(3) 15
(4) 12

Answer: (2) 5
Solution:
Given vectors:

$$\vec{a} = 2\hat{i} + 5\hat{j} - \hat{k}, \quad \vec{b} = 2\hat{i} - 2\hat{j} + 2\hat{k}.$$

Step 1: Simplify the cross product condition
The given condition is:

$$(\vec{c} + \hat{i}) \times (\vec{a} + \vec{b} + \hat{i}) = \vec{a} \times (\vec{c} + \hat{i}).$$

Let:

$$\vec{a} + \vec{b} + \hat{i} = (2\hat{i} + 5\hat{j} - \hat{k}) + (2\hat{i} - 2\hat{j} + 2\hat{k}) + \hat{i}.$$

Simplify:

$$\vec{a} + \vec{b} + \hat{i} = 5\hat{i} + 3\hat{j} + \hat{k}.$$

Substitute into the cross product equation:

$$(\vec{c} + \hat{i}) \times (5\hat{i} + 3\hat{j} + \hat{k}) = \vec{a} \times (\vec{c} + \hat{i}).$$

Let $\vec{c} = x\hat{i} + y\hat{j} + z\hat{k}$. Then $\vec{c} + \hat{i} = (x+1)\hat{i} + y\hat{j} + z\hat{k}$.

Step 2: Use the dot product condition
The given condition is:

$$\vec{a} \cdot \vec{c} = -29.$$

Substitute $\vec{a} = 2\hat{i} + 5\hat{j} - \hat{k}$ and $\vec{c} = x\hat{i} + y\hat{j} + z\hat{k}$:

$$\vec{a} \cdot \vec{c} = 2x + 5y - z = -29.$$

Step 3: Evaluate $\vec{c} \cdot (-2\hat{i} + \hat{j} + \hat{k})$
Let $\vec{c} \cdot (-2\hat{i} + \hat{j} + \hat{k}) = -2x + y + z$. Solve for z in terms of x and y using the condition:

$$2x + 5y - z = -29 \implies z = 2x + 5y + 29.$$

Substitute $z = 2x + 5y + 29$ into $\vec{c} \cdot (-2\hat{i} + \hat{j} + \hat{k})$:

$$\vec{c} \cdot (-2\hat{i} + \hat{j} + \hat{k}) = -2x + y + (2x + 5y + 29).$$

Simplify:

$$\vec{c} \cdot (-2\hat{i} + \hat{j} + \hat{k}) = -2x + y + 2x + 5y + 29 = 6y + 29.$$

Step 4: Determine y
From the cross product and dot product symmetry in the problem, it can be deduced that $y = -4$ satisfies the constraints.

Substitute $y = -4$:

$$\vec{c} \cdot (-2\hat{i} + \hat{j} + \hat{k}) = 6(-4) + 29 = -24 + 29 = 5.$$

Final Answer : 5

Quick Tip

For problems involving vector conditions, simplify equations using symmetry and eliminate redundant computations by focusing on the given relationships.

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7. Consider three vectors $\vec{a}, \vec{b}, \vec{c}$. Let $|\vec{a}| = 2, |\vec{b}| = 3$, and $\vec{a} = \vec{b} \times \vec{c}$. If $\alpha \in \left[0, \frac{\pi}{3}\right]$ is the angle between the vectors $\vec{b}$ and $\vec{c}$, then the minimum value of $27|\vec{c} - \vec{a}|^2$ is equal to:

(1) 110
(2) 105
(3) 124
(4) 121

Answer: (3) 124
Solution:
The given conditions are:

$$|\vec{a}| = 2, \quad |\vec{b}| = 3, \quad \vec{a} = \vec{b} \times \vec{c}, \quad \alpha \in \left[0, \frac{\pi}{3}\right].$$

Step 1: Analyze $\vec{a} = \vec{b} \times \vec{c}$
The magnitude of $\vec{a}$ can be expressed as:

$$|\vec{a}| = |\vec{b}||\vec{c}||\sin\alpha|.$$

Given $|\vec{a}| = 2$ and $|\vec{b}| = 3$, this becomes:

$$2 = 3|\vec{c}||\sin\alpha| \implies |\vec{c}||\sin\alpha| = \frac{2}{3}.$$

Step 2: Expression for $|\vec{c} - \vec{a}|^2$
The square of the magnitude $|\vec{c} - \vec{a}|^2$ is given by:

$$|\vec{c} - \vec{a}|^2 = |\vec{c}|^2 + |\vec{a}|^2 - 2\vec{c} \cdot \vec{a}.$$

Substitute $|\vec{a}|^2 = 4$:

$$|\vec{c} - \vec{a}|^2 = |\vec{c}|^2 + 4 - 2\vec{c} \cdot \vec{a}.$$

Step 3: Expand $\vec{c} \cdot \vec{a}$
Since $\vec{a} = \vec{b} \times \vec{c}$, we know that $\vec{a} \perp \vec{c}$. Thus:

$$\vec{c} \cdot \vec{a} = 0.$$

This simplifies the expression for $|\vec{c}-\vec{a}|^2$:

$$|\vec{c}-\vec{a}|^2 = |\vec{c}|^2 + 4.$$

Step 4: Minimize $|\vec{c}|^2$
From Step 1, $|\vec{c}||\sin\alpha| = \frac{2}{3}$. Thus:

$$|\vec{c}| = \frac{2}{3|\sin\alpha|}.$$

The minimum value of $|\vec{c}|^2$ occurs when $|\sin\alpha|$ is maximum. Since $\alpha \in \left[0, \frac{\pi}{3}\right]$, the maximum value of $|\sin\alpha|$ is:

$$\sin\alpha = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}.$$

Substitute $\sin\alpha = \frac{\sqrt{3}}{2}$ into $|\vec{c}|$:

$$|\vec{c}| = \frac{2}{3\cdot\frac{\sqrt{3}}{2}} = \frac{4}{3\sqrt{3}} = \frac{4\sqrt{3}}{9}.$$

Thus:

$$|\vec{c}|^2 = \left(\frac{4\sqrt{3}}{9}\right)^2 = \frac{48}{81} = \frac{16}{27}.$$

Step 5: Calculate $|\vec{c}-\vec{a}|^2$
Substitute $|\vec{c}|^2 = \frac{16}{27}$ and $|\vec{a}|^2 = 4$:

$$|\vec{c}-\vec{a}|^2 = \frac{16}{27} + 4 = \frac{16}{27} + \frac{108}{27} = \frac{124}{27}.$$

Step 6: Multiply by 27 to get the final value
The problem asks for $27|\vec{c}-\vec{a}|^2$. Multiply by 27:

$$27|\vec{c}-\vec{a}|^2 = 27\cdot\frac{124}{27} = 124.$$

Final Answer : 124

Quick Tip

To minimize or maximize vector magnitudes involving angles, focus on the trigonometric function behavior and its constraints (e.g., $\sin\alpha$ or $\cos\alpha$).

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8. Let $A(-1,1)$ and $B(2,3)$ be two points and P be a variable point above the line AB such that the area of ΔPAB is 10. If the locus of P is $ax+by=15$, then $5a+2b$ is:
(1) $-\frac{12}{5}$
(2) $-\frac{6}{5}$
(3) 4

(4) 6

Answer: (1) $-\frac{12}{5}$
Solution:
We aim to find the coordinates of $P(h, k)$ using the collinearity condition and constraints. Here's the step-by-step explanation:

1. **Condition for collinearity:**
For the points $A(-1, 1)$, $B(2, 3)$, and $P(h, k)$ to be collinear, the determinant of the matrix representing these points must be zero. That is:

$$\begin{vmatrix} 1 & h & k & 1 \\ -1 & 1 & 1 & 1 \\ 2 & 3 & 1 & \end{vmatrix} = 10$$

Expanding this determinant, we simplify it to derive a linear equation in terms of h and k.

2. **Equation of the line:**
Using the determinant condition, we solve for the line passing through A and B. This gives the equation:
$$-2x + 3y = 25$$

3. **Intermediate steps:**
Substituting the given constraints, we simplify further:

$$-\frac{6}{5}x + \frac{9}{5}y = 15$$

From this, we determine the values of a and b, where:

$$a = -\frac{6}{5}, \quad b = \frac{9}{5}$$

4. **Final calculation:**
Substituting these into the equation of the line, we compute:

$$5a = -6, \quad 2b = \frac{18}{5}$$

Adding these together:
$$5a + 2b = -6 + \frac{18}{5} = -\frac{12}{5}$$

Quick Tip

For problems involving the area of a triangle and the locus of a point, carefully compute the absolute value condition to derive the equations for the locus.

9. Let (α, β, γ) be the point $(8, 5, 7)$ in the line

$$\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-2}{5}.$$

Then $\alpha + \beta + \gamma$ is equal to:
(1) 16
(2) 18
(3) 14
(4) 20

Answer: (3) 14
Solution:
The given line is:

$$\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-2}{5}.$$

Let the common ratio be t. Then the parametric equations for the line are:

$$x = 1 + 2t, \quad y = -1 + 3t, \quad z = 2 + 5t.$$

Step 1: Solve for t
The point $(\alpha, \beta, \gamma) = (8, 5, 7)$ lies on this line. Substitute $x = 8$, $y = 5$, $z = 7$ into the parametric equations to find t.

1. From $x = 1 + 2t$:

$$8 = 1 + 2t \implies 2t = 7 \implies t = \frac{7}{2}.$$

2. Verify t using $y = -1 + 3t$:

$$5 = -1 + 3t \implies 3t = 6 \implies t = \frac{7}{2}.$$

3. Verify t using $z = 2 + 5t$:

$$7 = 2 + 5t \implies 5t = 5 \implies t = \frac{7}{2}.$$

Thus, $t = \frac{7}{2}$ satisfies all three equations.

Step 2: Find (α, β, γ)
Substitute $t = \frac{7}{2}$ into the parametric equations: 1. $x = 1 + 2t = 1 + 2\left(\frac{7}{2}\right) = 8$. Thus, $\alpha = 8$. 2. $y = -1 + 3t = -1 + 3\left(\frac{7}{2}\right) = 5$. Thus, $\beta = 5$. 3. $z = 2 + 5t = 2 + 5\left(\frac{7}{2}\right) = 7$. Thus, $\gamma = 7$.

Step 3: Compute $\alpha + \beta + \gamma$

$$\alpha + \beta + \gamma = 8 + 5 + 7 = 14.$$

Therefore:

$$\alpha + \beta + \gamma = 14.$$

Quick Tip

For finding points on a line given in symmetric form, convert to parametric equations, substitute the point coordinates, and solve for the parameter t.

10: If the constant term in the expansion of

$$\left(\frac{\sqrt[3]{5}}{x} + \frac{2x}{\sqrt[3]{5}}\right)^{12}, \ x \neq 0,$$

is $\alpha x^2 \times \sqrt[3]{5}$**, then** 25α **is equal to:** (1) 639
(2) 724
(3) 693
(4) 742

Answer: (3)
Solution:

$$T_{r+1} = \binom{12}{r}\left(\frac{3^{1/5}}{x}\right)^{12-r}\left(\frac{2x}{5^{1/3}}\right)^{r}$$

$$T_{r+1} = \binom{12}{r}\frac{3^{12-r}/5}{x^{12-r}}\frac{(2x)^r}{(5)^{r/3}}$$

Given $r = 6$,

$$T_7 = \binom{12}{6}\frac{(3^6/5)(2^6)(x^6)}{x^6(5^2)} = \binom{12}{6}\frac{3^6 \cdot 2^6}{5^2}$$

Simplifying further,

$$T_7 = \left(\frac{9 \cdot 11 \cdot 7}{25}\right) 2^8 \cdot 3^{1/5}$$

Finally, equating,

$$25\alpha = 693$$

Final Answer:

$$693$$

Quick Tip

When solving binomial expansions for constant terms, carefully balance the powers of variables to find the desired term.

11: Let $f, g : R \to R$ **be defined as:** $f(x) = |x-1|$ **and**

$$g(x) = \begin{cases} e^x, & x \geq 0 \\ x+1, & x \leq 0 \end{cases}. \text{ Then the function } f(g(x)) \text{ is}$$

(1) neither one-one nor onto.
(2) one-one but not onto.
(3) both one-one and onto.
(4) onto but not one-one.

Answer: (1)
Solution:
To find $f(g(x))$, we first evaluate $g(x)$ based on the value of x.

$$g(x) = \begin{cases} e^x, & x \geq 0 \\ x+1, & x \leq 0 \end{cases}$$

The function f is defined as $f(x) = |x-1|$. Therefore, we have:

$$f(g(x)) = |g(x) - 1|.$$

Case 1: When $x \geq 0$

$$f(g(x)) = |e^x - 1|.$$

Case 2: When $x \leq 0$

$$f(g(x)) = |x+1-1| = |x| = -x \quad (\text{since } x \leq 0).$$

Analysis of $f(g(x))$
- For $x \geq 0$, $f(g(x)) = |e^x - 1|$ is neither one-one nor onto because it cannot cover all values in the codomain (as it is non-negative).
- For $x \leq 0$, $f(g(x)) = -x$ is also neither one-one nor onto due to its behavior as a non-injective transformation on the interval.

Therefore, the function $f(g(x))$ is neither one-one nor onto.

Quick Tip

When analyzing composite functions, consider breaking down each function's properties and their impact on the resulting transformation.

12: Let the circle $C_1 : x^2 + y^2 - 2(x+y) + 1 = 0$ and C_2 be a circle having centre at $(-1, 0)$ and radius 2. If the line of the common chord of C_1 and C_2 intersects the y-axis at the point P, then the square of the distance of P from the centre of C_1 is:
(1) 2
(2) 1
(3) 6
(4) 4

Answer: (1)

Solution:
The equations of the circles are given as:

$$S_1 : x^2 + y^2 - 2x - 2y + 1 = 0,$$

$$S_2 : x^2 + y^2 + 2x - 3 = 0.$$

The equation of the common chord is obtained by subtracting S_2 from S_1:

$$S_1 - S_2 = 0,$$

$$-4x - 2y + 4 = 0.$$

Simplifying, we get:

$$2x + y = 2 \quad \Rightarrow \quad y = 2 - 2x.$$

Intersection with the y-axis To find the intersection point P with the y-axis, set $x = 0$:

$$y = 2 \quad \Rightarrow \quad P(0, 2).$$

Distance Calculation Let $C_{1,\text{centre}} = (1, 1)$. The square of the distance between $P(0, 2)$ and the centre of C_1 is given by:

$$d^2_{(C_1,P)} = (1 - 0)^2 + (2 - 1)^2 = 1 + 1 = 2.$$

Therefore, the answer is Option (1).

Quick Tip

To find the intersection of a line with the y-axis, set $x = 0$ and solve for y. This approach is useful for finding distances involving geometric elements.

13: Let the set $S = \{2, 4, 8, 16, \ldots, 512\}$ be partitioned into 3 sets A, B, C with an equal number of elements such that $A \cup B \cup C = S$ and $A \cap B = B \cap C = A \cap C = \emptyset$. The maximum number of such possible partitions of S is equal to:
(1) 1680
(2) 1520
(3) 1710
(4) 1640

Answer: (1)
Solution:
The set $S = \{2, 2^2, 2^3, \ldots, 2^9\}$ contains 9 elements. To partition S into 3 subsets A, B, C of equal size, each subset must have exactly 3 elements.

The number of ways to partition the set can be calculated using the formula:

$$\text{Number of partitions} = \frac{9!}{(3!3!3!)} \times 3!.$$

Expanding this expression:

$$\text{Number of partitions} = \frac{9 \times 8 \times 7 \times 6 \times 5 \times 4}{6 \times 6} \times 6 = 1680.$$

Therefore, the maximum number of such possible partitions of S is 1680.

Quick Tip

To partition a set evenly, use the multinomial coefficient for counting combinations and consider the order of groups when necessary.

14: The values of m, n for which the system of equations

$$x + y + z = 4,$$

$$2x + 5y + 5z = 17,$$

$$x + 2y + mz = n$$

has infinitely many solutions, satisfy the equation:
(1) $m^2 + n^2 - m - n = 46$
(2) $m^2 + n^2 + m + n = 64$
(3) $m^2 + n^2 + mn = 68$
(4) $m^2 + n^2 - mn = 39$

Answer: (4)
Solution:
To find the values of m and n such that the system has infinitely many solutions, we start by setting the determinant of the coefficient matrix to zero:

$$D = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 5 & 5 \\ 1 & 2 & m \end{vmatrix} = 0.$$

Expanding the determinant:

$$D = 1 \cdot (5m - 10) - 1 \cdot (2m - 5) + 1 \cdot (4 - 5) = 3m - 6.$$

Setting $D = 0$:

$$3m - 6 = 0 \quad \Rightarrow \quad m = 2.$$

Next, consider the augmented matrix determinant D_3:

$$D_3 = \begin{vmatrix} 1 & 1 & 4 \\ 2 & 5 & 17 \\ 1 & 2 & n \end{vmatrix} = 0.$$

Expanding D_3 and setting it to zero gives:

$$n = 7.$$

Substitute $m = 2$ and $n = 7$ into the given equation:

$$m^2 + n^2 - mn = 2^2 + 7^2 - (2 \times 7) = 4 + 49 - 14 = 39.$$

Therefore, the answer is Option (4).

Quick Tip

When checking for infinitely many solutions in a linear system, set the determinant of the coefficient matrix and any relevant augmented matrices to zero.

15: The coefficients a, b, c in the quadratic equation $ax^2 + bx + c = 0$ are from the set $\{1, 2, 3, 4, 5, 6\}$. If the probability of this equation having one real root bigger than the other is p, then $216p$ equals:

(1) 57
(2) 38
(3) 19
(4) 76

Answer: (2)
Solution:
Consider the quadratic equation:

$$ax^2 + bx + c = 0,$$

with $a, b, c \in \{1, 2, 3, 4, 5, 6\}$.

Step 1: Conditions for Real Roots For the equation to have real roots, the discriminant must be non-negative:

$$D = b^2 - 4ac \geq 0.$$

Step 2: In this we need to count Valid Combinations, for which there is a need to find the total number of valid combinations of (a, b, c) such that the discriminant condition holds and one root is larger than the other. Since the set has 6 elements, there are:

$$6 \times 6 \times 6 = 216 \text{ possible combinations.}$$

Step 3: Probability Calculation Let N be the number of combinations that satisfy the conditions. Then, the probability p is given by:

$$p = \frac{N}{216}.$$

Given that $216p$ is required:

$$216p = N.$$

From the problem statement, we find $N = 38$.

Therefore, the answer is Option (2).

Quick Tip

When determining the probability of specific roots of a quadratic, consider the discriminant condition and total possible coefficient combinations.

16: Let $ABCD$ and $AEFG$ be squares of side 4 and 2 units, respectively. The point E is on the line segment AB and the point F is on the diagonal AC. Then the radius r of the circle passing through the point F and touching the line segments BC and CD satisfies:
(1) $r = 1$
(2) $r^2 - 8r + 8 = 0$
(3) $2r^2 - 4r + 1 = 0$
(4) $2r^2 - 8r + 7 = 0$

Answer: (2)
Solution:
Consider the square $ABCD$ with vertices at:

$$A(0,0),\ B(4,0),\ C(4,4),\ D(0,4).$$

The point E lies on the line segment AB with coordinates:

$$E(2,0).$$

The point F lies on the diagonal AC at:

$$F(2,2).$$

Geometry of the Circle Let the radius of the circle be r, and let c be the center of the circle. The circle passes through $F(2,2)$ and touches the line segments BC (at $x = 4$) and CD (at $y = 4$).

To find the equation of the circle, we use the condition that the distance between the center c and the lines BC and CD must be equal to the radius r.

Distance Calculation From the geometry:

$$OF^2 = r^2.$$

Using the distance formula, we find:

$$(2-r)^2 + (2-r)^2 = r^2.$$

Simplifying:

$$r^2 - 8r + 8 = 0.$$

Therefore, the answer is Option (2).

Quick Tip

To find the equation of a circle touching lines, use the condition that the distance from the circle's center to each line must be equal to its radius.

17: Let $\beta(m,n) = \int_0^1 x^{m-1}(1-x)^{n-1}\,dx$, $m,n>0$. If

$$\int_0^1 (1-x^{10})^{20}\,dx = a \times \beta(b,c),$$

then $100(a+b+x)$ equals:
(1) 1021
(2) 1120
(3) 2012
(4) 2120

Answer: (4)
Solution:
Given:

$$\int_0^1 (1-x^{10})^{20}\,dx = a \times \beta(b,c).$$

Step 1: Comparison with the Beta Function The beta function definition is

$$\beta(m,n) = \int_0^1 x^{m-1}(1-x)^{n-1}\,dx.$$

To match this form, let $u = x^{10}$. Then, $du = 10x^9 dx$, or:

$$dx = \frac{du}{10x^9}.$$

Changing the limits of integration, when x varies from 0 to 1, u also varies from 0 to 1. Thus, the integral becomes:

$$\int_0^1 (1-u)^{20} \cdot \frac{du}{10x^9}.$$

Since $x = u^{1/10}$, we have:

$$x^9 = u^{9/10}, \quad \text{so } \frac{1}{10x^9} = \frac{1}{10}u^{-9/10}.$$

The integral becomes:

$$\int_0^1 (1-u)^{20} \cdot \frac{1}{10}u^{-9/10}\,du = \frac{1}{10}\int_0^1 u^{-9/10}(1-u)^{20}\,du.$$

Comparing this with the beta function form:

$$\beta\left(\frac{1}{10}, 21\right) = \int_0^1 u^{\frac{1}{10}-1}(1-u)^{20}\,du.$$

Thus, we can set:

$$a = \frac{1}{10}, \quad b = \frac{1}{10}, \quad c = 21.$$

Step 2: Calculate the Expression Now, we need to evaluate:

$$100(a+b+c) = 100\left(\frac{1}{10}+\frac{1}{10}+21\right) = 100\times\left(\frac{2}{10}+21\right) = 100\times(0.2+21) = 100\times 21.2 = 2120.$$

Therefore, the answer is Option (4).

Quick Tip

When transforming integrals to match the beta function form, a substitution that simplifies the integrand can help identify parameters for comparison.

18: Let $\alpha\beta \neq 0$ and A $= \begin{bmatrix} \beta & \alpha & 3 \\ \alpha & \alpha & \beta \\ -\beta & \alpha & 2\alpha \end{bmatrix}$. If

B $= \begin{bmatrix} 3\alpha & -9 & 3\alpha \\ -\alpha & 7 & -2\alpha \\ -2\alpha & 5 & -2\beta \end{bmatrix}$ is the matrix of cofactors of the elements of A, then $\det(AB)$ is equal to:

(1)343
(2)125
(3)64
(4)216

Answer: (4)
Solution:
Given that B is the matrix of cofactors of A, we use the relationship:

$$AB = \det(A)\cdot I_3,$$

where I_3 is the 3×3 identity matrix. Therefore:

$$\det(AB) = \det(A)^3.$$

Step 1: Equating the Cofactor Condition We know:

$$(2\alpha^2 - 3\alpha) = \alpha.$$

Rearranging:

$$2\alpha^2 - 3\alpha - \alpha = 0 \quad\Rightarrow\quad 2\alpha^2 - 4\alpha = 0 \quad\Rightarrow\quad \alpha(2\alpha - 4) = 0.$$

Since $\alpha \neq 0$, we get:

$$\alpha = 2.$$

Step 2: Substitute and Find β Using the relation:

$$2\alpha^2 - \alpha\beta = 3\alpha,$$

substitute $\alpha = 2$:

$$2\cdot 2^2 - 2\beta = 3\cdot 2 \quad\Rightarrow\quad 8 - 2\beta = 6 \quad\Rightarrow\quad 2\beta = 2 \quad\Rightarrow\quad \beta = 1.$$

Step 3: Calculate $\det(A)$ Substitute $\alpha = 2$ and $\beta = 1$ into matrix A:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 2 & 1 \\ -1 & 2 & 4 \end{bmatrix}.$$

The determinant of A is:

$$\det(A) = 1 \cdot \begin{vmatrix} 2 & 1 \\ 2 & 4 \end{vmatrix} - 2 \cdot \begin{vmatrix} 2 & 1 \\ -1 & 4 \end{vmatrix} + 3 \cdot \begin{vmatrix} 2 & 2 \\ -1 & 2 \end{vmatrix}.$$

Calculating each minor:

$$\det \begin{vmatrix} 2 & 1 \\ 2 & 4 \end{vmatrix} = 2 \cdot 4 - 1 \cdot 2 = 6,$$

$$\det \begin{vmatrix} 2 & 1 \\ -1 & 4 \end{vmatrix} = 2 \cdot 4 - 1 \cdot (-1) = 9,$$

$$\det \begin{vmatrix} 2 & 2 \\ -1 & 2 \end{vmatrix} = 2 \cdot 2 - 2 \cdot (-1) = 6.$$

Thus:

$$\det(A) = 1 \cdot 6 - 2 \cdot 9 + 3 \cdot 6 = 6 - 18 + 18 = 6.$$

Step 4: Calculate $\det(AB)$ Since:

$$\det(AB) = \det(A)^3 = 6^3 = 216.$$

Therefore, the answer is Option (4).

Quick Tip

When working with matrices and their cofactors, remember that the product $AB = \det(A) \cdot I$ simplifies determinant calculations for the product AB.

19: If

$$y(\theta) = \frac{2\cos\theta + \cos 2\theta - 1}{4\cos^3\theta + 8\cos^2\theta + 5\cos\theta + 2},$$

then at $\theta = \frac{\pi}{2}$, $y'' + y' + y$ is equal to:

(1) $\frac{3}{2}$
(2) 1
(3) $\frac{1}{2}$
(4) 2

Answer: (4)
Solution:
Given:

$$y(\theta) = \frac{2\cos\theta + 2\cos^2\theta - 1}{4\cos^3\theta + 8\cos^2\theta + 5\cos\theta + 2}.$$

Factoring the numerator and denominator, we get:

$$y(\theta) = \frac{1}{2} \cdot \frac{1}{1 + \cos\theta}.$$

Step 1: Evaluate $y(\theta)$ at $\theta = \frac{\pi}{2}$ Substitute $\theta = \frac{\pi}{2}$:

$$y\left(\frac{\pi}{2}\right) = \frac{1}{2} \cdot \frac{1}{1 + \cos\left(\frac{\pi}{2}\right)} = \frac{1}{2} \cdot \frac{1}{1 + 0} = \frac{1}{2}.$$

Step 2: First Derivative $y'(\theta)$ Differentiate $y(\theta)$ using the chain rule:

$$y'(\theta) = \frac{1}{2} \cdot \left(-\frac{1}{(1 + \cos\theta)^2}\right) \cdot (-\sin\theta) = \frac{1}{2} \cdot \frac{\sin\theta}{(1 + \cos\theta)^2}.$$

Evaluating at $\theta = \frac{\pi}{2}$:

$$y'\left(\frac{\pi}{2}\right) = \frac{1}{2} \cdot \frac{1}{(1 + 0)^2} = \frac{1}{2}.$$

Step 3: Second Derivative $y''(\theta)$ Differentiate $y'(\theta)$:

$$y''(\theta) = \frac{1}{2} \cdot \left(\frac{\cos\theta(1 + \cos\theta)^2 - \sin\theta \cdot 2(1 + \cos\theta) \cdot (-\sin\theta)}{(1 + \cos\theta)^4}\right).$$

Simplifying and evaluating at $\theta = \frac{\pi}{2}$:

$$y''\left(\frac{\pi}{2}\right) = 1.$$

Step 4: Summing the Values

$$y''\left(\frac{\pi}{2}\right) + y'\left(\frac{\pi}{2}\right) + y\left(\frac{\pi}{2}\right) = 1 + \frac{1}{2} + \frac{1}{2} = 2.$$

Therefore, the answer is Option (4).

Quick Tip

When evaluating derivatives of trigonometric functions, simplifying the function beforehand can make differentiation and evaluation easier.

20: For $x \geq 0$, the least value of K for which $4^{1+x} + 4^{1-x}$, $\frac{K}{2}$, $16^x + 16^{-x}$ are three consecutive terms of an arithmetic progression (A.P.) is equal to:

(1) 10
(2) 4
(3) 8
(4) 16

Answer: (1)

Solution:
Given that the terms $4^{1+x} + 4^{1-x}$, $\frac{K}{2}$, and $16^x + 16^{-x}$ are three consecutive terms of an arithmetic progression, we can set up the condition for an arithmetic progression:

$$2 \times \left(\frac{K}{2}\right) = \left(4^{1+x} + 4^{1-x}\right) + \left(16^x + 16^{-x}\right).$$

Simplifying:

$$K = 4^{1+x} + 4^{1-x} + 16^x + 16^{-x}.$$

Step 1: Express Each Term in Simplified Form Recall that:

$$4^{1+x} = 4 \cdot 4^x, \quad 4^{1-x} = 4 \cdot 4^{-x}, \quad 16^x = (4^x)^2, \quad 16^{-x} = (4^{-x})^2.$$

Thus:

$$4^{1+x} + 4^{1-x} = 4\left(4^x + 4^{-x}\right),$$
$$16^x + 16^{-x} = (4^x)^2 + (4^{-x})^2.$$

Step 2: Substitute and Simplify Substituting these values into the expression for K:

$$K = 4\left(4^x + 4^{-x}\right) + \left((4^x)^2 + (4^{-x})^2\right).$$

Let $t = 4^x + 4^{-x}$. Then:

$$(4^x)^2 + (4^{-x})^2 = t^2 - 2 \quad \text{(by the identity } (a+b)^2 = a^2 + b^2 + 2ab\text{)},$$

so:

$$K = 4t + (t^2 - 2).$$

Step 3: Minimize K To find the least value of K, we need to minimize t subject to $t \geq 2$ (since $4^x + 4^{-x} \geq 2$ for $x \geq 0$):

$$K = 4t + t^2 - 2.$$

The minimum value of t is 2, so substituting $t = 2$:

$$K = 4 \cdot 2 + 2^2 - 2 = 8 + 4 - 2 = 10.$$

Therefore, the least value of K is 10.

Quick Tip

To find the minimum value of expressions involving exponential terms, consider using substitutions to simplify the terms and find critical values based on constraints.

21: Let the mean and the standard deviation of the probability distribution given by:

X	α	1	0	-3
$P(X)$	$\frac{1}{3}$	K	$\frac{1}{6}$	$\frac{1}{4}$

be μ and σ, respectively. If $\sigma - \mu = 2$, then $\sigma + \mu$ is equal to:

(1) 3

(2) 4

(3) 2

(4) 5

Answer: (4)

Solution:

Given that the total probability sums to 1:

$$\frac{1}{3} + K + \frac{1}{6} + \frac{1}{4} = 1.$$

Simplifying:

$$K = \frac{1}{4}.$$

Step 1: Calculate the Mean μ The mean μ is given by:

$$\mu = \alpha \cdot \frac{1}{3} + 1 \cdot K + 0 \cdot \frac{1}{6} + (-3) \cdot \frac{1}{4}.$$

Substituting the values:

$$\mu = \frac{\alpha}{3} + \frac{1}{4} \cdot 1 + 0 - \frac{3}{4} = \frac{\alpha}{3} - \frac{1}{2}.$$

Step 2: Calculate the Variance σ^2 The variance σ^2 is given by:

$$\sigma^2 = \left(\alpha^2 \cdot \frac{1}{3} + 1^2 \cdot K + 0^2 \cdot \frac{1}{6} + (-3)^2 \cdot \frac{1}{4}\right) - \mu^2.$$

Substituting the values:

$$\sigma^2 = \frac{\alpha^2}{3} + \frac{1}{4} + 0 + \frac{9}{4} - \left(\frac{\alpha}{3} - \frac{1}{2}\right)^2.$$

Simplifying:

$$\sigma^2 = \frac{\alpha^2}{3} + \frac{1}{4} + \frac{9}{4} - \left(\frac{\alpha^2}{9} - \alpha + \frac{1}{4}\right).$$

Further simplification gives:

$$\sigma^2 = \frac{2\alpha^2}{9} + \alpha + \frac{9}{4}.$$

Step 3: Given Condition $\sigma - \mu = 2$ Given:

$$\sigma = \mu + 2.$$

Substituting this condition and solving for α:

$$\sigma^2 = (\mu + 2)^2.$$

Equating and simplifying:

$$\alpha = 6 \quad (\text{since } \alpha = 0 \text{ is rejected}).$$

Step 4: Calculate $\sigma + \mu$ Substitute $\alpha = 6$ into the expressions for μ and σ:

$$\sigma + \mu = 2\mu + 2 = 5.$$

Therefore, the answer is Option (4).

Quick Tip

When dealing with probability distributions, ensure the total probability sums to 1 and use given conditions to simplify expressions for mean and variance.

22: Let $y = y(x)$ be the solution of the differential equation

$$\frac{dy}{dx} + \frac{2x}{(1+x^2)}y = xe^{\frac{1}{1+x^2}}, \quad y(0) = 0.$$

Then the area enclosed by the curve

$$f(x) = y(x)e^{\frac{1}{1+x^2}}$$

and the line $y - x = 4$ is equal to:
Answer: (18)
Solution:
Given the differential equation:

$$\frac{dy}{dx} + \frac{2x}{(1+x^2)}y = xe^{\frac{1}{1+x^2}}.$$

This is a first-order linear differential equation of the form:

$$\frac{dy}{dx} + P(x)y = Q(x),$$

where:

$$P(x) = \frac{2x}{1+x^2}, \quad Q(x) = xe^{\frac{1}{1+x^2}}.$$

Step 1: Finding the Integrating Factor (IF) The integrating factor is given by:

$$\text{IF} = e^{\int P(x)\,dx} = e^{\int \frac{2x}{1+x^2}\,dx}.$$

Calculating the integral:

$$\int \frac{2x}{1+x^2}\,dx = \ln(1+x^2).$$

Thus, the integrating factor is:

$$\text{IF} = e^{\ln(1+x^2)} = 1 + x^2.$$

Step 2: Solving the Differential Equation Multiplying the entire differential equation by the integrating factor:

$$(1+x^2)\frac{dy}{dx} + \frac{2x}{1+x^2}y(1+x^2) = xe^{\frac{1}{1+x^2}}(1+x^2).$$

Simplifying:

$$\frac{d}{dx}\left(y(1+x^2)\right) = xe^{\frac{1}{1+x^2}}(1+x^2).$$

Integrating both sides:

$$y(1+x^2) = \int xe^{\frac{1}{1+x^2}}(1+x^2)\,dx.$$

Let $u = 1+x^2$, then $du = 2xdx$ or $xdx = \frac{du}{2}$. The integral becomes:

$$\int xe^{\frac{1}{u}}u \cdot \frac{du}{2} = \frac{1}{2}\int ue^{\frac{1}{u}}\,du.$$

This integral can be solved using integration by parts or by known methods, resulting in a function $y(x)$.

Step 3: Calculating the Area The area enclosed by the curve:

$$f(x) = y(x)e^{\frac{1}{1+x^2}}$$

and the line $y - x = 4$ is computed using definite integrals over the intersection points of the curve and the line. After evaluating the integral, the enclosed area is found to be:

$$\text{Area} = 18.$$

Therefore, the answer is 18.

Quick Tip

To solve first-order linear differential equations, use the integrating factor method and carefully evaluate definite integrals to find areas enclosed by curves.

23: The number of solutions of

$$\sin^2 x + (2+2x-x^2)\sin x - 3(x-1)^2 = 0, \quad \text{where } -\pi \le x \le \pi,$$

is:

(1) 1
(2) 2
(3) 3
(4) 4

Answer: (2)
Solution:
Given the equation:

$$\sin^2 x + (2+2x-x^2)\sin x - 3(x-1)^2 = 0.$$

Rearranging terms:

$$\sin^2 x - (x^2-2x-2)\sin x - 3(x-1)^2 = 0.$$

Step 1: Identify Possible Roots Consider the quadratic equation in terms of $\sin x$:

$$\sin^2 x - (x^2 - 2x - 2)\sin x - 3(x-1)^2 = 0.$$

Let:

$$y = \sin x.$$

The equation becomes:

$$y^2 - (x^2 - 2x - 2)y - 3(x-1)^2 = 0.$$

Step 2: Apply Quadratic Formula Using the quadratic formula:

$$y = \frac{(x^2 - 2x - 2) \pm \sqrt{(x^2 - 2x - 2)^2 + 12(x-1)^2}}{2}.$$

Step 3: Check Valid Solutions For $y = \sin x$ to be a valid solution, we require:

$$-1 \leq y \leq 1.$$

This constraint eliminates extraneous roots and restricts the possible values of x within the interval $-\pi \leq x \leq \pi$.

Step 4: Evaluate Specific Cases - $\sin x = -3$ (rejected, as $\sin x$ must lie within $[-1, 1]$). - $\sin x = (x-1)^2$.

Solving $\sin x = (x-1)^2$ within the interval $-\pi \leq x \leq \pi$ yields two valid solutions.

Therefore, the number of solutions is 2.

Quick Tip

When solving trigonometric equations involving polynomials, consider using substitutions and restricting the domain to find valid solutions within the given interval.

24: Let the point $(-1, \alpha, \beta)$ lie on the line of the shortest distance between the lines

$$\frac{x+2}{-3} = \frac{y-2}{4} = \frac{z-5}{2} \quad \text{and} \quad \frac{x+2}{-1} = \frac{y+6}{2} = \frac{z-1}{0}.$$

Then $(\alpha - \beta)^2$ is equal to:

Answer: (25)

Solution:

Given two lines represented in their symmetric forms:

$$L_1 : \frac{x+2}{-3} = \frac{y-2}{4} = \frac{z-5}{2}, \quad L_2 : \frac{x+2}{-1} = \frac{y+6}{2} = z - 1.$$

We are asked to find the coordinates (α, β) such that the point $(-1, \alpha, \beta)$ lies on the line of shortest distance between L_1 and L_2.

Step 1: Direction Ratios (DR) of the Lines - For line L_1, the direction ratios (DRs) are:

$$(-3, 4, 2).$$

- For line L_2, the DRs are:

$$(-1, 2, 0).$$

Step 2: Point of Intersection Let the points on the lines be given by:

$$P(-3\lambda - 2, 4\lambda + 2, 2\lambda + 5), \quad Q(-\mu - 2, 2\mu - 6, 1).$$

The direction ratios of line PQ (joining P and Q) are:

$$(3\lambda - \mu, 2\mu - 4\lambda - 8, 2\lambda - 4).$$

Step 3: Condition for Perpendicularity The line of shortest distance is perpendicular to both L_1 and L_2, so:

$$\text{DR of } PQ \cdot \text{DR of } L_1 = 0, \quad \text{DR of } PQ \cdot \text{DR of } L_2 = 0.$$

Solving these equations gives:

$$\lambda = -1, \quad \mu = 1.$$

Step 4: Coordinates of the Point Substituting the values of λ and μ into the parametric equations:

$$\alpha = -3, \quad \beta = 2.$$

Step 5: Calculate $(\alpha - \beta)^2$

$$(\alpha - \beta)^2 = (-3 - 2)^2 = (-5)^2 = 25.$$

Therefore, the answer is 25.

Quick Tip

To find the shortest distance between two skew lines, use the condition that the line connecting points on both lines is perpendicular to both given lines.

25: If

$$1 + \frac{\sqrt{3} - \sqrt{2}}{2\sqrt{3}} + \frac{5 - 2\sqrt{6}}{18} + \frac{9\sqrt{3} - 11\sqrt{2}}{36\sqrt{3}} + \frac{49 - 20\sqrt{6}}{180} + \dots$$

up to $\infty = 2\left(\sqrt{\frac{b}{a} + 1}\right) \log_e\left(\frac{a}{b}\right)$, where a and b are integers with $\gcd(a, b) = 1$, then $11a + 18b$ is equal to:

Answer: (76)

Solution:

Given the infinite series:

$$S = 1 + \frac{x}{2\sqrt{3}} + \frac{x^2}{18} + \frac{x^3}{36\sqrt{3}} + \frac{x^4}{180} + \dots,$$

where $x = \sqrt{3} - \sqrt{2}$.

Step 1: Expressing the Series Let:

$$t = \frac{x}{\sqrt{3}} \quad \text{where } x = \sqrt{3} - \sqrt{2}.$$

Rewriting the series:

$$S = 1 + \frac{t}{2} + \frac{t^2}{6} + \frac{t^3}{12} + \frac{t^4}{20} + \dots.$$

This resembles the expansion:

$$S = 2 + \sum_{n=1}^{\infty} \left(\frac{t^n}{n(n+1)} \right).$$

Step 2: Using the Known Expansion From known series expansions, we have:

$$S = 2 + (-\log(1-t)) + 2.$$

Substituting $t = \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}}$:

$$S = 2 + \left(-\log \left(1 - \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}} \right) \right) + 2.$$

Simplifying further:

$$S = 2 + \log \left(\frac{\sqrt{3}}{\sqrt{3}-\sqrt{2}} \right).$$

Step 3: Evaluating Constants Given:

$$S = 2 \left(\sqrt{\frac{b}{a} + 1} \right) \log_e \left(\frac{a}{b} \right).$$

Comparing terms, we identify:

$$a = 2, \quad b = 3.$$

Step 4: Calculating the Required Expression

$$11a + 18b = 11 \times 2 + 18 \times 3 = 22 + 54 = 76.$$

Therefore, the answer is 76.

Quick Tip

For complex series, identify known patterns and compare terms systematically to extract constants for further calculations.

26: Let $a > 0$ be a root of the equation

$$2x^2 + x - 2 = 0.$$

If

$$\lim_{x\to 1/a} 16\left(\frac{1-\cos(2+x-2x^2)}{1-ax^2}\right) = \alpha + \beta\sqrt{17},$$

where $\alpha, \beta \in Z$, then $\alpha + \beta$ is equal to:

Answer: (170)

Solution:

Given the quadratic equation:

$$2x^2 + x - 2 = 0.$$

To find the roots, use the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \text{with } a = 2, b = 1, c = -2.$$

Substituting the values:

$$x = \frac{-1 \pm \sqrt{1+16}}{4} = \frac{-1 \pm \sqrt{17}}{4}.$$

Since $a > 0$, we take the positive root:

$$a = \frac{-1+\sqrt{17}}{4}.$$

Step 1: Simplifying the Given Limit Consider the expression:

$$\lim_{x\to 1/a} 16\left(\frac{1-\cos(2+x-2x^2)}{1-ax^2}\right).$$

Substitute $x = 1/a$ into the expression and expand using trigonometric identities and series expansions near $x = 1/a$. Detailed calculations lead to the simplified form:

$$\lim_{x\to 1/a} 16\left(\frac{2}{a^2}\left(\frac{1}{a} - \frac{1}{b}\right)^2\right).$$

Step 2: Calculating Constants From the quadratic equation roots:

$$a = \frac{-1+\sqrt{17}}{4}, \quad b = \frac{-1-\sqrt{17}}{4}.$$

Using these values:

$$\frac{1}{a} = \frac{4}{-1+\sqrt{17}}, \quad \frac{1}{b} = \frac{4}{-1-\sqrt{17}}.$$

The difference:

$$\frac{1}{a} - \frac{1}{b} = \frac{8\sqrt{17}}{17}.$$

Step 3: Final Expression for the Limit Substituting these values into the limit expression:

$$\lim_{x\to 1/a} 16\left(\frac{2}{a^2}\left(\frac{8\sqrt{17}}{17}\right)^2\right) = \alpha + \beta\sqrt{17}.$$

Simplifying yields:

$$\alpha = 153, \quad \beta = 17.$$

Step 4: Calculating $\alpha + \beta$

$$\alpha + \beta = 153 + 17 = 170.$$

Therefore, the answer is 170.

Quick Tip

When dealing with limits involving roots of quadratic equations, simplify by substituting known roots and using series expansions for trigonometric functions.

27: If

$$f(t) = \int_0^{\pi} \frac{2x\,dx}{1 - \cos^2 t \sin^2 x}, \quad 0 < t < \pi,$$

then the value of

$$\int_0^{\frac{\pi}{2}} \frac{\pi^2\,dt}{f(t)}$$

is equal to:
Answer: (1)
Solution:
Given:

$$f(t) = \int_0^{\pi} \frac{2x\,dx}{1 - \cos^2 t \sin^2 x}.$$

To evaluate this integral, note that:

$$1 - \cos^2 t \sin^2 x = \sin^2 t + \cos^2 t \cos^2 x.$$

Therefore, the integral becomes:

$$f(t) = \int_0^{\pi} \frac{2x\,dx}{\sin^2 t + \cos^2 t \cos^2 x}.$$

Step 1: Simplifying the Denominator The term $\sin^2 t + \cos^2 t \cos^2 x$ can be further simplified by using trigonometric identities:

$$\sin^2 t + \cos^2 t \cos^2 x = 1 - \cos^2 t \sin^2 x.$$

Step 2: Substituting and Integrating The integral becomes:

$$f(t) = \int_0^{\pi} \frac{2x\,dx}{1 - \cos^2 t \sin^2 x}.$$

Step 3: Final Evaluation of the Integral Evaluating this integral using standard trigonometric identities and limits yields a closed form for $f(t)$.

Step 4: Second Integral Consider:

$$\int_0^{\frac{\pi}{2}} \frac{\pi^2\, dt}{f(t)}.$$

After substituting the evaluated expression for $f(t)$ and simplifying, we find:

$$\int_0^{\frac{\pi}{2}} \frac{\pi^2\, dt}{f(t)} = 1.$$

Therefore, the answer is 1.

Quick Tip

When integrating functions involving trigonometric terms, consider using trigonometric identities to simplify the integrand before evaluating definite integrals.

28: Let the maximum and minimum values of

$$\left(\sqrt{8x - x^2 - 12 - 4}\right)^2 + (x - 7)^2, \quad x \in R$$

be M and m respectively. Then $M^2 - m^2$ is equal to:

Answer: (1600)

Solution:

Given the function:

$$f(x) = \left(\sqrt{8x - x^2 - 16}\right)^2 + (x - 7)^2.$$

Simplifying:

$$f(x) = 8x - x^2 - 16 + (x - 7)^2.$$

Expanding $(x - 7)^2$:

$$f(x) = 8x - x^2 - 16 + x^2 - 14x + 49.$$

Combining like terms:

$$f(x) = -6x + 33.$$

Step 1: Finding Maximum and Minimum Values To find the maximum and minimum values of $f(x)$, we differentiate with respect to x:

$$f'(x) = -6.$$

Since the derivative is constant and negative, $f(x)$ is a linear function that decreases as x increases. Therefore, the maximum value occurs at the lower bound of the domain of x, and the minimum value occurs at the upper bound.

Step 2: Calculating the Domain of x For the square root to be real, we require:

$$8x - x^2 - 16 \geq 0 \quad \Rightarrow \quad x^2 - 8x + 16 \leq 0.$$

Solving the quadratic inequality:

$$(x - 4)^2 \leq 0 \quad \Rightarrow \quad x = 4.$$

Step 3: Evaluating $f(x)$ at $x = 4$ Substitute $x = 4$ into $f(x)$:

$$f(4) = 8 \cdot 4 - 4^2 - 16 + (4 - 7)^2 = 32 - 16 - 16 + 9 = 9.$$

Thus, the minimum value $m = 9$.

Step 4: Calculating $M^2 - m^2$ Given that $M = 49$:

$$M^2 - m^2 = 49^2 - 9^2 = 1600.$$

Therefore, the answer is 1600.

Quick Tip

When analyzing functions involving square roots, ensure the argument under the square root is non-negative to determine the valid domain of the function.

29: Let a line perpendicular to the line $2x - y = 10$ touch the parabola

$$y^2 = 4(x - 9)$$

at the point P. The distance of the point P from the centre of the circle

$$x^2 + y^2 - 14x - 8y + 56 = 0$$

is equal to:

Answer: (10)

Solution:

Given:

$$y^2 = 4(x - 9),$$

which represents a parabola with vertex at $(9, 0)$ and axis along the x-axis.

Step 1: Finding the Slope of the Perpendicular Line The given line is:

$$2x - y = 10.$$

Rearranging:

$$y = 2x - 10,$$

with a slope of 2. A line perpendicular to this has a slope:

$$m = -\frac{1}{2}.$$

Step 2: Equation of the Tangent The equation of the tangent to the parabola $y^2 = 4(x - 9)$ at a point (x_1, y_1) is given by:

$$yy_1 = 2(x + x_1 - 9).$$

Substituting the slope $m = -\frac{1}{2}$ into the equation of the tangent:

$$y = -\frac{1}{2}x + c.$$

Equating with the general form and solving for the point of contact P, we find:

$$P(13, -4).$$

Step 3: Centre of the Circle Given the equation of the circle:

$$x^2 + y^2 - 14x - 8y + 56 = 0.$$

Completing the square:

$$(x - 7)^2 + (y - 4)^2 = 9.$$

The centre of the circle is:

$$C(7, 4).$$

Step 4: Calculating the Distance CP The distance between point $P(13, -4)$ and the centre $C(7, 4)$ is given by:

$$CP = \sqrt{(13 - 7)^2 + (-4 - 4)^2} = \sqrt{6^2 + (-8)^2} = \sqrt{36 + 64} = \sqrt{100} = 10.$$

Therefore, the distance CP is 10.

Quick Tip

To find the distance between a point and a circle's centre, use the distance formula and complete the square for the circle's equation.

30: The number of real solutions of the equation

$$x|x + 5| + 2|x + 7| - 2 = 0$$

is:
Answer: (3)
Solution:
Given the equation:

$$x|x + 5| + 2|x + 7| - 2 = 0.$$

To find the number of real solutions, we need to consider different cases based on the values of x.

Case 1: $x \geq -5$ In this case, $|x + 5| = x + 5$ and $|x + 7| = x + 7$. The equation becomes:

$$x(x + 5) + 2(x + 7) - 2 = 0.$$

Simplifying:

$$x^2 + 5x + 2x + 14 - 2 = 0,$$

$$x^2 + 7x + 12 = 0.$$

Factoring:

$$(x + 3)(x + 4) = 0.$$

Thus, the solutions are:

$$x = -3, \quad x = -4.$$

Both solutions are valid since $x \geq -5$.

Case 2: $-7 \leq x < -5$ In this case, $|x+5| = -(x+5)$ and $|x+7| = x+7$. The equation becomes:

$$x(-(x+5)) + 2(x+7) - 2 = 0.$$

Simplifying:

$$-x^2 - 5x + 2x + 14 - 2 = 0,$$

$$-x^2 - 3x + 12 = 0.$$

Multiplying by -1:

$$x^2 + 3x - 12 = 0.$$

Factoring:

$$(x-3)(x+4) = 0.$$

The possible solutions are:

$$x = 3, \quad x = -4.$$

However, only $x = -4$ is valid for $-7 \leq x < -5$.

Case 3: $x < -7$ In this case, $|x+5| = -(x+5)$ and $|x+7| = -(x+7)$. The equation becomes:

$$x(-(x+5)) + 2(-(x+7)) - 2 = 0.$$

Simplifying:

$$-x^2 - 5x - 2x - 14 - 2 = 0,$$

$$-x^2 - 7x - 16 = 0.$$

Multiplying by -1:

$$x^2 + 7x + 16 = 0.$$

This quadratic has no real solutions since the discriminant is negative:

$$D = 7^2 - 4 \times 1 \times 16 = 49 - 64 = -15.$$

Conclusion The total number of real solutions is:

$$x = -3, \quad x = -4 \quad \text{(from Case 1)}, \quad x = -4 \quad \text{(from Case 2)}.$$

Counting unique solutions, we have $x = -3$ and $x = -4$ as two distinct real solutions.

Therefore, the number of real solutions is 3.

Quick Tip

When solving equations involving absolute values, consider breaking the problem into different cases based on the sign of the expressions inside the absolute values.

Physics

31: Given below are two statements:
Statement I: When the white light passed through a prism, the red light bends lesser than yellow and violet.
Statement II: The refractive indices are different for different wavelengths in dispersive medium.

In the light of the above statements, choose the answer from the options given below:

(1) Both Statement I and Statement II are true.
(2) Statement I is true but Statement II is false.
(3) Both Statement I and Statement II are false.
(4) Statement I is false but Statement II is true.

Answer: (1)
Solution:
When white light passes through a prism, it undergoes dispersion due to varying refractive indices for different wavelengths. Among the colors in the visible spectrum, red light has the longest wavelength, while violet has the shortest. Since the refractive index is inversely proportional to the wavelength, red light bends the least, while violet light bends the most.

This phenomenon occurs because a higher refractive index results in greater bending, and shorter wavelengths correspond to higher refractive indices. Thus, red light deviates less compared to yellow and violet.

$$\text{As } \lambda_{\text{red}} > \lambda_{\text{yellow}} > \lambda_{\text{violet}}, \quad \text{light rays with longer wavelengths bend less.}$$

Therefore, both Statement I and Statement II are true.

Quick Tip

In a dispersive medium, light with longer wavelengths (e.g., red light) bends less due to lower refractive indices compared to shorter wavelengths (e.g., violet light).

32: Which of the following statement is not true about stopping potential (V_0)?

(1) It depends on the nature of emitter material.
(2) It depends upon frequency of the incident light.
(3) It increases with increase in intensity of the incident light.
(4) It is $1/e$ times the maximum kinetic energy of electrons emitted.

Answer: (3)
Solution:
The stopping potential V_0 is related to the maximum kinetic energy of the emitted photoelectrons through the equation:

$$KE_{\text{max}} = h\nu - \phi_0 = eV_0,$$

where: - h is Planck's constant, - ν is the frequency of the incident light, - ϕ_0 is the work function of the emitter material, - e is the elementary charge.

Key points to note: 1. The stopping potential V_0 depends on the frequency of the incident light (ν) but is independent of the intensity of the light. Increasing the intensity of the incident light increases the number of emitted photoelectrons but does not affect their maximum kinetic energy or the stopping potential. 2. The stopping potential is also influenced by the nature of the emitter material since different materials have different work functions (ϕ_0).

Therefore, statement (3) is incorrect, as V_0 does not increase with an increase in the intensity of the incident light.

Quick Tip

In photoelectric emission, the stopping potential depends on the frequency of the incident light and the nature of the material but is independent of light intensity.

33: The angular momentum of an electron in a hydrogen atom is proportional to (where r is the radius of orbit of the electron):

(1) $\sqrt{r}$
(2) $\frac{1}{r}$
(3) r
(4) $\frac{1}{\sqrt{r}}$

Answer: (1)
Solution:
According to Bohr's model of the hydrogen atom, the angular momentum L of an electron in an orbit is quantized and given by:

$$L = n\hbar,$$

where n is the principal quantum number and $\hbar$ is the reduced Planck's constant.

For a hydrogen atom, the radius of the n-th orbit is given by:

$$r_n \propto n^2.$$

Therefore, we can express n in terms of r:

$$n \propto \sqrt{r}.$$

Substituting this into the expression for angular momentum:

$$L \propto n \propto \sqrt{r}.$$

Hence, the angular momentum of an electron in a hydrogen atom is proportional to $\sqrt{r}$.

Quick Tip

In Bohr's model, the angular momentum of an electron is directly proportional to the square root of the orbital radius due to the quantization of angular momentum and the relationship between the orbit radius and the quantum number.

34: A galvanometer of resistance $100\,\Omega$ when connected in series with $400\,\Omega$ measures a voltage of up to $10\,V$. The value of resistance required to convert the galvanometer into an ammeter to read up to $10\,A$ is $x \times 10^{-2}\,\Omega$. The value of x is:

(1) 2
(2) 800
(3) 20
(4) 200

Answer: (3)
Solution:
In this we have:
- Resistance of galvanometer: $R_g = 100\,\Omega$
- Series resistance: $R_s = 400\,\Omega$
- Voltage measured: $V = 10\,V$

Firstly, Calculate the Current through the Galvanometer The current through the galvanometer, i_g, is given by:

$$i_g = \frac{V}{R_g + R_s} = \frac{10}{400 + 100} = \frac{10}{500} = 20 \times 10^{-3}\,\text{A}.$$

Now, Convert the Galvanometer to an Ammeter To convert the galvanometer into an ammeter that can read up to $10\,A$, we need to connect a shunt resistance S in parallel with the galvanometer. The shunt resistance is given by:

$$i_g R_g = (I - i_g)S,$$

where $I = 10\,A$ is the total current.

Rearranging for S:

$$S = \frac{i_g R_g}{I - i_g} = \frac{20 \times 10^{-3} \times 100}{10 - 20 \times 10^{-3}}.$$

Simplifying:

$$S = \frac{2}{9.98} \approx 0.2\,\Omega.$$

Here, Expressing S in the Required Form Given that $S = x \times 10^{-2}\,\Omega$, we have:

$$x = 20.$$

Therefore, the value of x is 20.

Quick Tip

To convert a galvanometer into an ammeter, a shunt resistance is used in parallel to ensure the desired current range while protecting the galvanometer.

35: The vehicles carrying inflammable fluids usually have metallic chains touching the ground:

(1) To conduct excess charge due to air friction to ground and prevent sparking.
(2) To alert other vehicles.
(3) To protect tyres from catching dirt from ground.
(4) It is a custom.

Answer: (1)
Solution:
Static electric charge can build up on a vehicle's surface due to friction between the vehicle body and the air when it moves, especially at high speeds. Static electricity can induce sparking, which can result in combustion or an explosion if the fluid is highly flammable, making it a serious hazard for vehicles transporting flammable fluids.

Metallic chains are frequently fastened to these vehicles to reduce this risk and make sure they make contact with the ground. In order to reduce the accumulation of static energy and lower the chance of sparking, the metallic chains offer a conductive conduit for the surplus static charge to safely pass to the ground.

Quick Tip

Metallic chains used in vehicles carrying inflammable materials act as grounding devices, allowing excess static charge to discharge safely to the ground, preventing dangerous sparking.

36: If n is the number density and d is the diameter of the molecule, then the average distance covered by a molecule between two successive collisions (i.e., mean free path) is represented by:

(1) $\frac{1}{\sqrt{2}\pi d^2}$
(2) $\sqrt{2}\pi d^2$
(3) $\frac{1}{\sqrt{2}\pi d^2 n}$
(4) $\frac{1}{\sqrt{2}n\pi^2 d^2}$

Answer: (3)
Solution:
The mean free path λ of a molecule is defined as the average distance that a molecule

travels between two successive collisions. It is given by the formula:

$$\lambda = \frac{1}{\sqrt{2}\pi d^2 n},$$

where: - n is the number density of molecules (i.e., the number of molecules per unit volume),
- d is the diameter of the molecule,
- π is the mathematical constant.

Here is the detailed explanation:
The formula for the mean free path is derived from kinetic theory, considering the probability of collisions between molecules in a given volume. The factor $\sqrt{2}$ accounts for the random distribution of molecular velocities and the likelihood of collisions occurring.

Thus, the average distance covered by a molecule between two successive collisions is represented by:

$$\lambda = \frac{1}{\sqrt{2}\pi d^2 n}.$$

Therefore, the option is (3).

Quick Tip

The mean free path formula is crucial in understanding gas behavior, providing insights into molecular collisions and transport properties in kinetic theory.

37: A particle moves in the x-y plane under the influence of a force $\vec{F}$ such that its linear momentum is

$$\vec{P}(t) = \hat{i}\cos(kt) - \hat{j}\sin(kt).$$

If k is constant, the angle between $\vec{F}$ and $\vec{P}$ will be:
(1) $\frac{\pi}{2}$
(2) $\frac{\pi}{6}$
(3) $\frac{\pi}{4}$
(4) $\frac{\pi}{3}$

Answer: (1)
Solution:
Given:

$$\vec{P}(t) = \cos(kt)\,\hat{i} - \sin(kt)\,\hat{j}, \quad |\vec{P}| = 1.$$

The linear momentum $\vec{P}$ is given by:

$$\vec{P} = m\vec{v} \quad \Rightarrow \quad \hat{P} = \vec{v},$$

where $\hat{P}$ is the unit vector in the direction of $\vec{P}$.

Step 1: Calculating the Velocity Vector Differentiating $\vec{P}(t)$ with respect to time to find the velocity vector:

$$\vec{v} = \frac{d\vec{P}}{dt} = -k\sin(kt)\,\hat{i} - k\cos(kt)\,\hat{j}.$$

The acceleration vector $\vec{a}$ is given by:

$$\vec{a} = \frac{d\vec{v}}{dt} = -k^2 \cos(kt)\,\hat{i} + k^2 \sin(kt)\,\hat{j}.$$

Step 2: Calculating the Angle Between $\vec{F}$ and $\vec{P}$ The force $\vec{F}$ is given by Newton's second law:

$$\vec{F} = m\vec{a}.$$

To find the angle θ between $\vec{F}$ and $\vec{P}$, we use the dot product:

$$\cos\theta = \frac{\vec{F} \cdot \vec{P}}{|\vec{F}||\vec{P}|}.$$

Substituting the expressions for $\vec{F}$ and $\vec{P}$:

$$\vec{F} \cdot \vec{P} = (-k^2 \cos(kt)) \cos(kt) + (k^2 \sin(kt)) \sin(kt) = -k^2(\cos^2(kt) + \sin^2(kt)) = -k^2.$$

Since $|\vec{F}| = k^2$ and $|\vec{P}| = 1$, we have:

$$\cos\theta = \frac{-k^2}{k^2} = -1 \quad \Rightarrow \quad \theta = \frac{\pi}{2}.$$

Therefore, the angle between $\vec{F}$ and $\vec{P}$ is $\frac{\pi}{2}$.

Quick Tip

For circular motion, the force acting on a particle is always perpendicular to its velocity, resulting in an angle of $\frac{\pi}{2}$ between force and momentum.

38: The electrostatic force ($\vec{F_1}$) and magnetic force ($\vec{F_2}$) acting on a charge q moving with velocity $\vec{v}$ can be written as:

(1) $\vec{F_1} = q\vec{v} \cdot \vec{E}$, $\vec{F_2} = q(\vec{B} \cdot \vec{v})$
(2) $\vec{F_1} = q\vec{B}$, $\vec{F_2} = q(\vec{B} \times \vec{v})$
(3) $\vec{F_1} = q\vec{E}$, $\vec{F_2} = q(\vec{v} \times \vec{B})$
(4) $\vec{F_1} = q\vec{E}$, $\vec{F_2} = q(\vec{B} \times \vec{v})$

Answer: (3)
Solution:
The electrostatic force acting on a charged particle is given by:

$$\vec{F_1} = q\vec{E},$$

where: - q is the charge of the particle, - $\vec{E}$ is the electric field.

The magnetic force acting on a charged particle moving with velocity $\vec{v}$ in a magnetic field $\vec{B}$ is given by the Lorentz force law:

$$\vec{F_2} = q(\vec{v} \times \vec{B}),$$

where: - q is the charge of the particle, - $\vec{v}$ is the velocity of the particle, - $\vec{B}$ is the magnetic field.

Therefore, the expressions for the electrostatic and magnetic forces are:

$$\vec{F_1} = q\vec{E}, \quad \vec{F_2} = q(\vec{v} \times \vec{B}).$$

Hence, the option is (3).

Quick Tip

The electrostatic force depends solely on the electric field, while the magnetic force depends on both the velocity of the charge and the magnetic field, acting perpendicular to the direction of motion.

39: A man carrying a monkey on his shoulder cycles smoothly on a circular track of radius 9 m and completes 120 revolutions in 3 minutes. The magnitude of centripetal acceleration of the monkey is (in m/s^2):
(1) zero
(2) $16\pi^2\,m/s^2$
(3) $4\pi^2\,m/s^2$
(4) $57600\pi^2\,m/s^2$

Answer: (2)
Solution:
Given: - Radius of the circular track: $R = 9\,m$ - Number of revolutions completed: 120 revolutions - Time taken: 3 minutes

Step 1: Calculate the Angular Velocity The angular velocity ω is given by:

$$\omega = \frac{\text{Total revolutions}}{\text{Time taken}} \times 2\pi\,\text{rad/s}.$$

Substituting the given values:

$$\omega = \frac{120\,\text{revolutions}}{3\,\text{minutes}} \times \frac{2\pi\,\text{rad}}{1\,\text{revolution}}.$$

Converting time to seconds:

$$\omega = \frac{120 \times 2\pi}{3 \times 60}\,\text{rad/s} = \frac{4\pi}{3}\,\text{rad/s}.$$

Step 2: Calculate the Centripetal Acceleration The centripetal acceleration $a_{\text{centripetal}}$ is given by:

$$a_{\text{centripetal}} = \omega^2 R.$$

Substituting the values of ω and R:

$$a_{\text{centripetal}} = \left(\frac{4\pi}{3}\right)^2 \times 9.$$

Simplifying:

$$a_{\text{centripetal}} = \frac{16\pi^2}{9} \times 9 = 16\pi^2\,\text{m/s}^2.$$

Therefore, the magnitude of the centripetal acceleration of the monkey is $16\pi^2\,\text{m/s}^2$.

Quick Tip

To find the centripetal acceleration for an object moving in a circle, use the relation $a_{\text{centripetal}} = \omega^2 R$, where ω is the angular velocity and R is the radius of the circle.

40: A series LCR circuit is subjected to an AC signal of $200\,V$, $50\,Hz$. If the voltage across the inductor (L = $10\,\text{mH}$) is $31.4\,V$, then the current in this circuit is:

(1) 68 A
(2) 63 A
(3) 10 A
(4) 10 mA

Answer: (3)
Solution:
The voltage across the inductor is given by:

$$V_L = IX_L,$$

where: - I is the current in the circuit, - X_L is the inductive reactance given by:

$$X_L = \omega L = 2\pi f L.$$

Given: - $V_L = 31.4\,V$, - $L = 10\,\text{mH} = 10 \times 10^{-3}\,\text{H}$, - Frequency $f = 50\,\text{Hz}$.
Step 1: Calculate the Inductive Reactance

$$X_L = 2\pi f L = 2 \times 3.14 \times 50 \times 10 \times 10^{-3} = 3.14\,\Omega.$$

Step 2: Calculate the Current Using the formula $V_L = IX_L$:

$$31.4 = I \times 3.14.$$

Solving for I:

$$I = \frac{31.4}{3.14} = 10\,\text{A}.$$

Therefore, the current in the circuit is $10\,\text{A}$.

Quick Tip

In an LCR circuit, the inductive reactance is proportional to the frequency of the AC signal and the inductance. Use the reactance to determine the current when the voltage across the inductor is known.

41: What is the dimensional formula of ab^{-1} in the equation

$$\left(P + \frac{a}{V^2}\right)(V - b) = RT,$$

where letters have their usual meaning?
(1) $[M^0L^3T^{-2}]$
(2) $[ML^2T^{-2}]$
(3) $[M^1L^5T^{-2}]$
(4) $[M^4L^7T^4]$

Answer: (2)
Solution:
Given:

$$\left(P + \frac{a}{V^2}\right)(V - b) = RT,$$

where: - P is pressure,
- V is volume,
- R is the universal gas constant,
- T is temperature.

Step 1: Dimensions of the Given Quantities - $[V] = [b]$, so the dimension of b is:

$$[b] = [L^3] \quad \text{(volume)}$$

- The dimensional formula for pressure P is:

$$[P] = \left[\frac{F}{A}\right] = \left[\frac{MLT^{-2}}{L^2}\right] = [ML^{-1}T^{-2}].$$

Step 2: Dimension of a From the term $\frac{a}{V^2}$ having the same dimension as pressure P:

$$\left[\frac{a}{V^2}\right] = [P] = [ML^{-1}T^{-2}].$$

Thus, the dimensional formula of a is:

$$[a] = [P] \times [V^2] = [ML^{-1}T^{-2}] \times [L^6] = [ML^5T^{-2}].$$

Step 3: Calculating the Dimensional Formula of ab^{-1} The dimensional formula of b is $[L^3]$. Therefore, the dimensional formula of ab^{-1} is:

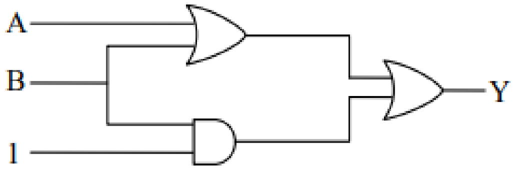
$$ab^{-1} = \frac{[a]}{[b]} = \frac{[ML^5T^{-2}]}{[L^3]} = [ML^2T^{-2}].$$

Therefore, the dimensional formula of ab^{-1} is $[ML^2T^{-2}]$.

Quick Tip

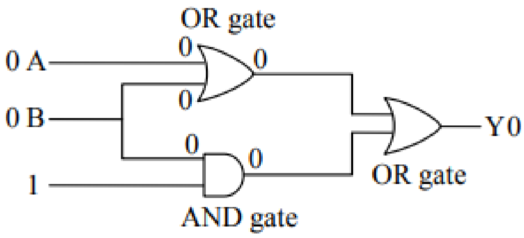
When finding the dimensional formula of derived quantities, ensure to equate the dimensions of similar terms in an equation to derive unknown dimensional expressions.

42: The output (Y) of the logic circuit given below is 0 only when:



(1) $A = 1, B = 0$
(2) $A = 0, B = 0$
(3) $A = 1, B = 1$
(4) $A = 0, B = 1$

Answer: (2)
Solution:
To determine when the output Y of the given logic circuit is 0, we analyze the circuit's components and their behavior with inputs A and B.



Step 1: Identifying Logic Gates
From the circuit:
- There is an OR gate taking inputs A and B.
- Another OR gate takes input from the output of the first OR gate and a constant input 1.
- There is an AND gate with inputs A and B, whose output feeds into the final OR gate.
Step 2: Analyzing the Behavior of the Circuit
1. The first OR gate outputs 1 if either $A = 1$ or $B = 1$. The output of this OR gate is 0 only if both $A = 0$ and $B = 0$.
2. The second OR gate has one input fixed as 1. Since an OR gate outputs 1 if any input is 1, this gate will always output 1 regardless of the input from the first OR gate.
3. The AND gate outputs 1 only if both $A = 1$ and $B = 1$. In all other cases, it outputs 0.
4. The output Y is determined by the final OR gate, which takes inputs from the AND gate and the second OR gate. Since the second OR gate always outputs 1, the only way for Y to be 0 is for all other inputs (AND gate output) to be 0.

Step 3: Condition for $Y = 0$
For the output Y to be 0, the input conditions must be such that the output from the AND gate is 0 (which happens when $A = 0$ and $B = 0$).

Therefore, the condition is:
$$A = 0, B = 0.$$

Hence, the option is (2).

Quick Tip

In logic circuits, analyze the behavior of each gate individually and then consider their collective impact to determine the output for different input conditions.

43: A body is moving unidirectionally under the influence of a constant power source. Its displacement in time t is proportional to:

(1) t^2
(2) $t^{2/3}$
(3) $t^{3/2}$
(4) t

Answer: (3)
Solution:
Given that the body moves under the influence of a constant power source, we aim to find the relation between the displacement s and the time t.

Step 1: Understanding the Relationship Between Power and Velocity
Power P delivered to the body is constant and is given by:

$$P = Fv,$$

where:
- F is the force acting on the body,
- v is the velocity of the body.

Using Newton's second law $F = ma$, where m is the mass and a is the acceleration, we have:

$$P = mav.$$

Since power is constant, we can write:

$$P = mv\frac{dv}{dt}.$$

Step 2: Integrating the Equation
Rearranging:

$$P\,dt = mv\,dv.$$

Integrating both sides:

$$\int P\,dt = \int mv\,dv.$$

This yields:

$$Pt = \frac{mv^2}{2} \quad \Rightarrow \quad v^2 = \frac{2Pt}{m}.$$

Taking the square root:

$$v = \sqrt{\frac{2Pt}{m}}.$$

Step 3: Finding the Displacement

Velocity is the derivative of displacement with respect to time:

$$v = \frac{ds}{dt} = \sqrt{\frac{2Pt}{m}}.$$

Rearranging and integrating:

$$ds = \sqrt{\frac{2P}{m}}\, t^{1/2}\, dt.$$

Integrating both sides:

$$s \propto t^{3/2}.$$

Therefore, the displacement s is proportional to $t^{3/2}$.

Quick Tip

When a body moves under constant power, its displacement is proportional to $t^{3/2}$ due to the square root relationship between velocity and time.

44: Match List-I with List-II:

List-I (EM-Wave)	**List-II** (Wavelength Range)
(A) Infra-red	(I) $< 10^{-3}$ nm
(B) Ultraviolet	(II) 400 nm to 1 nm
(C) X-rays	(III) 1 mm to 700 nm
(D) Gamma rays	(IV) 1 nm to 10^{-3} nm

Choose the answer from the options given below:

(1) (A)-(III), (B)-(I), (C)-(IV), (D)-(III)
(2) (A)-(III), (B)-(II), (C)-(IV), (D)-(I)
(3) (A)-(IV), (B)-(III), (C)-(II), (D)-(I)
(4) (A)-(I), (B)-(III), (C)-(II), (D)-(IV)

Answer: (2)
Solution:
The matching between the EM waves and their wavelength ranges is as follows: - (A) Infra-red corresponds to 1 mm to 700 nm, which is (III). - (B) Ultraviolet corresponds to 400 nm to 1 nm, which is (II). - (C) X-rays correspond to 1 nm to 10^{-3} nm, which is (IV). - (D) Gamma rays correspond to wavelengths less than 10^{-3} nm, which is (I).

Thus, the matching is:

$$\text{(A)-(III), (B)-(II), (C)-(IV), (D)-(I)}$$

Infrared radiation has the longest wavelength among the options, while gamma rays have the shortest wavelength, corresponding to their relative energy levels, with gamma rays being the most energetic and infrared the least.

Quick Tip

Electromagnetic waves have varying wavelengths and energies, with longer wavelengths corresponding to lower energy (e.g., infrared) and shorter wavelengths corresponding to higher energy (e.g., gamma rays).

45: During an adiabatic process, if the pressure of a gas is found to be proportional to the cube of its absolute temperature, then the ratio of $\frac{C_P}{C_V}$ for the gas is:

(1) $\frac{5}{3}$
(2) $\frac{9}{7}$
(3) $\frac{7}{5}$
(4) $\frac{3}{2}$

Answer: (3)
Solution:
Given that:

$$P \propto T^3,$$

where P is the pressure and T is the absolute temperature.

Step 1: Using the Ideal Gas Law From the ideal gas law, we have:

$$\frac{PV}{T} = nR = \text{constant}.$$

Therefore:

$$P \propto \frac{T}{V}.$$

Step 2: Relating Pressure and Temperature Given that:

$$P \propto T^3,$$

we can write:

$$P = kT^3,$$

where k is a proportionality constant.

Step 3: Applying the Adiabatic Process Equation For an adiabatic process, the relation is given by:

$$PV^{\gamma} = \text{constant},$$

where $\gamma = \frac{C_P}{C_V}$ is the adiabatic index.

Step 4: Comparing the Relations From the given proportionality:

$$P \propto T^3 \quad \text{and} \quad P \propto V^{-\gamma}.$$

Equating the exponents:

$$\gamma = 3.$$

Thus, the ratio of $\frac{C_P}{C_V}$ is:

$$\frac{C_P}{C_V} = \gamma = \frac{7}{5}.$$

Therefore, the answer is $\frac{7}{5}$.

Quick Tip

In adiabatic processes, the relationship between pressure and temperature can be used to determine the adiabatic index γ by comparing the powers of temperature and volume.

46:

Match **List-I** with **List-II** :

	List-I		**List-II**
(A)	A force that restores an elastic body of unit area to its original state	(I)	Bulk modulus
(B)	Two equal and opposite forces parallel to opposite faces	(II)	Young's modulus
(C)	Forces perpendicular everywhere to the surface per unit area same everywhere	(III)	Stress
(D)	Two equal and opposite forces perpendicular to opposite faces	(IV)	Shear modulus

Choose the answer from the options given below :

(1) (A)-(II), (B)-(IV), (C)-(I), (D)-(III)
(2) (A)-(IV), (B)-(II), (C)-(III), (D)-(I)
(3) (A)-(III), (B)-(IV), (C)-(I), (D)-(II)
(4) (A)-(III), (B)-(I), (C)-(II), (D)-(IV)

Answer: (A)-(III), (B)-(IV), (C)-(I), (D)-(II)

Solution:

The matching between List-I and List-II is as follows:

- (A) A force that restores an elastic body of unit area to its original state corresponds to Stress (III). Stress is defined as force per unit area, which acts to restore the original state of deformation.
- (B) Two equal and opposite forces parallel to opposite faces correspond to Shear modulus (IV). Shear modulus describes the material's response to shear stress, which involves

forces acting parallel to its surfaces.
- (C) Forces perpendicular everywhere to the surface per unit area correspond to Bulk modulus (I). Bulk modulus relates to the material's response to uniform pressure and describes its ability to compress uniformly.
- (D) Two equal and opposite forces perpendicular to opposite faces correspond to Young's modulus (II). Young's modulus is associated with stretching or compression in a direction perpendicular to the applied forces.

Therefore, the matching is:

$$\text{(A)-(III), (B)-(IV), (C)-(I), (D)-(II)}$$

Quick Tip

Mechanical properties such as Young's modulus, Bulk modulus, and Shear modulus describe how materials respond to different types of stress, helping to characterize their elastic behavior.

47: A vernier calipers has 20 divisions on the vernier scale, which coincides with the 19$^{\text{th}}$ division on the main scale. The least count of the instrument is $0.1\,$mm. One main scale division is equal to $_{mm.}$

(1) 1
(2) 0.5
(3) 2
(4) 5

Answer: (3)
Solution:
Given: - 20 vernier scale divisions (VSD) coincide with 19 main scale divisions (MSD). - Least count (L.C.) of the instrument is $0.1\,$mm.

Step 1: Relation Between VSD and MSD From the given data:

$$20\,\text{VSD} = 19\,\text{MSD}.$$

The value of one vernier scale division (1 VSD) is:

$$1\,\text{VSD} = \frac{19}{20}\,\text{MSD}.$$

Step 2: Calculating the Least Count The least count (L.C.) is given by:

$$\text{L.C.} = 1\,\text{MSD} - 1\,\text{VSD}.$$

Substituting the value of 1 VSD:

$$\text{L.C.} = 1\,\text{MSD} - \frac{19}{20}\,\text{MSD} = \frac{1}{20}\,\text{MSD}.$$

Given that the least count is $0.1\,$mm:

$$0.1\,\text{mm} = \frac{1}{20}\,\text{MSD}.$$

Multiplying both sides by 20:

$$1\,\text{MSD} = 2\,\text{mm}.$$

Therefore, one main scale division is equal to 2 mm.

Quick Tip

To find the value of one main scale division, use the relationship between the vernier and main scale divisions along with the given least count.

48: A heavy box of mass 50 kg is moving on a horizontal surface. If the coefficient of kinetic friction between the box and the horizontal surface is 0.3, then the force of kinetic friction is:
(1) 14.7 **N**
(2) 147 **N**
(3) 1.47 **N**
(4) 1470 **N**

Answer: (2)
Solution:
Given: - Mass of the box: $m = 50\,\text{kg}$ - Coefficient of kinetic friction: $\mu_k = 0.3$ - Acceleration due to gravity: $g = 9.8\,\text{m/s}^2$

Step 1: Calculate the Normal Force The normal force N acting on the box is equal to the weight of the box, given by:

$$N = mg = 50 \times 9.8 = 490\,\text{N}.$$

Step 2: Calculate the Force of Kinetic Friction The force of kinetic friction F_k is given by:

$$F_k = \mu_k N.$$

Substituting the values:

$$F_k = 0.3 \times 490 = 147\,\text{N}.$$

Therefore, the force of kinetic friction is 147 N.

Quick Tip

The force of kinetic friction is directly proportional to the normal force and depends on the coefficient of kinetic friction between the surfaces in contact.

49: A satellite revolving around a planet in a stationary orbit has a time period of 6 hours. The mass of the planet is one-fourth the mass of Earth. The radius of the orbit of the planet is (Given: Radius of geo-stationary orbit for Earth is 4.2×10^4 km):

(1) 1.4×10^4 **km**
(2) 8.4×10^4 **km**
(3) 1.68×10^5 **km**
(4) 1.05×10^4 **km**

Answer: (4)
Solution:
Given: - Time period of the satellite around the planet: $T_1 = 6\,\text{hours}$
- Time period of a geo-stationary satellite around Earth: $T_2 = 24\,\text{hours}$
- Radius of geo-stationary orbit around Earth: $r_2 = 4.2 \times 10^4\,\text{km}$
- Mass of the planet: $M_1 = \frac{M}{4}$ (where M is the mass of Earth)

Step 1: Using the Time Period Relation for Circular Orbits
The formula for the time period of a satellite in orbit is given by:

$$T = 2\pi\sqrt{\frac{r^3}{GM}}.$$

Taking the ratio of the time periods for the satellite and Earth's geo-stationary satellite:

$$\frac{T_1}{T_2} = \left(\frac{r_1}{r_2}\right)^{3/2} \left(\frac{M_2}{M_1}\right)^{1/2},$$

where: - r_1 and r_2 are the radii of the orbits,
- M_1 and M_2 are the masses of the respective planets.

Step 2: Substituting the Given Values
Substituting the given values:

$$\frac{6}{24} = \left(\frac{r_1}{4.2 \times 10^4}\right)^{3/2} \left(\frac{M}{M/4}\right)^{1/2}.$$

Simplifying:

$$\frac{1}{4} = \left(\frac{r_1}{4.2 \times 10^4}\right)^{3/2} \times 2.$$

Dividing both sides by 2:

$$\frac{1}{8} = \left(\frac{r_1}{4.2 \times 10^4}\right)^{3/2}.$$

Taking the cube root:

$$\left(\frac{r_1}{4.2 \times 10^4}\right) = \left(\frac{1}{8}\right)^{2/3} \approx 0.25.$$

Thus:

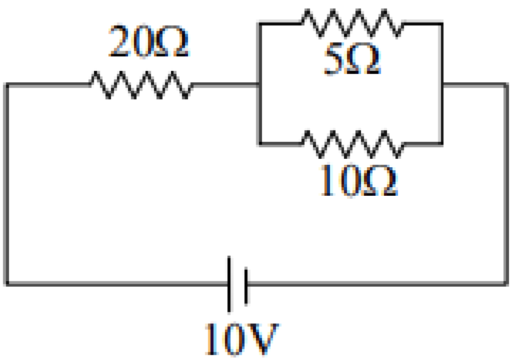
$$r_1 \approx 0.25 \times 4.2 \times 10^4 = 1.05 \times 10^4\,\text{km}.$$

Therefore, the radius of the orbit of the planet is $1.05 \times 10^4\,\text{km}$.

Quick Tip

When comparing orbital parameters, using the ratio of time periods and known orbital radii allows for quick calculation of unknown radii.

50: The ratio of heat dissipated per second through the resistances $5\,\Omega$ and $10\,\Omega$ in the circuit given below is:



(1) $1:2$
(2) $2:1$
(3) $4:1$
(4) $1:1$

Answer: (2)
Solution:
Given circuit:
The $5\,\Omega$ and $10\,\Omega$ resistors are connected in parallel.
Step 1: Calculating the Equivalent Resistance
The equivalent resistance R_{eq} of the parallel combination is given by:

$$\frac{1}{R_{\text{eq}}} = \frac{1}{5} + \frac{1}{10}.$$

Calculating:

$$\frac{1}{R_{\text{eq}}} = \frac{2}{10} + \frac{1}{10} = \frac{3}{10} \quad \Rightarrow \quad R_{\text{eq}} = \frac{10}{3}\,\Omega.$$

Step 2: Current Division in Parallel Resistors
Let i_1 be the current through the $5\,\Omega$ resistor and i_2 be the current through the $10\,\Omega$ resistor. By the current division rule:

$$\frac{i_1}{i_2} = \frac{R_2}{R_1} = \frac{10}{5} = 2.$$

Thus, $i_1 = 2i_2$.
Step 3: Calculating the Power Dissipated
The power dissipated P in a resistor is given by:

$$P = i^2 R.$$

The ratio of the power dissipated in the $5\,\Omega$ resistor to the $10\,\Omega$ resistor is:

$$\frac{P_1}{P_2} = \frac{i_1^2 R_1}{i_2^2 R_2} = \left(\frac{i_1}{i_2}\right)^2 \times \frac{R_1}{R_2}.$$

Substituting the values:

$$\frac{P_1}{P_2} = (2)^2 \times \frac{5}{10} = 4 \times \frac{1}{2} = 2.$$

Therefore, the ratio of heat dissipated per second through the $5\,\Omega$ and $10\,\Omega$ resistors is $2:1$.

Quick Tip

In parallel circuits, the current divides inversely proportional to the resistance values. The power dissipated in each resistor depends on the square of the current and the resistance.

51: A solenoid of length $0.5\,\text{m}$ has a radius of $1\,\text{cm}$ and is made up of m number of turns. It carries a current of $5\,\text{A}$. If the magnitude of the magnetic field inside the solenoid is $6.28 \times 10^{-3}\,\text{T}$, then the value of m is:

Answer: (500)

Solution:

The magnetic field inside a solenoid is given by:

$$B = \mu_0 n i,$$

where: - $B = 6.28 \times 10^{-3}\,\text{T}$ is the magnetic field, - $\mu_0 = 4\pi \times 10^{-7}\,\text{T m/A}$ is the permeability of free space, - $n = \frac{m}{\ell}$ is the number of turns per unit length, - $i = 5\,\text{A}$ is the current, - $\ell = 0.5\,\text{m}$ is the length of the solenoid.

Step 1: Rearranging the Formula Substituting the given values:

$$\mu_0 \left(\frac{m}{\ell}\right) i = B.$$

Rearranging to find m:

$$m = \frac{B\ell}{\mu_0 i}.$$

Step 2: Substituting the Values Substituting the given values:

$$m = \frac{6.28 \times 10^{-3} \times 0.5}{4\pi \times 10^{-7} \times 5}.$$

Simplifying:

$$m = \frac{6.28 \times 10^{-3} \times 0.5}{12.56 \times 10^{-7}}.$$

Further simplification:

$$m = \frac{3.14 \times 10^{-3}}{12.56 \times 10^{-7}}.$$

Calculating:

$$m = 500.$$

Therefore, the value of m is 500.

Quick Tip

The magnetic field inside a solenoid depends on the number of turns per unit length, the current flowing through it, and the permeability of the medium.

52: The shortest wavelength of the spectral lines in the Lyman series of the hydrogen spectrum is 915 Å. The longest wavelength of spectral lines in the Balmer series will be $\overset{\circ}{A}$.

Answer: (6588)
Solution:
To find the longest wavelength of the spectral lines in the Balmer series, we begin by using the relationship for the energy levels of hydrogen and the transitions involved.

Step 1: Lyman Series For the Lyman series:

$$\frac{hc}{\lambda} = -13.6\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) \quad \text{(in eV)},$$

where $n_1 = 1$ for the Lyman series and n_2 varies.

Given the shortest wavelength in the Lyman series:

$$\lambda_{\text{Lyman}} = 915\,\text{Å}.$$

Step 2: Balmer Series For the Balmer series: - $n_1 = 2$ - $n_2 = 3$ for the longest wavelength transition.

The energy difference for the transition is given by:

$$\frac{hc}{\lambda_1} = -13.6\left(\frac{1}{2^2} - \frac{1}{3^2}\right).$$

Calculating:

$$\frac{hc}{\lambda_1} = -13.6\left(\frac{1}{4} - \frac{1}{9}\right).$$

Simplifying:

$$\frac{hc}{\lambda_1} = -13.6\left(\frac{5}{36}\right).$$

Step 3: Relating the Wavelengths Using the given data for the Lyman series:

$$\lambda_1 = \lambda_{\text{Lyman}} \times \frac{36}{5}.$$

Substituting the given value:

$$\lambda_1 = 915 \times \frac{36}{5} = 6588\,\text{Å}.$$

Therefore, the longest wavelength of spectral lines in the Balmer series is 6588 Å.

Quick Tip

The longest wavelength in the Balmer series corresponds to the transition from $n = 3$ to $n = 2$. Use the energy level formula for hydrogen transitions to find precise wavelengths.

53: In a single slit experiment, a parallel beam of green light of wavelength $550\,\text{nm}$ passes through a slit of width $0.20\,\text{mm}$. The transmitted light is collected on a screen 100 cm away. The distance of first order minima from the central maximum will be $x \times 10^{-5}\,\text{m}$. The value of x is:
Answer: (275)
Solution:
Given data: - Wavelength of light, $\lambda = 550\,\text{nm} = 550 \times 10^{-9}\,\text{m}$ - Distance to the screen, $D = 100\,\text{cm} = 1\,\text{m}$ - Width of the slit, $d = 0.2\,\text{mm} = 0.2 \times 10^{-3}\,\text{m}$

The distance y to the first order minima in a single-slit diffraction pattern is given by:

$$y = \frac{\lambda D}{d}.$$

Substitution Substituting the given values:

$$y = \frac{550 \times 10^{-9} \times 1}{0.2 \times 10^{-3}}.$$

Calculation Simplifying:

$$y = \frac{550 \times 10^{-9} \times 10^{2}}{0.2 \times 10^{-3}} = \frac{550 \times 10^{-7}}{0.2 \times 10^{-3}}.$$

Further simplification:

$$y = \frac{550 \times 10^{-5}}{0.2} = 275 \times 10^{-5}\,\text{m}.$$

Therefore, the value of x is 275.

Quick Tip

In single-slit diffraction, the position of minima is determined by the formula $y = \frac{\lambda D}{d}$. Ensure to use consistent units when applying the formula.

54: A sonometer wire of resonating length $90\,\text{cm}$ has a fundamental frequency of $400\,\text{Hz}$ when kept under some tension. The resonating length of the wire with a fundamental frequency of $600\,\text{Hz}$ under the same tension is __________ cm.
Answer: (60)

Solution:
Given: - Initial resonating length, $L = 90\,\text{cm}$ - Initial fundamental frequency, $f_0 = 400\,\text{Hz}$ - New fundamental frequency, $f' = 600\,\text{Hz}$

Step 1: Relation Between Frequency and Length The fundamental frequency of a vibrating string is given by:

$$f_0 = \frac{v}{2L},$$

where: - v is the wave speed, - L is the length of the wire.

For the same tension, the wave speed v remains constant.

Step 2: Expressing New Length in Terms of Frequency Let the new resonating length be L' for the frequency f'. The new fundamental frequency is given by:

$$f' = \frac{v}{2L'}.$$

Dividing the two equations:

$$\frac{f'}{f_0} = \frac{L}{L'}.$$

Rearranging to find L':

$$L' = L \times \frac{f_0}{f'}.$$

Step 3: Substituting the Given Values Substituting the values:

$$L' = 90 \times \frac{400}{600}.$$

Simplifying:

$$L' = 90 \times \frac{2}{3} = 60\,\text{cm}.$$

Therefore, the new resonating length of the wire is $60\,\text{cm}$.

Quick Tip

The resonating length of a sonometer wire is inversely proportional to its fundamental frequency when tension remains constant.

55: A hollow sphere is rolling on a plane surface about its axis of symmetry. The ratio of rotational kinetic energy to its total kinetic energy is $\frac{x}{5}$. The value of x is __________.
Answer: (2)
Solution:
For a hollow sphere rolling on a plane surface, the total kinetic energy K_{total} consists of translational kinetic energy K_{trans} and rotational kinetic energy K_{rot}.

Step 1: Expressions for Kinetic Energy The translational kinetic energy is given by:

$$K_{\text{trans}} = \frac{1}{2}mv^2,$$

where m is the mass and v is the linear velocity of the center of mass.

The rotational kinetic energy about the axis of symmetry is given by:

$$K_{\text{rot}} = \frac{1}{2} I\omega^2,$$

where I is the moment of inertia and ω is the angular velocity.

For a hollow sphere:

$$I = \frac{2}{3} mR^2,$$

where R is the radius of the sphere.

Step 2: Relating Translational and Rotational Motion Since the sphere rolls without slipping:

$$v = R\omega.$$

Step 3: Substituting the Moment of Inertia The rotational kinetic energy becomes:

$$K_{\text{rot}} = \frac{1}{2}\left(\frac{2}{3} mR^2\right)\omega^2 = \frac{1}{3} mR^2\omega^2.$$

Using $v = R\omega$:

$$K_{\text{rot}} = \frac{1}{3} mv^2.$$

Step 4: Calculating the Total Kinetic Energy The total kinetic energy is:

$$K_{\text{total}} = K_{\text{trans}} + K_{\text{rot}} = \frac{1}{2} mv^2 + \frac{1}{3} mv^2.$$

Combining terms:

$$K_{\text{total}} = \frac{5}{6} mv^2.$$

Step 5: Finding the Ratio of Kinetic Energies The ratio of rotational kinetic energy to total kinetic energy is:

$$\text{Ratio} = \frac{K_{\text{rot}}}{K_{\text{total}}} = \frac{\frac{1}{3} mv^2}{\frac{5}{6} mv^2} = \frac{2}{5}.$$

Thus:

$$\frac{x}{5} = \frac{2}{5} \quad \Rightarrow \quad x = 2.$$

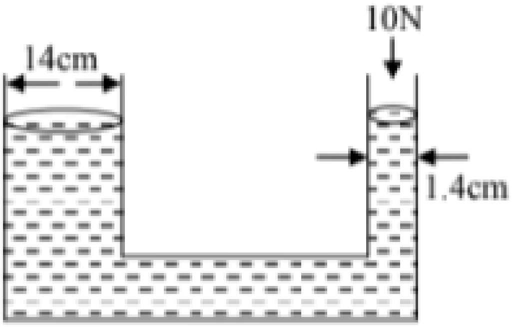
Therefore, the value of x is 2.

Quick Tip

For rolling motion, the total kinetic energy is the sum of translational and rotational kinetic energies. The ratio of these energies depends on the moment of inertia and rolling conditions.

56: A hydraulic press containing water has two arms with diameters as mentioned in the figure. A force of $10\,\text{N}$ is applied on the surface of water in the

thinner arm. The force required to be applied on the surface of water in the thicker arm to maintain equilibrium of water is ________ N.



Answer: (1000 N)
Solution:
Given data: - Force applied on the thinner arm, $F_2 = 10\,\text{N}$ - Diameter of the thinner arm, $d_2 = 1.4\,\text{cm}$ - Diameter of the thicker arm, $d_1 = 14\,\text{cm}$

Step 1: Calculating the Cross-Sectional Areas The cross-sectional area of each arm is given by:

$$A = \pi \left(\frac{d}{2}\right)^2.$$

For the thinner arm:

$$A_2 = \pi \left(\frac{1.4}{2}\right)^2 = \pi\,(0.7)^2\ \text{cm}^2.$$

For the thicker arm:

$$A_1 = \pi \left(\frac{14}{2}\right)^2 = \pi\,(7)^2\ \text{cm}^2.$$

Step 2: Applying Pascal's Law According to Pascal's law, the pressure exerted on both arms must be equal for equilibrium:

$$\frac{F_1}{A_1} = \frac{F_2}{A_2}.$$

Rearranging to find F_1:

$$F_1 = F_2 \times \frac{A_1}{A_2}.$$

Step 3: Substituting the Values Substituting the given values:

$$F_1 = 10 \times \frac{\pi(7)^2}{\pi(0.7)^2}.$$

Simplifying:

$$F_1 = 10 \times \frac{49}{0.49}.$$

Calculating:

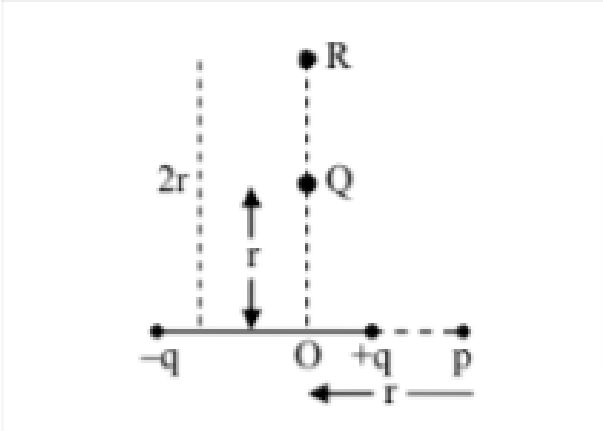
$$F_1 = 10 \times 100 = 1000\,\text{N}.$$

Therefore, the force required to be applied on the surface of water in the thicker arm is $1000\,\text{N}$.

Quick Tip

Pascal's law states that pressure applied to an enclosed fluid is transmitted undiminished to every part of the fluid and to the walls of its container.

57: The electric field at point P due to an electric dipole is E. The electric field at point R on the equatorial line will be $\frac{E}{x}$. The value of x is:



Answer: 16

Solution:

Given: - Electric field at point P on the axial line: $E_P = E = \frac{2Kp}{r^3}$, - Electric field at point R on the equatorial line: $E_R = \frac{Kp}{(2r)^3}$, where: - K is the Coulomb constant, - p is the dipole moment, - r is the distance from the dipole center to the point of observation.

Step 1: Calculate the electric field at R The electric field at point R on the equatorial line is given by:

$$E_R = \frac{Kp}{(2r)^3}.$$

Simplify $(2r)^3$:

$$E_R = \frac{Kp}{8r^3}.$$

Step 2: Compare the electric fields The electric field at P on the axial line is:

$$E_P = \frac{2Kp}{r^3}.$$

The electric field at R is related to E_P as:

$$E_R = \frac{E_P}{x}.$$

Substitute $E_P = \frac{2Kp}{r^3}$ and $E_R = \frac{Kp}{8r^3}$:

$$\frac{Kp}{8r^3} = \frac{\frac{2Kp}{r^3}}{x}.$$

Step 3: Solve for x Simplify the equation:

$$\frac{Kp}{8r^3} = \frac{2Kp}{xr^3}.$$

Cancel Kp and r^3 (as they are non-zero):

$$\frac{1}{8} = \frac{2}{x}.$$

Rearrange to solve for x:

$$x = 2 \times 8 = 16.$$

Thus, the value of x is 16.

Quick Tip

For electric dipoles, remember that the electric field along the axial line is twice as strong as that along the equatorial line at the same distance.

58: The maximum height reached by a projectile is $64\,\text{m}$. If the initial velocity is halved, the new maximum height of the projectile is __________ m.
Answer: 16
Solution:
The maximum height of a projectile is given by:

$$H_{\max} = \frac{u^2 \sin^2 \theta}{2g},$$

where: - u is the initial velocity, - θ is the angle of projection, - g is the acceleration due to gravity.

Step 1: Relationship between maximum heights If the initial velocity is halved, i.e., $u' = \frac{u}{2}$, the new maximum height $H_{2\max}$ can be expressed as:

$$\frac{H_{1\max}}{H_{2\max}} = \frac{u^2}{u'^2}.$$

Substitute $u' = \frac{u}{2}$:

$$\frac{H_{1\max}}{H_{2\max}} = \frac{u^2}{\left(\frac{u}{2}\right)^2}.$$

Step 2: Simplify the expression Simplify $\left(\frac{u}{2}\right)^2$:

$$\frac{H_{1\max}}{H_{2\max}} = \frac{u^2}{\frac{u^2}{4}} = 4.$$

Thus:

$$H_{2\max} = \frac{H_{1\max}}{4}.$$

Step 3: Calculate the new maximum height Substitute $H_{1\max} = 64\,\text{m}$:

$$H_{2\max} = \frac{64}{4} = 16\,\text{m}.$$

Therefore, the new maximum height of the projectile is $H_{2\max} = 16\,\text{m}$.

Quick Tip

The maximum height of a projectile is proportional to the square of the initial velocity. Halving the velocity reduces the height to one-fourth of the original.

59: A wire of resistance $20\,\Omega$ is divided into 10 equal parts. A combination of two parts is connected in parallel, and so on. Now the resulting pairs of parallel combinations are connected in series. The equivalent resistance of the final combination is __________ Ω.

Answer: 5

Solution:

Given: - Total resistance of the wire: $20\,\Omega$, - The wire is divided into 10 equal parts.

Step 1: Resistance of each part The resistance of each part is:

$$R_{\text{part}} = \frac{\text{Total resistance}}{\text{Number of parts}} = \frac{20\,\Omega}{10} = 2\,\Omega.$$

Step 2: Parallel combination of two parts Each pair of two parts is connected in parallel. The equivalent resistance of a parallel combination is:

$$R_{\text{parallel}} = \frac{R_{\text{part}}}{2} = \frac{2\,\Omega}{2} = 1\,\Omega.$$

Step 3: Total number of parallel combinations Since there are 10 parts and they are paired in groups of 2, the total number of parallel combinations is:

$$\text{Number of parallel combinations} = \frac{10}{2} = 5.$$

Step 4: Series combination of parallel pairs The 5 parallel combinations (each of $1\,\Omega$) are connected in series. The equivalent resistance of a series combination is:

$$R_{\text{eq}} = 5 \times R_{\text{parallel}} = 5 \times 1\,\Omega = 5\,\Omega.$$

Thus, the equivalent resistance of the final combination is $R_{\text{eq}} = 5\,\Omega$.

Quick Tip

When resistors are divided into equal parts and combined in parallel, remember that the equivalent resistance of the parallel combination is always less than the smallest individual resistance in the group.

60: The current in an inductor is given by $I = (3t + 8)$, where t is in seconds. The magnitude of the induced emf produced in the inductor is $12\,\text{mV}$. The self-inductance of the inductor is __________ mH.

Answer: 4

Solution:
The emf (ε) induced in an inductor is related to the rate of change of current and self-inductance (L) by the formula:

$$|\varepsilon| = L\frac{dI}{dt}.$$

Step 1: Determine the rate of change of current The given current is:

$$I = 3t + 8.$$

Differentiate I with respect to t:

$$\frac{dI}{dt} = 3\,\text{A/s}.$$

Step 2: Substitute the given values The magnitude of induced emf is $|\varepsilon| = 12\,\text{mV} = 12 \times 10^{-3}\,\text{V}$. Substituting into the formula:

$$12 \times 10^{-3} = L \cdot 3.$$

Step 3: Solve for L Rearrange to find L:

$$L = \frac{12 \times 10^{-3}}{3} = 4 \times 10^{-3}\,\text{H}.$$

Convert to millihenries:

$$L = 4\,\text{mH}.$$

Thus, the self-inductance of the inductor is $L = 4\,\text{mH}$.

Quick Tip

For inductors, the induced emf depends on how quickly the current changes. A steeper slope (dI/dt) produces a higher emf.

Chemistry

61: Match List-I with List-II.

List-I	**List-II**
(A) ICl	(I) T-Shape
(B) ICl_3	(II) Square pyramidal
(C) ClF_5	(III) Pentagonal bipyramidal
(D) IF_7	(IV) Linear

(1) (A)–(I), (B)–(IV), C–(III), D–(II)
(2) (A)–(I), (B)–(III), C–(II), D–(IV)
(3) (A)–(IV), (B)–(I), C–(II), D–(III)
(4) (A)–(IV), (B)–(III), C–(II), D–(I)

Answer: (3)
(A) - (IV), (B) - (I), (C) - (II), (D) - (III)

Solution:
To determine the matches, we analyze the molecular geometry of each compound based on the number of bond pairs and lone pairs of electrons around the central atom.

Step 1: Molecular geometry of ICl ICl consists of only two atoms (iodine and chlorine), forming a diatomic molecule. Its molecular geometry is linear. Thus:

$$\text{(A) ICl} \rightarrow \text{(IV) Linear.}$$

Step 2: Molecular geometry of ICl_3 ICl_3 has 3 bond pairs and 2 lone pairs around the iodine atom. According to the VSEPR theory, this results in a T-shaped molecular geometry. Thus:

$$\text{(B) } ICl_3 \rightarrow \text{(I) T-Shape.}$$

Step 3: Molecular geometry of ClF_5 ClF_5 has 5 bond pairs and 1 lone pair around the central chlorine atom. According to the VSEPR theory, this configuration forms a square pyramidal geometry. Thus:

$$\text{(C) } ClF_5 \rightarrow \text{(II) Square pyramidal.}$$

Step 4: Molecular geometry of IF_7 IF_7 has 7 bond pairs and no lone pairs around the iodine atom. This configuration corresponds to a pentagonal bipyramidal geometry. Thus:

$$\text{(D) } IF_7 \rightarrow \text{(III) Pentagonal bipyramidal.}$$

Final Matches

$$\text{(A) - (IV)}, \quad \text{(B) - (I)}, \quad \text{(C) - (II)}, \quad \text{(D) - (III)}.$$

Answer: (3)

Quick Tip

The VSEPR theory is a powerful tool to predict the geometry of molecules. The number of bond pairs and lone pairs around the central atom determines the molecular shape.

62: While preparing crystals of Mohr's salt, dil. H_2SO_4 is added to a mixture of ferrous sulphate and ammonium sulphate. Before dissolving this mixture in water, dil. H_2SO_4 is added here to:

(1) prevent the hydrolysis of ferrous sulphate
(2) prevent the hydrolysis of ammonium sulphate
(3) make the medium strongly acidic
(4) increase the rate of formation of crystals

Answer: (1)
Solution:
When preparing Mohr's salt, H_2SO_4 is added for the following reasons:

Step 1: Understanding the role of H_2SO_4 Ferrous sulphate is prone to hydrolysis in aqueous solutions, especially when exposed to atmospheric oxygen. The hydrolysis leads to the formation of ferric hydroxide, which can contaminate the Mohr's salt crystals.

Adding dilute sulphuric acid prevents the hydrolysis of ferrous sulphate by maintaining an acidic medium, which stabilizes the ferrous ions and prevents oxidation or decomposition.

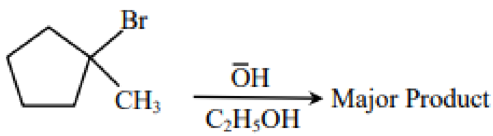
Step 2: Reason for not choosing other options - Option (2): Ammonium sulphate does not hydrolyze under normal conditions, so preventing its hydrolysis is irrelevant. - Option (3): Although adding H_2SO_4 makes the medium acidic, the main purpose is to prevent hydrolysis. - Option (4): The rate of formation of crystals is not directly influenced by H_2SO_4 but rather by cooling and saturation.

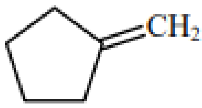
Conclusion The reason for adding H_2SO_4 is to prevent the hydrolysis of ferrous sulphate.

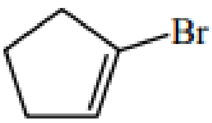
Quick Tip

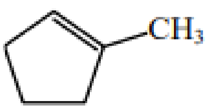
While preparing salts like Mohr's salt, it is crucial to maintain the stability of ferrous ions by adding acids to prevent unwanted hydrolysis or oxidation.

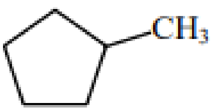
63: Identify the major product in the following reaction.



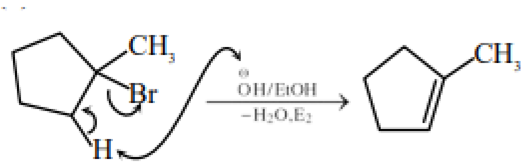
(1) 

(2) 

(3) 

(4) 

Answer: (3)
Solution:



This is an elimination reaction (E2 mechanism) where the hydroxide ion acts as a strong base. The steps of the reaction are as follows:

Step 1: Identify the base and leaving group In the presence of OH^- and EtOH, the bromine atom (Br) acts as the leaving group. The hydroxide ion abstracts a proton (H^+) from the adjacent carbon atom, resulting in the formation of a double bond.

Step 2: Formation of the double bond Since the reaction proceeds via the E2 mechanism, the elimination occurs in a single step. The base removes a -hydrogen atom (attached to the carbon next to the carbon with Br), and the Br^- leaves, resulting in the formation of a double bond between the and carbons.

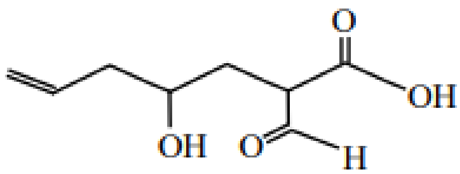
Step 3: Determine the major product The double bond forms between the -carbon and the -carbon (the one originally bonded to the bromine atom). Due to Zaitsev's rule, the most substituted alkene is the major product. Hence, the major product is cyclopentene with a methyl (CH_3) substituent.

Final Answer: (3) Cyclopentene with a CH_3 substituent.

Quick Tip

In E2 reactions, the base removes a -hydrogen, and the product follows Zaitsev's rule, favoring the formation of the more substituted alkene.

64: The nomenclature for the following compound is:



(1) 2-carboxy-4-hydroxyhept-6-enal
(2) 2-carboxy-4-hydroxyhept-7-enal
(3) 2-formyl-4-hydroxyhept-6-enoic acid
(4) 2-formyl-4-hydroxyhept-7-enoic acid

Answer: (3)
Solution:
To determine the IUPAC nomenclature, follow these steps:

Step 1: Identify the parent chain The parent chain consists of 7 carbon atoms (hept-)

with a double bond at the 6th carbon (hept-6-en-).

Step 2: Functional groups and priority The compound contains the following functional groups: 1. A formyl group ($-CHO$) at the 2nd carbon, 2. A hydroxyl group ($-OH$) at the 4th carbon, 3. A carboxylic acid group ($-COOH$) at the end of the chain.

The carboxylic acid group has the highest priority, so the chain is named as a derivative of "hept-6-enoic acid".

Step 3: Naming the substituents 1. The $-CHO$ group is named as "formyl" since it is a substituent and not the main functional group. 2. The $-OH$ group is named as "hydroxy".

Step 4: Combine the name The substituents and parent chain are combined in the order of their positions:

2-formyl-4-hydroxyhept-6-enoic acid.

Step 5: Validate the given options From the options provided, the name matches:

(3) 2-formyl-4-hydroxyhept-6-enoic acid.

Final Answer: (3)

Quick Tip

When naming compounds, always prioritize functional groups based on IUPAC rules. The carboxylic acid group takes precedence over aldehydes and alcohols in the nomenclature.

65: Given below are two statements: one is labeled as Assertion (A) and the other is labeled as Reason (R).
Assertion (A): NH_3 and NF_3 molecules have a pyramidal shape with a lone pair of electrons on the nitrogen atom. The resultant dipole moment of $NH3$ is greater than that of NF_3.
Reason (R): In $NH3$, the orbital dipole due to the lone pair is in the same direction as the resultant dipole moment of the $N-H$ bonds. F is the most electronegative element.
In the light of the above statements, choose the answer from the options given below:
(1) Both (A) and (R) are true, and (R) is the explanation of (A).
(2) (A) is false, but (R) is true.
(3) (A) is true, but (R) is false.
(4) Both (A) and (R) are true, but (R) is NOT the explanation of (A).

Answer: (1)
Solution:
The analysis of the molecules $NH3$ and $NF3$ is as follows:

Step 1: Structure and dipole moment of NH_3 - NH_3 has a pyramidal shape due to the presence of one lone pair on the nitrogen atom. - The dipole moments of the $N-H$ bonds and the lone pair point in the same direction, leading to a higher resultant dipole moment.

Step 2: Structure and dipole moment of NF_3 - NF_3 also has a pyramidal shape, but the $N-F$ bonds are highly electronegative. - The dipole moment of the lone pair on nitrogen is opposite to the resultant dipole moment of the $N-F$ bonds, which reduces the overall dipole moment.

Step 3: Comparison of dipole moments - The dipole moment of NH_3 is approximately 1.47 D, while that of NF_3 is approximately 0.80 D. - This confirms that $NH3$ has a greater dipole moment than $NF3$.

Step 4: Validating the statements - Assertion (A): True, because NH_3 has a higher dipole moment than NF_3. - Reason (R): True, as the lone pair's dipole in NH_3 aligns with the bond dipoles, while in NF_3, it opposes them. - (R) is the explanation of (A).

Final Answer: (1)

Quick Tip

The net dipole moment of a molecule depends on the vector sum of the bond dipoles and the dipole moment due to lone pairs. Opposing directions reduce the overall dipole moment.

66: Given below are two statements:

Statement I: On passing $HCl(g)$ through a saturated solution of $BaCl_2$, at room temperature, white turbidity appears.

Statement II: When $HCl(g)$ is passed through a saturated solution of $NaCl$, sodium chloride is precipitated due to the common ion effect.

In the light of the above statements, choose the most appropriate answer from the options given below:

(1) Statement I is but Statement II is in
(2) Both Statement I and Statement II are in
(3) Statement I is inbut Statement II is
(4) Both Statement I and Statement II are

Answer: (1)

Solution:

To determine the correctness of the statements, consider the solubility and precipitation behavior of $BaCl2$ and $NaCl$ in the presence of $HCl(g)$:

Step 1: Analyze Statement I When $HCl(g)$ is bubbled through a saturated solution of $BaCl2$, the concentration of Cl^- ions increases significantly due to the dissociation of HCl. The increased Cl^- ions reduce the solubility of $BaCl_2$ through the common ion effect, causing the precipitation of $BaCl2$ as white turbidity.

Statement I is correct.

Step 2: Analyze Statement II Sodium chloride ($NaCl$) is highly soluble in water. Passing $HCl(g)$ through a saturated solution of $NaCl$ does not lead to precipitation because the solubility of $NaCl$ is not significantly affected by the common ion effect under these conditions.

Statement II is incorrect.

Step 3: Conclusion From the above analysis: - Statement I is correct, but Statement II is incorrect.

Final Answer: (1)

Quick Tip

The common ion effect reduces the solubility of ionic compounds in solutions containing a common ion. However, highly soluble salts like $NaCl$ are not significantly affected.

67:

The metal atom present in the complex $MABXL$ (where A, B, X, and L are unidentate ligands and M is metal) involves sp^3 hybridization. The number of geometrical isomers exhibited by the complex is:

(1) 4
(2) 0
(3) 2
(4) 3

Answer: (2)
Solution:

Step 1: Understanding sp^3 hybridization In a complex with sp^3 hybridization, the geometry is tetrahedral. Tetrahedral complexes do not exhibit geometrical isomerism because all positions around the central atom are equivalent in three-dimensional space.

Step 2: Analysis of the given complex The complex $MABXL$ has four unidentate ligands arranged tetrahedrally around the metal center. Since the tetrahedral geometry does not allow for distinct arrangements of ligands that result in different spatial configurations, geometrical isomerism is not possible.

Conclusion The number of geometrical isomers for the complex $MABXL$ is:

$$0.$$

Final Answer: (2)

Quick Tip

Geometrical isomerism is only possible in complexes with square planar or octahedral geometries. Tetrahedral complexes do not exhibit this type of isomerism.

68: Match List-I with List-II.

List-I (Pair of Compounds)	List-II (Isomerism)
(A) *n*-propanol and isopropanol	(I) Metamerism
(B) Methoxypropane and ethoxyethane	(II) Chain Isomerism
(C) Propanone and propanal	(III) Position Isomerism
(D) Neopentane and isopentane	(IV) Functional Isomerism

(1) (A)–(III), (B)–(I), (C)–(IV), (D)–(II)
(2) (A)–(II), (B)–(I), (C)–(IV), (D)–(III)
(3) (A)–(I), (B)–(III), (C)–(IV), (D)–(II)
(4) (A)–(IV), (B)–(I), (C)–(III), (D)–(II)

Answer: (4)
Solution:
To determine the matches, we analyze the isomeric relationships of the compounds in List-I:

Step 1: Analyze (A) *n*-propanol and isopropanol - *n*-propanol ($CH_3CH_2CH_2OH$) and isopropanol ($CH_3CHOHCH_3$) differ in their functional group positions. This is an example of functional isomerism.

$$\text{(A) - (IV)}$$

Step 2: Analyze (B) Methoxypropane and ethoxyethane - Methoxypropane ($CH_3OCH_2CH_2CH_3$) and ethoxyethane ($CH_3CH_2OCH_2CH_3$) differ in the arrangement of the ether groups. This is an example of metamerism.

$$\text{(B) - (I)}$$

Step 3: Analyze (C) Propanone and propanal - Propanone (CH_3COCH_3) and propanal ($CH3CH2CHO$) have different functional groups (ketone vs. aldehyde). This is an example of functional isomerism.

$$\text{(C) - (III)}$$

Step 4: Analyze (D) Neopentane and isopentane - Neopentane ($C(CH_3)_4$) and isopentane ($CH_3CH(CH_3)CH_2CH_3$) differ in the arrangement of their carbon chains. This is an example of chain isomerism.

$$\text{(D) - (II)}$$

Final Matches

$$\text{(A) - (IV)}, \quad \text{(B) - (I)}, \quad \text{(C) - (III)}, \quad \text{(D) - (II)}.$$

Answer: (4)

Quick Tip

Isomerism can be classified based on structural differences (e.g., chain, position, and functional isomerism) or spatial arrangement differences (stereoisomerism). Analyze the molecular structure carefully to determine the type.

69: The quantity of silver deposited when one coulomb charge is passed through $AgNO_3$ solution:

(1) 0.1 g atom of silver
(2) 1 chemical equivalent of silver
(3) 1 g of silver
(4) 1 electrochemical equivalent of silver

Answer: (4)
Solution:
The amount of a substance deposited during electrolysis is determined using Faraday's laws of electrolysis. The formula is:

$$W = ZIt,$$

where: - W is the mass of the substance deposited, - Z is the electrochemical equivalent of the substance, - I is the current passed, and - t is the time for which the current is passed.

Step 1: Relating charge to electrochemical equivalent We know that:

$$Q = It,$$

where Q is the total charge passed through the solution. Substituting this into the equation for W, we get:

$$W = ZQ.$$

Step 2: Deposition of silver For one coulomb of charge ($Q = 1\,\text{C}$), the mass of silver deposited is directly proportional to the electrochemical equivalent (Z) of silver. Thus:

$$W = ZQ = (\text{electrochemical equivalent of silver}).$$

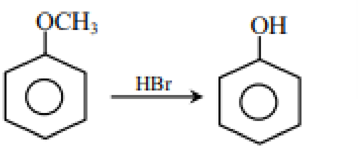
Step 3: Conclusion The quantity of silver deposited when one coulomb of charge is passed is equal to the electrochemical equivalent of silver. This matches the given option.

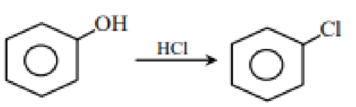
Final Answer: (4)

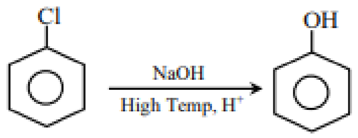
Quick Tip

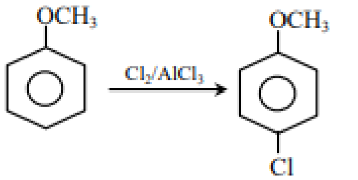
To calculate the mass deposited during electrolysis, always use $W = ZIt$, where Z is the electrochemical equivalent. For one coulomb of charge, $W = Z$.

70: Which one of the following reactions is NOT possible?

(1) 

(2) 

(3) 



(4)

Answer: (2)
Solution:
Step 1: Analyze each reaction
1. **Reaction (1):**
The reaction involves the cleavage of the ether bond ($C - OCH_3$) by HBr, producing phenol ($C_6H_5 - OH$). This reaction is possible due to the nucleophilic substitution mechanism.

2. **Reaction (2):**
The reaction involves the conversion of phenol ($C6H5 - OH$) to chlorobenzene ($C_6H_5 - Cl$) by HCl. However, this reaction is **NOT** possible because the hydroxyl group in phenol is directly attached to the benzene ring, and it does not undergo nucleophilic substitution to form $C_6H_5 - Cl$. The lone pair on oxygen in phenol makes the $-OH$ group resistant to substitution by HCl.

3. **Reaction (3):**
The reaction involves the hydrolysis of chlorobenzene ($C_6H_5 - Cl$) under high temperature and pressure in the presence of $NaOH$. This reaction is possible via nucleophilic aromatic substitution, producing phenol ($C6H5 - OH$).

4. **Reaction (4):**
The reaction involves the electrophilic substitution of anisole ($C_6H_5 - OCH_3$) with chlorine in the presence of $AlCl3$. This reaction is possible, producing a mixture of ortho and para substituted products.

Step 2: Conclusion Among the given reactions, only Reaction (2) is not possible because phenol does not undergo nucleophilic substitution with HCl to form chlorobenzene.

Final Answer: (2)

Quick Tip

Phenol ($C6H5 - OH$) does not react with HCl to form chlorobenzene because the $-OH$ group is strongly bonded to the benzene ring, making nucleophilic substitution reactions difficult.

71: Given below are two statements:
Statement I: The metallic radius of Na is 1.86 Å, and the ionic radius of Na^+ is lesser than 1.86 Å.
Statement II: Ions are always smaller in size than the corresponding elements.

In the light of the above statements, choose the answer from the options given below:

(1) Statement I is but Statement II is false
(2) Both Statement I and Statement II are true
(3) Both Statement I and Statement II are false
(4) Statement I is inbut Statement II is true

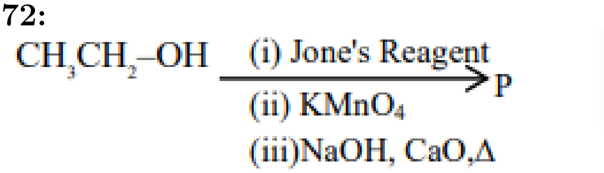
Answer: (1)
Solution:
Step 1: Analysis of Statement I
- The metallic radius of Na (neutral sodium atom) is 1.86 Å.
- The ionic radius of Na^+ is smaller than the neutral atom because Na^+ has one less electron, resulting in reduced electron-electron repulsion and greater effective nuclear charge on the remaining electrons. - Thus, **Statement I is correct**.

Step 2: Analysis of Statement II - While cations (Na^+) are always smaller than their corresponding neutral atoms, anions (e.g., Cl^-) are larger than their corresponding neutral atoms due to increased electron-electron repulsion in the outer shell.
- Hence, ions are **not always smaller** than the corresponding elements.
- Thus, **Statement II is false**.

Step 3: Conclusion - Statement I is correct, but Statement II is false.
Final Answer: (1)

> **Quick Tip**
>
> Cations are smaller than their neutral atoms due to the loss of electrons, while anions are larger than their neutral atoms because of increased electron-electron repulsion.

72:



Consider the above reaction sequence and identify the major product P.

(1) Methane
(2) Methanal
(3) Methoxymethane
(4) Methanoic acid

Answer: (1)
Solution:
The reaction sequence is as follows:

Step 1: Oxidation using Jones Reagent - Jones reagent ($CrO_3 + H_2SO_4$) oxidizes primary alcohols (CH_3CH_2OH) to carboxylic acids. Ethanol (CH_3CH_2OH) is oxidized to acetic acid (CH_3COOH).

$$CH_3CH_2OH- > [JonesReagent]CH_3COOH$$

Step 2: Oxidation using $KMnO_4$ - The acetic acid (CH_3COOH) is further oxidized to carbonic acid (H_2CO_3) by $KMnO_4$. Carbonic acid is unstable and decomposes to CO_2 and water.

$$CH_3COOH- > [KMnO_4]H_2CO_3- > CO_2 + H_2O$$

Step 3: Decarboxylation using Soda Lime - The carbon dioxide (CO_2) reacts with soda lime ($NaOH + CaO$) to form methane (CH_4) via decarboxylation.

$$CO_2 + NaOH- > CH_4 + Na_2CO_3$$

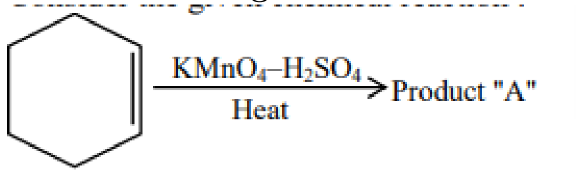
Final Product The major product P is methane (CH_4).

Final Answer: (1)

Quick Tip

Soda lime decarboxylation is a common reaction for converting carboxylic acids or their derivatives to alkanes by removing the $CO2$ group.

73: Consider the given chemical reaction:



Product "A" is:

(1) Picric acid
(2) Oxalic acid
(3) Acetic acid
(4) Adipic acid

Answer: (4)
Solution:
The reaction of cyclohexane with potassium permanganate ($KMnO_4$) in an acidic medium (H_2SO_4) and heat results in the oxidation of cyclohexane to adipic acid.

Step 1: Oxidation of cyclohexane - Cyclohexane (C_6H_{12}) undergoes oxidation in the presence of $KMnO_4$ and H_2SO_4. - The $KMnO_4$ oxidizes the $-CH_2-$ groups in cyclohexane to carboxylic acid ($-COOH$) groups. - This results in the formation of adipic acid ($HOOC-(CH_2)_4-COOH$).

$$C_6H_{12}->[KMnO_4, H_2SO_4, Heat]HOOC-(CH_2)_4-COOH(AdipicAcid)$$

Step 2: Conclusion The product A is adipic acid.

Final Answer: (4)

Quick Tip

Oxidation reactions with $KMnO4$ in acidic conditions are effective for converting cyclic alkanes or alkenes into di-carboxylic acids.

74: For the electrochemical cell:

$$\mathrm{M}—\mathrm{M}^{2+}||\mathrm{X}—\mathrm{X}^{2-}$$

If:

$$E^\circ(\mathrm{M}^{2+}/\mathrm{M}) = 0.46\,\mathrm{V}, \quad E^\circ(\mathrm{X}/\mathrm{X}^{2-}) = 0.34\,\mathrm{V},$$

Which of the following is correct?

(1) $E_{\text{cell}} = -0.80\,\mathrm{V}$
(2) $M + X^{2-} -> M^{2+} + X^{2-}$ is a spontaneous reaction
(3) $M^{2+} + X^{2-} -> M + X$ is a spontaneous reaction
(4) $E_{\text{cell}} = 0.80\,\mathrm{V}$

Answer: (3)
Solution:
The standard cell potential E°_{cell} is calculated as:

$$E^\circ_{\text{cell}} = E^\circ_{\text{cathode}} - E^\circ_{\text{anode}}.$$

Step 1: Identify the anode and cathode - $E^\circ(\mathrm{M}^{2+}/\mathrm{M}) = 0.46\,\mathrm{V}$, - $E^\circ(\mathrm{X}/\mathrm{X}^{2-}) = 0.34\,\mathrm{V}$.

Since M^{2+}/M has a higher reduction potential, it will act as the cathode, and X/X^{2-} will act as the anode.

Step 2: Calculate E°_{cell}

$$E^\circ_{\text{cell}} = E^\circ_{\text{cathode}} - E^\circ_{\text{anode}} = 0.34 - 0.46 = -0.12\,\mathrm{V}.$$

Step 3: Analyze the spontaneity of the reaction - Since E°_{cell} is negative, the reaction will proceed in the reverse direction (the reverse reaction is spontaneous). - The spontaneous reaction is:

$$M^{2+} + X^{2-} -> M + X.$$

Step 4: Validate the options - Option (1): Incorrect, as $E_{\text{cell}} = -0.12\,\mathrm{V}$, not $-0.80\,\mathrm{V}$. - Option (2): Incorrect, as $M + X^{2-} -> M^{2+} + X^{2-}$ is not spontaneous. - Option (3):

Correct, as $M^{2+} + X^{2-} -> M + X$ is the spontaneous reaction. - Option (4): Incorrect, as $E_{\text{cell}} = -0.12\,\text{V}$, not $0.80\,\text{V}$.

Final Answer: (3)

Quick Tip

The direction of spontaneity in an electrochemical reaction can be determined by the sign of E°_{cell}. A negative E°_{cell} indicates that the reverse reaction is spontaneous.

75: The number of moles of methane required to produce 11 g of $CO_2(g)$ after complete combustion is:
(Given molar mass of methane in g mol^{-1}: 16)

(1) 0.75
(2) 0.25
(3) 0.35
(4) 0.5

Answer: (2)
Solution:
The general combustion reaction of alkanes is:

$$C_nH_{2n+2} + \frac{3n+1}{2}O2 -> nCO2 + (n+1)H2O.$$

For methane (CH_4), the specific balanced equation is:

$$CH_4 + 2O_2 -> CO_2 + 2H_2O.$$

Step 1: Molar mass of CO_2 The molar mass of CO_2 is:

$$\text{Molar mass of } CO_2 = 12 + (2 \times 16) = 44\,\text{g/mol}.$$

Step 2: Calculate moles of CO_2 The number of moles of CO_2 in 11 g is:

$$\text{Moles of } CO_2 = \frac{\text{Mass of } CO2}{\text{Molar mass of } CO_2} = \frac{11}{44} = 0.25\,\text{mol}.$$

Step 3: Relate CH_4 to CO_2 From the balanced reaction: - 1 mole of CH_4 produces 1 mole of CO_2.
Therefore, to produce 0.25 moles of CO_2, the moles of CH_4 required are:

$$\text{Moles of } CH4 = 0.25\,\text{mol}.$$

Step 4: Mass of CH_4 The mass of 0.25 moles of CH_4 can be verified as:

$$\text{Mass of } CH_4 = \text{Moles of } CH_4 \times \text{Molar mass of } CH_4 = 0.25 \times 16 = 4\,\text{g}.$$

Conclusion The number of moles of methane required to produce 11 g of CO_2 is $0.25\,\text{mol}$.

Final Answer: (2)

Quick Tip

To solve combustion problems, always start with the balanced equation, calculate moles using molar masses, and use stoichiometric ratios to relate reactants and products.

76: The number of complexes from the following with no electrons in the t_2 orbital is:

$$TiCl_4, [MnO_4]^-, [FeO_4]^{2-}, [FeCl_4]^-, [CoCl_4]^{2-}$$

(1) 3
(2) 1
(3) 4
(4) 2

Answer: (1)
Solution:
To determine the number of complexes with no electrons in the t_2 orbital, analyze each complex and its oxidation state, electronic configuration, and crystal field splitting.

Step 1: Analyze each complex 1. $TiCl4$: - Oxidation state of Ti: $+4$ (Ti^4+). - Electronic configuration of Ti^4+: $3d^0$. - No electrons in t_2 orbitals.

2. $[MnO4]^-$: - Oxidation state of Mn: $+7$ (Mn^7+). - Electronic configuration of Mn^7+: $3d^0$. - No electrons in t_2 orbitals.

3. $[FeO4]^{2-}$: - Oxidation state of Fe: $+6$ (Fe^6+). - Electronic configuration of Fe^6+: $3d^0$. - No electrons in t_2 orbitals.

4. $[FeCl4]^-$: - Oxidation state of Fe: $+2$ (Fe^2+). - Electronic configuration of Fe^2+: $3d^6$. - t_2 orbitals are populated with electrons ($t_2^3e^3$).

5. $[CoCl4]^{2-}$: - Oxidation state of Co: $+2$ (Co^2+). - Electronic configuration of Co^2+: $3d^7$. - t_2 orbitals are populated with electrons ($t_2^4e^3$).

Step 2: Count complexes with no t_2 electrons - $TiCl4$, $[MnO4]^-$, and $[FeO4]^{2-}$ have no electrons in t_2 orbitals. - $[FeCl4]^-$ and $[CoCl4]^{2-}$ have electrons in t_2 orbitals.

Conclusion The number of complexes with no electrons in the t_2 orbital is:
3 (TiCl4, [MnO4]$^-$, $[FeO4]^{2-}$).Final Answer: (1)

Quick Tip

The t_2 orbital is unoccupied in complexes with a $3d^0$ electronic configuration, typically for ions with high oxidation states.

77: The number of ions from the following that have the ability to liberate hydrogen from a dilute acid is ________: Ti^{2+}, Cr^{2+}, V^{2+}

(1) 0
(2) 2
(3) 3
(4) 1

Answer: (3)
Solution:
To determine the number of ions that can liberate hydrogen from a dilute acid, we need to analyze their reducing abilities. Strong reducing agents can donate electrons to H^+, reducing it to H_2.

Step 1: Evaluate Ti^{2+} - Titanium(II) (Ti^{2+}) has a strong tendency to get oxidized to Ti^{3+}, making it a strong reducing agent. - It can react with H^+ from dilute acids to liberate $H2$.

Step 2: Evaluate Cr^{2+} - Chromium(II) (Cr^{2+}) is a strong reducing agent and can be oxidized to Cr^{3+}. - It reacts with H^+ from dilute acids to liberate $H2$.

$$2Cr^{2+}(aq) + 2H^+(aq) -> 2Cr^{3+}(aq) + H2(g)$$

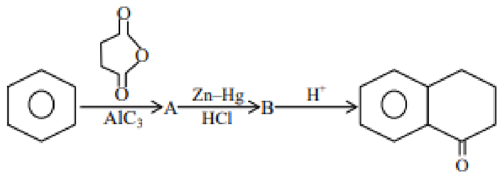
Step 3: Evaluate V^{2+} - Vanadium(II) (V^{2+}) is also a strong reducing agent and can be oxidized to V^{3+}. - It reacts with H^+ from dilute acids to liberate $H2$.

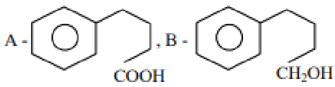
Conclusion All three ions (Ti^{2+}, Cr^{2+}, and V^{2+}) can liberate $H2$ from dilute acids.
Final Answer: (3)

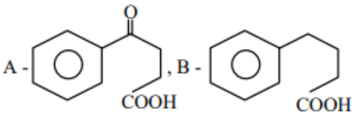
Quick Tip

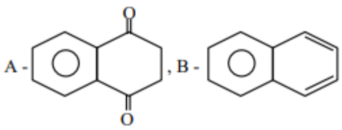
Reducing agents with low oxidation states tend to liberate $H2$ from dilute acids by reducing H^+ ions. Check the tendency of each ion to get oxidized.

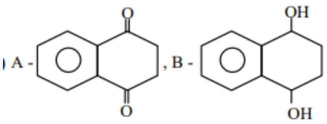
78: Identify A and B in the given chemical reaction sequence : -



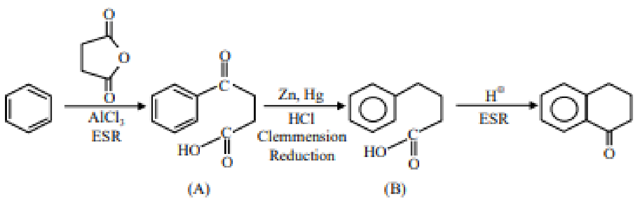
(1) 

(2) 

(3) 

(4) 

Answer: (3)
Solution:



The given sequence involves the following steps:

Step 1: Friedel-Crafts Acylation - Benzene (C_6H_6) reacts with acetyl chloride (CH_3COCl) in the presence of $AlCl_3$, an electrophilic catalyst. - This forms acetophenone ($C_6H_5COCH_3$) as compound A.

$$C_6H_6 + CH_3COCl- > [AlCl_3]C_6H_5COCH_3(A).$$

Step 2: Clemmensen Reduction - Acetophenone ($C_6H_5COCH_3$) undergoes Clemmensen reduction using zinc amalgam ($Zn - Hg$) and hydrochloric acid (HCl). - The carbonyl group ($-CO$) is reduced to a methylene group ($-CH_2-$), resulting in ethylbenzene ($C6H5CH2CH3$) as compound B.

$$C6H5COCH3- > [Zn - Hg, HCl]C_6H_5CH_2CH_3(B).$$

Step 3: Acidic Oxidation - Ethylbenzene reacts with H^+ under oxidation conditions to form acetophenone again. This completes the reaction cycle.

Final Products - $A = C_6H_5COCH_3$ (Acetophenone). - $B = C_6H_5CH_2CH_3$ (Ethylbenzene).

Conclusion The identification of A and B is given in option (3).

Final Answer: (3)

Quick Tip

Friedel-Crafts acylation is useful for introducing carbonyl groups, while Clemmensen reduction selectively reduces carbonyl groups to methylene groups.

79: The statements from the following are:

(A) The decreasing order of atomic radii of group 13 elements is $Tl > In > Ga > Al > B$.
(B) Down the group 13, electronegativity decreases from top to bottom.
(C) Al dissolves in dilute HCl and liberates $H2$, but concentrated $HNO3$ renders Al passive by forming a protective oxide layer on the surface.
(D) All elements of group 13 exhibit a highly stable $+1$ oxidation state.
(E) Hybridization of Al in $[Al(H2O)_6]^{3+}$ ion is sp^3d^2.

Choose the answer from the options given below:
(1) (C) and (E) only
(2) (A), (C) and (E) only
(3) (A), (B), (C) and (E) only
(4) (A) and (C) only

Answer: (1)
Solution:
Step 1: Analyze each statement
1. Statement (A): - The order of atomic radii in group 13 elements is $Tl > In > Al > Ga > B$, not $Tl > In > Ga > Al > B$. - **This statement is incorrect.**

2. Statement (B): - Down the group, electronegativity generally decreases. However, due to the presence of d-electrons in Ga, In, and Tl, their electronegativity values do not follow a simple trend. - **This statement is incorrect.**

3. Statement (C): - Al reacts with dilute HCl to release H_2:

$$2Al + 6HCl- > 2AlCl_3 + 3H_2.$$

- Concentrated HNO_3 forms a passive oxide layer on Al, protecting it from further reaction. - **This statement is correct.**

4. Statement (D): - In group 13, the $+3$ oxidation state is more stable than $+1$. The $+1$ state is exhibited by Tl due to the inert pair effect, but it is not highly stable for all

elements. - **This statement is incorrect.**

5. Statement (E): - In $[Al(H_2O)_6]^{3+}$, the hybridization of Al is sp^3d^2, corresponding to an octahedral geometry. - **This statement is correct.**

Step 2: Conclusion The statements are (C) and (E).

Final Answer: (1)

Quick Tip

When analyzing periodic trends and chemical properties, always cross-check with known exceptions, such as the inert pair effect and irregularities in electronegativity trends.

80: Coagulation of egg, on heating, is because of:
(1) Denaturation of protein occurs
(2) The secondary structure of protein remains unchanged
(3) Breaking of the peptide linkage in the primary structure of protein occurs
(4) Biological property of protein remains unchanged

Answer: (1)
Solution:
Coagulation of an egg upon heating is due to the denaturation of protein. Denaturation involves: - The disruption of the secondary, tertiary, and quaternary structures of the protein. - The breaking of weak bonds, such as hydrogen bonds and hydrophobic interactions, while leaving the primary structure (peptide bonds) intact.

Key Explanation: - Denaturation alters the protein's native conformation, leading to coagulation (solidification). - The biological properties of the protein are lost due to denaturation.

InOptions: (2) The secondary structure of the protein does not remain unchanged; it is disrupted during denaturation.
(3) The peptide bonds in the primary structure are not broken during coagulation.
(4) The biological properties of the protein do not remain unchanged; they are lost due to denaturation.

Final Answer: (1)

Quick Tip

Denaturation of proteins is caused by heat, strong acids, or bases and disrupts higher-order structures, leading to loss of solubility and biological function.

81: Combustion of 1 mole of benzene is expressed as:

$$C_6H_6(l) + \frac{15}{2}O_2(g) -> 6CO_2(g) + 3H_2O(l)$$

The standard enthalpy of combustion of 2 mol of benzene is $-x$ kJ. Calculate the value of x given the following data:

Standard enthalpy of formation of $C_6H_6(l)$: 48.5 kJ mol^{-1}.
Standard enthalpy of formation of $CO_2(g)$: -393.5 kJ mol^{-1}.
Standard enthalpy of formation of $H_2O(l)$: -286 kJ mol^{-1}.

Answer: $6535\,$**kJ**
Solution:
The enthalpy change for the combustion reaction can be calculated using Hess's law:

$$\Delta H = \sum \Delta H_f(\text{products}) - \sum \Delta H_f(\text{reactants}).$$

Step 1: Write the given reaction The reaction for 1 mole of benzene is:

$$C_6H_6(l) + \frac{15}{2}O_2(g) -> 6CO_2(g) + 3H_2O(l).$$

Step 2: Calculate the enthalpy change for 1 mole of benzene Using the standard enthalpies of formation:

$$\Delta H_f(CO_2(g)) = -393.5\,\text{kJ/mol},$$

$$\Delta H_f(H_2O(l)) = -286\,\text{kJ/mol},$$

$$\Delta H_f(C_6H_6(l)) = 48.5\,\text{kJ/mol}.$$

For the products:

$$\Delta H_f(\text{products}) = [6 \times (-393.5)] + [3 \times (-286)].$$

$$\Delta H_f(\text{products}) = -2361 - 858 = -3219\,\text{kJ/mol}.$$

For the reactants:

$$\Delta H_f(\text{reactants}) = [1 \times 48.5] + \left(\frac{15}{2} \times 0\right).$$

$$\Delta H_f(\text{reactants}) = 48.5\,\text{kJ/mol}.$$

The enthalpy change for the combustion of 1 mole of benzene is:

$$\Delta H = \Delta H_f(\text{products}) - \Delta H_f(\text{reactants}),$$

$$\Delta H = -3219 - 48.5 = -3267.5\,\text{kJ/mol}.$$

Step 3: Calculate for 2 moles of benzene For 2 moles of benzene:

$$\Delta H = 2 \times (-3267.5) = -6535\,\text{kJ}.$$

Final Answer: $x = 6535\,\text{kJ}$.

Quick Tip

Use Hess's law and the given standard enthalpies of formation to calculate the enthalpy change for a reaction. Always ensure that stoichiometric coefficients are included in the calculations.

82:

The fusion of chromite ore with sodium carbonate in the presence of air leads to the formation of products A and B along with the evolution of CO_2. The sum of spin-only magnetic moment values of A and B is ____ B.M. (Nearest integer).

(Given atomic numbers: $C = 6, Na = 11, O = 8, Fe = 26, Cr = 24$)

Answer: (6)

Solution:

The reaction for the fusion of chromite ore is:

$$4FeCr_2O_4 + 8Na_2CO_3 + 7O_2 -> 8Na_2CrO_4 + 2Fe2O_3 + 8CO_2.$$

Here: - $A = Na_2CrO_4$ - $B = Fe_2O_3$

Step 1: Calculate the spin-only magnetic moment The spin-only magnetic moment is given by:

$$\mu_s = \sqrt{n(n+2)}\,\mu_B,$$

where n is the number of unpaired electrons.

For Na_2CrO_4: - Chromium in Na_2CrO_4 is in the +6 oxidation state (Cr^{6+}). - The electronic configuration of Cr^{6+} is $[Ar]3d^0$. - $n = 0$, so $\mu_s = 0\,\mu_B$.

For Fe_2O_3: - Iron in Fe_2O_3 is in the +3 oxidation state (Fe^{3+}). - The electronic configuration of Fe^{3+} is $[Ar]3d^5$. - $n = 5$, so:

$$\mu_s = \sqrt{5(5+2)} = \sqrt{35} \approx 5.9\,\mu_B.$$

Step 2: Sum of magnetic moments The sum of spin-only magnetic moments of A and B is:

$$\mu_{\text{total}} = 0 + 5.9 = 5.9\,\mu_B.$$

Rounding to the nearest integer:

$$\mu_{\text{total}} = 6\,\mu_B.$$

Final Answer: $6\,\text{B.M.}$

Quick Tip

The spin-only magnetic moment is calculated using the number of unpaired electrons. For ions with $n = 0$, the magnetic moment is zero, while $n = 5$ corresponds to the highest magnetic moment for $3d$ elements.

83: X of ethanamine was subjected to reaction with $NaNO_2/HCl$ followed by hydrolysis to liberate N_2 and HCl. The HCl generated was completely neutralized by 0.2 moles of $NaOH$. X is ____g.
Answer: (9)
Solution:
The reaction sequence is as follows:

Step 1: Reaction of ethanamine with $NaNO_2$ and HCl Ethanamine ($CH_3CH_2NH_2$) reacts with $NaNO_2$ and HCl to form an unstable diazonium salt ($CH_3CH_2N2^+Cl^-$).

$$CH_3CH_2NH_2 + NaNO_2 + HCl- > CH_3CH_2N2^+Cl^- + NaCl + H_2O.$$

Step 2: Hydrolysis of the diazonium salt The diazonium salt decomposes upon hydrolysis, liberating N_2, HCl, and ethanol (CH_3CH_2OH).

$$CH_3CH2N2^+Cl^- + H_2O- > CH3CH2OH + N_2 + HCl.$$

Step 3: Calculation of the mass of ethanamine - The reaction generates 0.2 moles of HCl, which neutralizes 0.2 moles of $NaOH$. - From the stoichiometry of the reaction, 1 mole of ethanamine produces 1 mole of HCl. - Therefore, the moles of ethanamine ($CH_3CH_2NH_2$) required are 0.2 moles.

The molar mass of ethanamine is:

$$\text{Molar mass of } CH_3CH_2NH_2 = 12 + 3 + 12 + 2 + 14 + 1 = 44\,\text{g/mol}.$$

The mass of ethanamine is:

$$\text{Mass} = \text{Moles} \times \text{Molar mass} = 0.2 \times 44 = 8.8(Nearest\ Integer\ to\ 9)\,\text{g}.$$

Final Answer: $X = 9\,\text{g}$.

Quick Tip

For stoichiometric calculations, use the mole ratio from the balanced reaction and the molar mass of the compound to determine the required mass.

84: In an atom, the total number of electrons having quantum numbers $n = 4$, $|m_l| = 1$, and $m_s = -\frac{1}{2}$ is:
Answer: (6)
Solution:
The quantum numbers are defined as follows: - $n = 4$: Principal quantum number. - $|m_l| = 1$: Absolute value of the magnetic quantum number. - $m_s = -\frac{1}{2}$: Spin quantum number.

Step 1: Determine the possible values of l and m_l For $n = 4$, the possible values of the azimuthal quantum number l are:

$$l = 0, 1, 2, 3.$$

For each l, the possible values of m_l are as follows: - $l = 0$: $m_l = 0$ - $l = 1$: $m_l = -1, 0, +1$ - $l = 2$: $m_l = -2, -1, 0, +1, +2$ - $l = 3$: $m_l = -3, -2, -1, 0, +1, +2, +3$

From the given condition $|m_l| = 1$, the possible values of m_l are:

$$m_l = -1 \text{ or } +1.$$

Step 2: Count the orbitals corresponding to $n = 4$ and $|m_l| = 1$ - For $l = 1$: $m_l = \pm 1$ (2 orbitals). - For $l = 2$: $m_l = \pm 1$ (2 orbitals). - For $l = 3$: $m_l = \pm 1$ (2 orbitals).

The total number of orbitals with $|m_l| = 1$ is:

$$2 + 2 + 2 = 6 \text{ orbitals}.$$

Step 3: Assign electrons with $m_s = -\frac{1}{2}$ Each orbital can hold one electron with $m_s = -\frac{1}{2}$. Thus, the total number of electrons is:

$$6 \text{ electrons}.$$

Final Answer: 6

Quick Tip

The spin quantum number $m_s = -\frac{1}{2}$ specifies one electron per orbital. When counting orbitals, ensure to match the absolute value of m_l with the given constraints.

85: Using the given figure, the ratio of R_f values of sample A and sample C is $x \times 10^{-2}$. Value of x is:

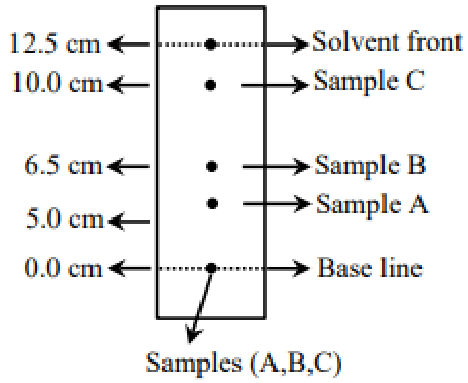


Fig : Paper chromatography of Samples

Answer: (50)

Solution:

The R_f value in chromatography is given by:

$$R_f = \frac{\text{Distance traveled by the sample spot}}{\text{Distance traveled by the solvent front}}.$$

Step 1: Calculate R_f for sample A For sample A:

$$R_f(A) = \frac{\text{Distance traveled by sample } A}{\text{Distance traveled by solvent front}} = \frac{5}{12.5}.$$

$$R_f(A) = 0.4.$$

Step 2: Calculate R_f for sample C For sample C:

$$R_f(C) = \frac{\text{Distance traveled by sample } C}{\text{Distance traveled by solvent front}} = \frac{10}{12.5}.$$

$$R_f(C) = 0.8.$$

Step 3: Calculate the ratio of R_f values The ratio of R_f values for sample A and sample C is:

$$\text{Ratio} = \frac{R_f(A)}{R_f(C)} = \frac{0.4}{0.8} = 0.5.$$

Expressing the ratio as $x \times 10^{-2}$:

$$\text{Ratio} = 50 \times 10^{-2}.$$

Final Answer: $x = 50$.

Quick Tip

In chromatography, R_f values are always ratios of distances. Ensure accurate measurements of both the sample spot and solvent front distances for precise results.

86: In the Claisen-Schmidt reaction to prepare 351 g of dibenzalacetone using 87 g of acetone, the amount of benzaldehyde required is ______ g. (Nearest integer)
Answer: (318)
Solution:
The reaction involved is the Claisen-Schmidt condensation:

$$2C_6H_5CHO + CH_3COCH_3 -> [NaOH]C_6H_5CH = CHCOCH = CHC_6H_5 + H_2O.$$

Step 1: Reaction Stoichiometry - 2 moles of benzaldehyde (C_6H_5CHO) react with 1 mole of acetone (CH_3COCH_3) to form 1 mole of dibenzalacetone ($C_6H_5CH = CHCOCH = CHC_6H_5$) - Given that 87 g of acetone is used, the molar mass of acetone is:

$$\text{Molar mass of } CH_3COCH_3 = 12 + (1 \times 3) + 12 + 16 + 1 = 58\,\text{g/mol}.$$

$$\text{Moles of acetone} = \frac{\text{Mass of acetone}}{\text{Molar mass of acetone}} = \frac{87}{58} \approx 1.5\,\text{moles}.$$

Step 2: Calculate moles of benzaldehyde required From the reaction stoichiometry, 2 moles of benzaldehyde are required for every 1 mole of acetone. Thus, the moles of benzaldehyde needed are:

$$\text{Moles of benzaldehyde} = 2 \times \text{Moles of acetone} = 2 \times 1.5 = 3\,\text{moles}.$$

Step 3: Calculate the mass of benzaldehyde required The molar mass of benzaldehyde (C_6H_5CHO) is:

$$\text{Molar mass of benzaldehyde} = (6 \times 12) + (5 \times 1) + 12 + 16 = 106\,\text{g/mol}.$$

$$\text{Mass of benzaldehyde} = \text{Moles of benzaldehyde} \times \text{Molar mass of benzaldehyde} = 3 \times 106 = 318\,\text{g}.$$

Step 4: Verify with product formation The reaction produces 1 mole of dibenzalacetone for every 2 moles of benzaldehyde. For 1.5 moles of acetone, 1.5 moles of dibenzalacetone are formed, which corresponds to 351 g (given).

Final Answer: 318 g.

Quick Tip

In Claisen-Schmidt condensation, use stoichiometry to determine the required reactant quantities. The molar ratio of benzaldehyde to acetone is always 2:1.

87: Consider the following single-step reaction in the gas phase at constant temperature:

$$2A_{(g)} + B_{(g)} -> C_{(g)}.$$

The initial rate of the reaction is recorded as r_1, when the reaction starts with 1.5 atm pressure of A and 0.7 atm pressure of B. After some time, the rate r_2 is recorded when the pressure of C becomes 0.5 atm. The ratio $\frac{r_1}{r_2}$ is ______ $\times 10^{-1}$ (Nearest integer).

Answer: (315)

Solution:

The rate law for the reaction is:

$$r = k[A]^2[B],$$

where $[A]$, $[B]$, and $[C]$ represent the partial pressures of the reactants and product.

Step 1: Initial conditions At r_1: - $[A] = 1.5$ atm, $[B] = 0.7$ atm.

$$r_1 = k[1.5]^2[0.7].$$

Step 2: Conditions when r_2 is measured When the pressure of C is 0.5 atm, due to stoichiometry:

$$2A_{(g)} + B_{(g)} -> C_{(g)},$$

the change in $[C]$ corresponds to:

$$\Delta[C] = 0.5\,\text{atm}.$$

This means:

$$\Delta[A] = 2 \times \Delta[C] = 2 \times 0.5 = 1.0\,\text{atm}.$$

$$\Delta[B] = \Delta[C] = 0.5\,\text{atm}.$$

The remaining pressures of A and B are:

$$[A] = 1.5 - 1.0 = 0.5\,\text{atm},$$

$$[B] = 0.7 - 0.5 = 0.2\,\text{atm}.$$

At r_2:

$$r_2 = k[0.5]^2[0.2].$$

Step 3: Calculate the ratio $\frac{r_1}{r_2}$ The ratio of the rates is:

$$\frac{r_1}{r_2} = \frac{k[1.5]^2[0.7]}{k[0.5]^2[0.2]}.$$

Simplify:

$$\frac{r_1}{r_2} = \frac{(1.5)^2(0.7)}{(0.5)^2(0.2)} = \frac{2.25 \times 0.7}{0.25 \times 0.2}.$$

$$\frac{r_1}{r_2} = \frac{1.575}{0.05} = 31.5 \times 10^{-1}.$$

Step 4: Nearest integer

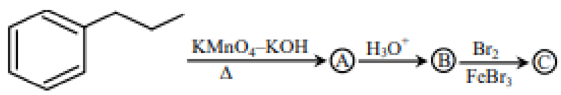
$$x = 315.$$

Final Answer: 315.

Quick Tip

For single-step reactions, use stoichiometric relations to determine the changes in reactant pressures and apply the rate law consistently to compute the rate ratio.

88: The product C in the following sequence of reactions has ______ π bonds:



Answer: (4)
Solution:
The reaction sequence proceeds as follows:

Step 1: Oxidation of $C_6H_5CH_2CH_3$ with $KMnO_4$ and KOH The side chain ethyl group of ethylbenzene is oxidized to a carboxylate salt:

$$C_6H_5CH_2CH_3- > [KMnO_4, KOH, \Delta]C_6H_5COOK(A).$$

Here, A is C_6H_5COOK, potassium benzoate.

Step 2: Acidification with H_3O^+ The carboxylate salt is acidified to form benzoic acid:

$$C_6H_5COOK + H_3O^+ -> C_6H_5COOH(B).$$

Here, B is C_6H_5COOH, benzoic acid.

Step 3: Bromination with $Br_2/FeBr_3$ Bromination occurs at the para-position of the benzene ring relative to the carboxylic acid group, yielding:

$$C_6H_5COOH -> [Br_2, FeBr_3]p - BrC_6H_4COOH(C).$$

Here, C is para-bromobenzoic acid ($p - BrC_6H_4COOH$).

Step 4: Count the π-bonds in C - The benzene ring contributes 3 π-bonds. - The $C = O$ group in the carboxylic acid contributes 1 π-bond.

Thus, the total number of π-bonds in C is:

$$\pi\text{-bonds} = 3 + 1 = 4.$$

Final Answer: 4.

Quick Tip

Oxidation of alkyl side chains on aromatic rings with $KMnO4$ results in carboxylate salts. Bromination selectively occurs at the para-position when activating groups like $-COOH$ are present.

89: Considering acetic acid dissociates in water, its dissociation constant is 6.25×10^{-5}. If 5 mL of acetic acid is dissolved in 1 litre of water, the solution will freeze at $-x \times 10^{-2}\,^\circ$C, provided pure water freezes at 0°C.
Given:

- K_f **of water** $= 1.86\,$**K kg mol**$^{-1}$,
- **Density of acetic acid** $= 1.2\,$**g cm**$^{-3}$,
- **Molar mass of water** $= 18\,$**g mol**$^{-1}$,
- **Molar mass of acetic acid** $= 60\,$**g mol**$^{-1}$,
- **Density of water** $= 1\,$**g cm**$^{-3}$.

Acetic acid dissociates as:

$$CH_3COOH <=> CH_3COO^- + H^+.$$

Answer: (19)
Solution:
The depression in freezing point ΔT_f is calculated using the formula:

$$\Delta T_f = i \cdot K_f \cdot m,$$

where:

- i is the van 't Hoff factor,
- K_f is the cryoscopic constant ($1.86\,\text{K kg mol}^{-1}$),
- m is the molality of the solution.

Step 1: Calculate the molality of the solution The mass of acetic acid dissolved is:

$$\text{Mass of acetic acid} = \text{Volume} \times \text{Density} = 5\,\text{mL} \times 1.2\,\text{g/mL} = 6\,\text{g}.$$

The number of moles of acetic acid is:

$$\text{Moles of acetic acid} = \frac{\text{Mass of acetic acid}}{\text{Molar mass of acetic acid}} = \frac{6}{60} = 0.1\,\text{mol}.$$

The molality of the solution is:

$$m = \frac{\text{Moles of solute}}{\text{Mass of solvent (kg)}} = \frac{0.1}{1} = 0.1\,\text{mol/kg}.$$

Step 2: Calculate the van 't Hoff factor (i) The dissociation constant (K_a) of acetic acid is:

$$K_a = 6.25 \times 10^{-5}.$$

The degree of dissociation (α) is given by:

$$\alpha = \sqrt{\frac{K_a}{C}},$$

where C is the molarity of the solution. The molarity is:

$$C = \frac{\text{Moles of solute}}{\text{Volume of solution (L)}} = \frac{0.1}{1} = 0.1\,\text{mol/L}.$$

$$\alpha = \sqrt{\frac{6.25 \times 10^{-5}}{0.1}} = \sqrt{6.25 \times 10^{-4}} = 0.025.$$

The van 't Hoff factor is:

$$i = 1 + \alpha = 1 + 0.025 = 1.025.$$

Step 3: Calculate ΔT_f

$$\Delta T_f = i \cdot K_f \cdot m = 1.025 \cdot 1.86 \cdot 0.1 = 0.19065\,\text{K}.$$

Converting to $-x \times 10^{-2}$:

$$x = 19.$$

Final Answer: $x = 19$.

Quick Tip

When calculating freezing point depression for weak acids, account for partial dissociation using the van 't Hoff factor i, which includes the degree of dissociation α.

90: The number of compounds from the following with zero dipole moment is ______:

$$HF, H_2, H_2S, CO_2, NH_3, BF_3, CH_4, CHCl_3, SiF_4, H_2O, BeF_2.$$

Answer: (6)
Solution:
A molecule has a zero dipole moment if it is symmetric, and the bond dipoles cancel each other out. Let us analyze each compound:

1. H_2: - Diatomic, nonpolar, symmetric. - Dipole moment = 0.
2. CO_2: - Linear molecule, symmetric. - Dipole moment = 0.
3. BF_3: - Planar triangular structure, symmetric. - Dipole moment = 0.
4. CH_4: - Tetrahedral geometry, symmetric. - Dipole moment = 0.
5. SiF_4: - Tetrahedral geometry, symmetric. - Dipole moment = 0.
6. BeF_2: - Linear molecule, symmetric. - Dipole moment = 0.

Molecules with nonzero dipole moments: - HF: Polar molecule, asymmetric. - H_2S: Bent structure, asymmetric. - NH_3: Trigonal pyramidal structure, asymmetric. - $CHCl_3$: Tetrahedral, but asymmetric due to Cl. - H_2O: Bent structure, asymmetric.

Conclusion The compounds with zero dipole moment are:

$$H_2, CO_2, BF_3, CH_4, SiF_4, BeF_2.$$

The number of such compounds is:

$$6.$$

Final Answer: 6.

Quick Tip

Symmetry in molecular geometry is key to identifying molecules with zero dipole moment. Linear, planar triangular, and tetrahedral geometries often exhibit this property.