JEE Main - 8 April (Shift-1) Question Paper with Solutions

Question 1. The value of $k \in \mathbb{N}$ for which the integral

$$I_n = \int_0^1 (1 - x^k)^n \, dx, \; n \in \mathbb{N}, \text{ satisfies } 147 \, I_{20} = 148 \, I_{21}$$

is:

1. 10
2. 8
3. 14
4. 7

Correct Answer: (4)

Solution:

The given integral is:

$$I_n = \int_0^1 (1 - x^k)^n \, dx.$$

Using integration by parts, we get:

$$I_n = \frac{nk}{nk+1} I_{n-1}.$$

Iterating this formula, the relationship becomes:

$$\frac{I_n}{I_{n-1}} = \frac{nk}{nk+1}.$$

Given:

$$\frac{I_{21}}{I_{20}} = \frac{147}{148},$$

we substitute into the formula:

$$\frac{21k}{21k+1} = \frac{147}{148}.$$

Cross-multiplying and solving:

$$148 \cdot 21k = 147 \cdot (21k + 1),$$

$$148 \cdot 21k = 147 \cdot 21k + 147,$$

$$21k = 147 \implies k = 7.$$

Quick Tip

For problems involving recursive integrals, always check if simplifications using ratios between successive terms can lead to easier equations.

Question 2. The sum of all the solutions of the equation

$$(8)^{2x} - 16 \cdot (8)^x + 48 = 0$$

is:

1. $1 + \log_6(8)$
2. $\log_8(6)$
3. $1 + \log_8(6)$
4. $\log_8(4)$

Correct Answer: (3)

Solution:

The given equation is:

$$(8)^{2x} - 16 \cdot (8)^x + 48 = 0.$$

Substitute $t = (8)^x$, which simplifies the equation to:

$$t^2 - 16t + 48 = 0.$$

Solve the quadratic equation:

$$t^2 - 16t + 48 = 0 \implies (t-4)(t-12) = 0.$$

Hence:

$$t = 4 \quad \text{or} \quad t = 12.$$

Back-substituting $t = (8)^x$, we get:

$$(8)^x = 4 \implies x = \log_8(4),$$

$$(8)^x = 12 \implies x = \log_8(12).$$

The sum of the solutions is:

$$\text{Sum} = \log_8(4) + \log_8(12).$$

Using the logarithmic property $\log_a(m) + \log_a(n) = \log_a(m \cdot n)$:

$$\text{Sum} = \log_8(4 \cdot 12) = \log_8(48).$$

Express 48 as $8 \cdot 6$:

$$\log_8(48) = \log_8(8 \cdot 6) = \log_8(8) + \log_8(6).$$

Since $\log_8(8) = 1$, we have:

$$\log_8(48) = 1 + \log_8(6).$$

Final Answer: 1 +$\log_8(6)$.

Quick Tip

For problems involving logarithms and exponents, use substitutions to simplify the equation into a quadratic or linear form. This often makes solving much easier.

Question 3. Let the circles $C_1 : (x-\alpha)^2 + (y-\beta)^2 = r_1^2$ **and**

$$C_2 : (x-8)^2 + \left(y - \frac{15}{2}\right)^2 = r_2^2$$

touch each other externally at the point $(6, 6)$**. If the point** $(6, 6)$ **divides the line segment joining the centers of the circles** C_1 **and** C_2 **internally in the ratio** $2 : 1$**, then:**

$$(\alpha + \beta) + 4 \cdot (r_1^2 + r_2^2)$$

equals:

1. **110**
2. **130**
3. **125**
4. **145**

Correct Answer: (2)

Solution: The centers of the circles are (α, β) **for** C_1 **and** $(8, \frac{15}{2})$ **for** C_2**. Since the point**

$(6, 6)$ **divides the line segment joining the centers in the ratio** $2 : 1$**, applying the section formula:**

$$\frac{16+\alpha}{3} = 6 \quad \Longrightarrow \quad 16+\alpha = 18 \quad \Longrightarrow \quad \alpha = 2,$$
$$\frac{15+\beta}{3} = 6 \quad \Longrightarrow \quad 15+\beta = 18 \quad \Longrightarrow \quad \beta = 3.$$

Thus, the center of C_1 **is** $(\alpha, \beta) = (2, 3)$**.**

The circles touch externally at the point $(6, 6)$**, so the distance between the centers equals the sum of the radii:**

$$C_1C_2 = r_1 + r_2.$$

Using the distance formula:

$$C_1C_2 = \sqrt{(2-8)^2 + \left(3 - \frac{15}{2}\right)^2},$$

$$C_1C_2 = \sqrt{(-6)^2 + \left(-\frac{9}{2}\right)^2} = \sqrt{36 + \frac{81}{4}} = \sqrt{\frac{144}{4} + \frac{81}{4}} = \sqrt{\frac{225}{4}} = \frac{15}{2}.$$

Thus:

$$r_1 + r_2 = \frac{15}{2}.$$

Now, since the point $(6, 6)$ **lies on both circles, for** C_1**:**

$$(6-\alpha)^2 + (6-\beta)^2 = r_1^2,$$

$$(6-2)^2 + (6-3)^2 = r_1^2 \quad \Longrightarrow \quad 4^2 + 3^2 = r_1^2 \quad \Longrightarrow \quad r_1^2 = 16 + 9 = 25.$$

So, $r_1 = 5$**. Substituting** $r_1 = 5$ **into** $r_1 + r_2 = \frac{15}{2}$**:**

$$5 + r_2 = \frac{15}{2} \quad \Longrightarrow \quad r_2 = \frac{15}{2} - 5 = \frac{5}{2}.$$

Finally, calculate $(\alpha + \beta) + 4(r_1^2 + r_2^2)$**:**

$$\alpha + \beta = 2 + 3 = 5,$$

$$r_1^2 + r_2^2 = 25 + \left(\frac{5}{2}\right)^2 = 25 + \frac{25}{4} = \frac{100}{4} + \frac{25}{4} = \frac{125}{4},$$
$$4(r_1^2 + r_2^2) = 4 \cdot \frac{125}{4} = 125.$$

Thus:

$$(\alpha + \beta) + 4(r_1^2 + r_2^2) = 5 + 125 = 130.$$

Quick Tip

For geometry problems involving ratios, use the section formula to simplify the problem. Always verify the conditions for tangency or contact between circles.

Question 4. Let $P(x, y, z)$ be a point in the first octant, whose projection in the xy-plane is the point Q. Let $OP = \gamma$; the angle between OQ and the positive x-axis be θ; and the angle between OP and the positive z-axis be ϕ, where O is the origin. Then the distance of P from the x-axis is:

1. $\gamma\sqrt{1 - \sin^2\phi\cos^2\theta}$
2. $\gamma\sqrt{1 + \cos^2\theta\sin^2\phi}$
3. $\gamma\sqrt{1 - \sin^2\theta\cos^2\phi}$
4. $\gamma\sqrt{1 + \cos^2\phi\sin^2\theta}$

Correct Answer: (1)

Solution:

P(x, y, z), Q(x, y, O); $\quad x^2 + y^2 + z^2 = \gamma^2$

OQ $= x\mathbf{i} + y\mathbf{j}$

$$\cos\theta = \frac{x}{\sqrt{x^2 + y^2}}$$

$$\cos\phi = \frac{z}{\sqrt{x^2 + y^2 + z^2}}$$

$$\implies \sin^2\phi = \frac{x^2 + y^2}{x^2 + y^2 + z^2}$$

Distance of P from x-axis $= \sqrt{y^2 + z^2}$

$$\implies \sqrt{\gamma^2 - x^2} \implies \gamma\sqrt{1 - \frac{x^2}{\gamma^2}}$$

$$= \gamma\sqrt{1 - \cos^2\theta\sin^2\phi}$$

Quick Tip

In 3D geometry problems, use projections and trigonometric identities for simplification. Visualize the axes and angles for clarity.

Question 5. The number of critical points of the function $f(x) = (x-2)^{2/3}(2x+1)$ is:

1. 2
2. 0
3. 1
4. 3

Correct Answer: (1)

Solution:

The derivative of $f(x)$ is:

$$f'(x) = \frac{2}{3}(x-2)^{-1/3}(2x+1) + (x-2)^{2/3}(2),$$

$$f'(x) = \frac{2}{3} \cdot \frac{(2x+1) + 3(x-2)}{(x-2)^{1/3}}.$$

Simplify the numerator:

$$(2x+1) + 3(x-2) = 5x - 5.$$

So:

$$f'(x) = \frac{2(5x-5)}{3(x-2)^{1/3}}.$$

Setting $f'(x) = 0$, solve:

$$5x - 5 = 0 \quad \implies \quad x = 1.$$

Also, $f'(x)$ is undefined at $x = 2$. Thus, the critical points are:

$$x = 1 \quad \textbf{and} \quad x = 2.$$

Final Answer: 2.

Quick Tip

For critical points, always check where the derivative is zero and where it is undefined. Include both cases for completeness.

Question 6.Let $f(x)$ be a positive function such that the area bounded by $y = f(x), y = 0$ from $x = 0$ to $x = a > 0$ is:

$$e^{-a} + 4a^2 + a - 1.$$

The differential equation, whose general solution is $y = c_1 f(x) + c_2$, where c_1 and c_2 are arbitrary constants, is:

1. $(8e^x - 1)\frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$
2. $(8e^x + 1)\frac{d^2y}{dx^2} - \frac{dy}{dx} = 0$
3. $(8e^x + 1)\frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$
4. $(8e^x - 1)\frac{d^2y}{dx^2} - \frac{dy}{dx} = 0$

Correct Answer: (3)

Solution:

The integral is:

$$\int_0^a f(x)dx = e^{-a} + 4a^2 + a - 1.$$

Differentiating with respect to a:

$$f(a) = -e^{-a} + 8a + 1.$$

Differentiating again:

$$f'(a) = e^{-a} + 8.$$

The general solution is:

$$y = c_1 f(x) + c_2 \quad \Longrightarrow \quad \frac{dy}{dx} = c_1 f'(x), \quad \frac{d^2y}{dx^2} = c_1 f''(x).$$

Substitute:

$$f''(x) = -e^{-x}, \quad f'(x) = e^{-x} + 8.$$

The differential equation becomes:

$$(8e^x + 1)\frac{d^2y}{dx^2} + \frac{dy}{dx} = 0.$$

Final Answer: (8e^x $+ 1)\frac{d^2y}{dx^2} + \frac{dy}{dx} = 0.$

Quick Tip

When dealing with integrals and differential equations, differentiate step by step and simplify at each stage. Always verify the final equation by substitution.

Question 7. Let $f(x) = 4\cos^3(x) + 3\sqrt{3}\cos^2(x) - 10$. The number of points of local maxima of f in the interval $(0, 2\pi)$ is:

1. 1
2. 2
3. 3
4. 4

Correct Answer: (2)

Solution:

The given function is:

$$f(x) = 4\cos^3(x) + 3\sqrt{3}\cos^2(x) - 10, \quad x \in (0, 2\pi).$$

Taking the derivative:

$$f'(x) = 12\cos^2(x)[-\sin(x)] + 3\sqrt{3}[2\cos(x)(-\sin(x))],$$

$$f'(x) = -6\sin(x)\cos(x)[2\cos(x) + \sqrt{3}].$$

The critical points occur when:

$$\sin(x) = 0 \quad \textbf{or} \quad 2\cos(x) + \sqrt{3} = 0.$$

Solving these equations:

$$\sin(x) = 0 \quad \Longrightarrow \quad x = 0, \pi, 2\pi,$$

$$\cos(x) = -\frac{\sqrt{3}}{2} \quad \Longrightarrow \quad x = \frac{5\pi}{6}, \frac{7\pi}{6}.$$

Checking the interval $(0, 2\pi)$, the local maxima occur at:

$$x = \frac{5\pi}{6}, \frac{7\pi}{6}.$$

Final Answer: 2.

Quick Tip

To find local maxima or minima, solve $f'(x) = 0$ and analyze sign changes in $f'(x)$ around the critical points.

Question 8. Let $A = \begin{bmatrix} 2 & a & 0 \\ 1 & 3 & 1 \\ 0 & 5 & b \end{bmatrix}$. **If** $A^3 = 4A^2 - A - 21I$, **where** I **is the identity matrix of order** 3×3, **then** $2a + 3b$ **is equal to:**

1. -10
2. -13
3. -9
4. -12

Correct Answer: (2)

Solution:

From the matrix equation:

$$A^3 - 4A^2 + A + 21I = 0.$$

Taking the trace:

$$\mathbf{tr}(A^3) - 4\mathbf{tr}(A^2) + \mathbf{tr}(A) + 21 \cdot \mathbf{tr}(I) = 0.$$

Since tr$(I) = 3$, **we find:**

$$\mathbf{tr}(A) = 4 + 5 + b = b - 1.$$

The determinant:

$$|A| = -16 + a = -21 \quad \Longrightarrow \quad a = -5.$$

Finally:

$$2a + 3b = 2(-5) + 3(-1) = -13.$$

Final Answer: -13.

Quick Tip

For matrix problems involving trace or determinant equations, focus on simplifying the conditions to directly find the variables.

Question 9. If the shortest distance between the lines:

$$L_1 : \mathbf{r} = (2+\lambda)\hat{i} + (1-3\lambda)\hat{j} + (3+4\lambda)\hat{k},\ \lambda \in \mathbb{R},$$

$$L_2 : \mathbf{r} = 2(1+\mu)\hat{i} + 3(1+\mu)\hat{j} + 5(1+\mu)\hat{k},\ \mu \in \mathbb{R},$$

is $\frac{m}{\sqrt{n}}$, where $\gcd(m,n) = 1$, then the value of $m+n$ is:

1. **384**
2. **387**
3. **377**
4. **390**

Correct Answer: (2)

Solution:

The shortest distance between skew lines is given by:

$$\textbf{Shortest Distance} = \frac{|\mathbf{AB} \cdot (\mathbf{p} \times \mathbf{q})|}{|\mathbf{p} \times \mathbf{q}|}.$$

Here:

$$\mathbf{p} = \begin{bmatrix} 1 \\ -3 \\ 4 \end{bmatrix}, \quad \mathbf{q} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \quad \mathbf{AB} = \begin{bmatrix} 0 \\ 2 \\ 2 \end{bmatrix}.$$

Compute:

$$\mathbf{p} \times \mathbf{q} = \begin{bmatrix} -4 \\ -3 \\ 4 \end{bmatrix}, \quad |\mathbf{p} \times \mathbf{q}| = \sqrt{55}.$$

Finally:

$$\textbf{Shortest Distance} = \frac{32}{\sqrt{355}} \implies m = 32,\ n = 355,\ \gcd(m,n) = 1.$$

$$m + n = 387.$$

Final Answer: 387.

Quick Tip

For shortest distance between lines, carefully compute $\mathrm{p} \times \mathrm{q}$ and verify orthogonality conditions.

Question 10. Let the sum of two positive integers be 24. If the probability, that their product is not less than $\frac{3}{4}$ times their greatest positive product, is $\frac{m}{n}$, where $\gcd(m,n) = 1$, then $n - m$ equals:

1. 9
2. 11
3. 8
4. 10

Correct Answer: (4)

Solution:

Given $x + y = 24$, $x, y \in \mathbb{N}$, the greatest product occurs at:

$$x = y = 12 \implies \textbf{Maximum Product} = 144.$$

The condition:

$$xy \geq \frac{3}{4} \cdot 144 \implies xy \geq 108.$$

The favorable pairs are:

$$(13,11), (12,12), (14,10), (15,9), (16,8), (17,7), (18,6), (6,18), (7,17), (8,16), (9,15), (10,14), (11,13).$$

There are 13 favorable cases out of 23 total cases. Hence:

$$\textbf{Probability} = \frac{13}{23}.$$

Thus:

$$m = 13,\ n = 23 \implies n - m = 10.$$

Final Answer: 10.

Quick Tip

For probability-based number theory problems, calculate the favorable and total cases carefully. Simplify fractions to ensure the $\gcd(m,n) = 1$ condition.

Question 11. If $\sin x = -\frac{3}{5}$, where $\pi < x < \frac{3\pi}{2}$, then $80(\tan^2 x - \cos x)$ is equal to:

1. **109**

2. **108**

3. **18**

4. **19**

Correct Answer: (1)

Solution:

Given:

$$\sin x = -\frac{3}{5}, \quad \pi < x < \frac{3\pi}{2}.$$

Using the Pythagorean identity:

$$\cos^2 x = 1 - \sin^2 x = 1 - \left(-\frac{3}{5}\right)^2 = 1 - \frac{9}{25} = \frac{16}{25}.$$

Since $\cos x < 0$ **in the third quadrant:**

$$\cos x = -\frac{4}{5}.$$

Similarly:

$$\tan x = \frac{\sin x}{\cos x} = \frac{-\frac{3}{5}}{-\frac{4}{5}} = \frac{3}{4}.$$

Now, calculate $80(\tan^2 x - \cos x)$**:**

$$\tan^2 x = \left(\frac{3}{4}\right)^2 = \frac{9}{16}, \quad 80\left(\tan^2 x - \cos x\right) = 80\left(\frac{9}{16} - \left(-\frac{4}{5}\right)\right).$$

Simplify:

$$80\left(\frac{9}{16} + \frac{4}{5}\right) = 80\left(\frac{45}{80} + \frac{64}{80}\right) = 80 \cdot \frac{109}{80} = 109.$$

Final Answer: 109.

Quick Tip

For trigonometric problems, always identify the correct quadrant to determine the signs of the trigonometric ratios.

Question 12. Let $I(x) = \int \frac{6}{\sin^2 x(1-\cot x)^2}\, dx$**. If** $I(0) = 3$**, then** $I\left(\frac{\pi}{12}\right)$ **is equal to:**

1. $\sqrt{3}$

2. $3\sqrt{3}$

3. $6\sqrt{3}$

4. $2\sqrt{3}$

Correct Answer: (2)

Solution:

Given:

$$I(x) = \int \frac{6}{\sin^2 x(1 - \cot x)^2}\, dx.$$

Substitute:

$$t = 1 - \cot x, \quad \csc^2 x\, dx = dt.$$

The integral becomes:

$$I = \int \frac{6\, dt}{t^2} = -\frac{6}{t} + c = -\frac{6}{1 - \cot x} + c.$$

Using $I(0) = 3$:

$$I(0) = 3 = -\frac{6}{1 - \cot(0)} + c \quad \Longrightarrow \quad c = 3.$$

Now:

$$I(x) = 3 - \frac{6}{1 - \cot x}.$$

Evaluate $I\left(\frac{\pi}{12}\right)$:

$$\cot\left(\frac{\pi}{12}\right) = 2 + \sqrt{3}, \quad I\left(\frac{\pi}{12}\right) = 3 - \frac{6}{1 - (2 + \sqrt{3})}.$$

Simplify:

$$I\left(\frac{\pi}{12}\right) = 3 + \frac{6}{2 + \sqrt{3} - 1} = 3 + \frac{6}{1 + \sqrt{3}}.$$

Rationalizing the denominator:

$$\frac{6}{1 + \sqrt{3}} = \frac{6(1 - \sqrt{3})}{1 - 3} = \frac{6(\sqrt{3} - 1)}{2}.$$

$$I\left(\frac{\pi}{12}\right) = 3 + 3\sqrt{3} - 3 = 3\sqrt{3}.$$

Final Answer: 3$\sqrt{3}$.

Quick Tip

In integrals involving trigonometric substitutions, use identities and substitutions to simplify terms systematically.

Question 13. The equations of two sides AB and AC of a triangle ABC are $4x + y = 14$ and $3x - 2y = 5$, respectively. The point $\left(2, -\frac{4}{3}\right)$ divides the third side BC internally in the ratio $2:1$. The equation of the side BC is:

1. $x - 6y - 10 = 0$
2. $x - 3y - 6 = 0$
3. $x + 3y + 2 = 0$
4. $x + 6y + 6 = 0$

Correct Answer: (3)

Solution:

The given equations are:

$$AB : 4x + y = 14, \quad AC : 3x - 2y = 5.$$

The coordinates of B on AB are:

$$B(x_1, y_1) \quad \textbf{where} \quad y_1 = 14 - 4x_1.$$

The coordinates of C on AC are:

$$C(x_2, y_2) \quad \textbf{where} \quad y_2 = \frac{3x_2 - 5}{2}.$$

The point $P(2, -\frac{4}{3})$ divides BC in the ratio $2:1$. Using the section formula:

$$x_P = \frac{2x_2 + x_1}{3}, \quad y_P = \frac{2y_2 + y_1}{3}.$$

Substitute $x_P = 2$ and $y_P = -\frac{4}{3}$ into these equations:

For $x_P = 2$:

$$2 = \frac{2x_2 + x_1}{3}.$$

Rearrange:

$$6 = 2x_2 + x_1 \quad \Longrightarrow \quad x_1 = 6 - 2x_2.$$

For $y_P = -\frac{4}{3}$: Substitute $y_1 = 14 - 4x_1$ and $y_2 = \frac{3x_2 - 5}{2}$ into:

$$-\frac{4}{3} = \frac{2y_2 + y_1}{3}.$$

Simplify the numerator:

$$-\frac{4}{3} = \frac{2\left(\frac{3x_2 - 5}{2}\right) + (14 - 4x_1)}{3}.$$

Multiply through by 3:

$$-4 = 3x_2 - 5 + 14 - 4x_1.$$

Combine terms:

$$-4 = 3x_2 + 9 - 4x_1 \implies 3x_2 - 4x_1 = -13.$$

We now have two equations:

$$x_1 = 6 - 2x_2, \quad 3x_2 - 4x_1 = -13.$$

Substitute $x_1 = 6 - 2x_2$ into the second equation:

$$3x_2 - 4(6 - 2x_2) = -13.$$

Simplify:

$$3x_2 - 24 + 8x_2 = -13 \implies 11x_2 = 11 \implies x_2 = 1.$$

Substitute $x_2 = 1$ into $x_1 = 6 - 2x_2$:

$$x_1 = 6 - 2(1) = 4.$$

Substitute $x_1 = 4$ into $y_1 = 14 - 4x_1$:

$$y_1 = 14 - 4(4) = -2.$$

Substitute $x_2 = 1$ into $y_2 = \frac{3x_2-5}{2}$:

$$y_2 = \frac{3(1) - 5}{2} = \frac{-2}{2} = -1.$$

Thus, $B(4, -2)$ and $C(1, -1)$.

The slope of BC is:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1 - (-2)}{1 - 4} = \frac{1}{-3} = -\frac{1}{3}.$$

Using the point-slope form at $B(4, -2)$:

$$y - (-2) = -\frac{1}{3}(x - 4).$$

Simplify:

$$y + 2 = -\frac{1}{3}x + \frac{4}{3}.$$

Multiply through by 3:

$$3y + 6 = -x + 4 \implies x + 3y + 2 = 0.$$

Final Answer: x + 3y + 2 = 0.

Quick Tip

In geometry problems involving section formula, carefully calculate intermediate points and verify with the line equations.

Question 14. Let $[t]$ be the greatest integer less than or equal to t. Let A be the set of all prime factors of 2310 and

$$f : A \to \mathbb{Z},\ f(x) = \left[\log_2\left(x^2 + \frac{x^3}{5}\right)\right].$$

The number of one-to-one functions from A to the range of f is:

1. 20
2. 120
3. 25
4. 24

Correct Answer: (2)

Solution:

The prime factorization of 2310 is:

$$2310 = 2 \cdot 3 \cdot 5 \cdot 7 \cdot 11.$$

Thus, $A = \{2, 3, 5, 7, 11\}$.

For each $x \in A$, compute:

$$f(x) = \left[\log_2\left(x^2 + \frac{x^3}{5}\right)\right].$$

For $x = 2$,

$$f(2) = \left[\log_2\left(2^2 + \frac{2^3}{5}\right)\right] = \left[\log_2\left(4 + \frac{8}{5}\right)\right] = \left[\log_2\left(\frac{28}{5}\right)\right] = [\log_2(5.6)] = 2.$$

For $x = 3$,

$$f(3) = \left[\log_2\left(3^2 + \frac{3^3}{5}\right)\right] = \left[\log_2\left(9 + \frac{27}{5}\right)\right] = \left[\log_2\left(\frac{72}{5}\right)\right] = [\log_2(14.4)] = 3.$$

For $x = 5$,

$$f(5) = \left[\log_2\left(5^2 + \frac{5^3}{5}\right)\right] = [\log_2(25+25)] = [\log_2(50)] = 5.$$

For $x = 7$,

$$f(7) = \left[\log_2\left(7^2 + \frac{7^3}{5}\right)\right] = \left[\log_2\left(49 + \frac{343}{5}\right)\right] = \left[\log_2\left(\frac{588}{5}\right)\right] = [\log_2(117.6)] = 6.$$

For $x = 11$,

$$f(11) = \left[\log_2\left(11^2 + \frac{11^3}{5}\right)\right] = \left[\log_2\left(121 + \frac{1331}{5}\right)\right] = \left[\log_2\left(\frac{1936}{5}\right)\right] = [\log_2(387.2)] = 8.$$

The range of f is:

$$\textbf{Range of } f = \{2, 3, 5, 6, 8\}.$$

The number of one-to-one functions from A to the range of f is:

$$5! = 120.$$

Final Answer: 120.

Quick Tip

To compute the number of one-to-one functions, ensure that the domain and range have the same size and use permutations.

Question 15. Let z be a complex number such that $|z+2| = 1$ and

$$\textbf{Im}\left(\frac{z+1}{z+2}\right) = \frac{1}{5}.$$

Then the value of $\left|\textbf{Re}\overline{(z+2)}\right|$ is:

1. $\frac{\sqrt{6}}{5}$
2. $1 + \frac{\sqrt{6}}{5}$
3. $\frac{24}{5}$
4. $\frac{2\sqrt{6}}{5}$

Correct Answer: (4)

Solution:

Let:

$$z + 2 = \cos\theta + i\sin\theta \quad \implies \quad \frac{1}{z+2} = \cos\theta - i\sin\theta.$$

Now:

$$\frac{z+1}{z+2} = 1 - \frac{1}{z+2} = 1 - (\cos\theta - i\sin\theta).$$

Simplify:

$$\frac{z+1}{z+2} = (1-\cos\theta) + i\sin\theta.$$

The imaginary part is:

$$\textbf{Im}\left(\frac{z+1}{z+2}\right) = \sin\theta = \frac{1}{5}.$$

Using $\sin^2\theta + \cos^2\theta = 1$:

$$\cos^2\theta = 1 - \sin^2\theta = 1 - \left(\frac{1}{5}\right)^2 = 1 - \frac{1}{25} = \frac{24}{25}.$$

$$\cos\theta = \pm\frac{\sqrt{24}}{5} = \pm\frac{2\sqrt{6}}{5}.$$

Now, the real part of $z+2$ is:

$$\textbf{Re}\overline{(z+2)} = \cos\theta.$$

The magnitude of $\textbf{Re}\overline{(z+2)}$ is:

$$|\textbf{Re}(z+2)| = \frac{2\sqrt{6}}{5}.$$

Final Answer: $2\sqrt{6}_{\overline{5.}}$

Quick Tip

For complex numbers, carefully separate the real and imaginary parts and apply trigonometric identities.

Question 16. If the set $R = \{(a,b) : a + 5b = 42, a, b \in \mathbb{N}\}$ has m elements and

$$\sum_{n=1}^{m}(1 - i^{n!}) = x + iy,$$

where $I = \sqrt{-1}$, then the value of $m + x + y$ is:

1. 8

2. 12

3. 4

4. 5

Correct Answer: (2)

Solution:

From $a + 5b = 42$, where $a, b \in \mathbb{N}$, we have:

$$a = 42 - 5b.$$

Since $a > 0$, we require:

$$42 - 5b > 0 \quad \implies \quad b < \frac{42}{5} \quad \implies \quad b \leq 8.$$

The possible values of (a, b) are:

$$(37, 1), (32, 2), (27, 3), (22, 4), (17, 5), (12, 6), (7, 7), (2, 8).$$

Thus, $m = 8$.

The sum is:

$$\sum_{n=1}^{8} (1 - i^n).$$

For $n \geq 4$, i^n repeats every 4 terms:

$$i^1 = i,\ i^2 = -1,\ i^3 = -i,\ i^4 = 1.$$

Compute:

$$\sum_{n=1}^{8} (1-i^n) = (1-i)+(1-(-1))+(1-(-i))+(1-1)+(1-i)+(1-(-1))+(1-(-i))+(1-1).$$

Simplify:

$$= (1 - i) + 2 + (1 + i) + 0 + (1 - i) + 2 + (1 + i) + 0 = 5 - i + i = 5.$$

Thus:

$$x + y = 5,\ m = 8,\ m + x + y = 8 + 5 - 1 = 12.$$

Final Answer: 12.

Quick Tip

For problems involving complex numbers and modular arithmetic, identify repeating patterns to simplify calculations.

Question 17. For the function $f(x) = (\cos x) - x + 1,\ x \in \mathbb{R}$, between the following two statements:

(S1) $f(x) = 0$ **for only one value of** x **in** $[0, \pi]$.

(S2) $f(x)$ **is decreasing in** $\left[0, \frac{\pi}{2}\right]$ **and increasing in** $\left[\frac{\pi}{2}, \pi\right]$.

The correct answer is:

1. **Both (S1) and (S2) are correct**
2. **Only (S1) is correct**
3. **Both (S1) and (S2) are incorrect**
4. **Only (S2) is correct**

Correct Answer: (2)

Solution:

The function is:

$$f(x) = \cos x - x + 1.$$

The derivative is:

$$f'(x) = -\sin x - 1.$$

Since $-\sin x - 1 < 0$ **for all** $x \in \mathbb{R}$**,** $f(x)$ **is strictly decreasing in** $[0, \pi]$**.**

At $x = 0$**,**

$$f(0) = \cos(0) - 0 + 1 = 2.$$

At $x = \pi$**,**

$$f(\pi) = \cos(\pi) - \pi + 1 = -\pi < 0.$$

By the intermediate value theorem, $f(x) = 0$ **has exactly one root in** $[0, \pi]$**. Thus, (S1) is correct.**

(S2) is incorrect because $f(x)$ **is strictly decreasing in** $[0, \pi]$**.**

Final Answer: Only (S1) is correct.

Quick Tip

To determine monotonicity, analyze the derivative of the function. Strictly monotonic functions have at most one root in an interval.

Question 18. The set of all α, for which the vector

$$\vec{a} = \alpha t\hat{i} + 6\hat{j} - 3\hat{k}, \quad \vec{b} = t\hat{i} - 2\hat{j} - 2\alpha t\hat{k}$$

are inclined at an obtuse angle for all $t \in \mathbb{R}$, is:

1. $[0, 1)$
2. $(-2, 0]$
3. $\left(-\frac{4}{3}, 0\right]$
4. $\left(-\frac{4}{3}, 1\right)$

Correct Answer: (3)

Solution:

The dot product of $\vec{a}$ and $\vec{b}$ is:

$$\vec{a} \cdot \vec{b} = \alpha t + 6(-2) + (-3)(-2\alpha t) = \alpha t - 12 + 6\alpha t.$$

$$\vec{a} \cdot \vec{b} = (\alpha + 6\alpha)t - 12 = 7\alpha t - 12.$$

For the angle to be obtuse:

$$\vec{a} \cdot \vec{b} < 0.$$

This gives:

$$7\alpha t - 12 < 0 \quad \implies \quad t(7\alpha) - 12 < 0.$$

For all $t \in \mathbb{R}$, this inequality holds only if:

$$\alpha < 0 \quad \textbf{and} \quad -12 < 0.$$

To ensure obtuse angles:

$$-\frac{4}{3} < \alpha < 0.$$

Final Answer: $\left(-\frac{4}{3}, 0\right]$.

Quick Tip

For vectors to form an obtuse angle, ensure the dot product is negative for all permissible parameter values.

Question 19. Let y = $y(x)$ be the solution of the differential equation:

$$(1 + y^2)e^{\tan x}dx + \cos^2 x(1 + e^{2\tan x})dy = 0,$$

with $y(0) = 1$. Then $y\left(\frac{\pi}{4}\right)$ is equal to:

1. $\frac{2}{e}$

2. $\frac{1}{e^2}$

3. $\frac{1}{e}$

4. $\frac{2}{e^2}$

Correct Answer: (3)

Solution:

The given differential equation is:

$$(1 + y^2)e^{\tan^{-1} x}dx + \cos^2 x(1 + e^{2\tan^{-1} x})dy = 0.$$

Separate the variables:

$$\frac{\sec^2 x \cdot e^{\tan^{-1} x}dx}{1 + e^{2\tan^{-1} x}} + \frac{dy}{1 + y^2} = 0.$$

Integrating both sides:

$$\tan^{-1}(e^{\tan^{-1} x}) + \tan^{-1}(y) = C.$$

Using the initial condition $y(0) = 1$:

$$\tan^{-1}(e^{\tan^{-1}(0)}) + \tan^{-1}(1) = C.$$

Simplify:

$$\tan^{-1}(e^0) + \tan^{-1}(1) = C \quad \Longrightarrow \quad \tan^{-1}(1) + \tan^{-1}(1) = C \quad \Longrightarrow \quad C = \frac{\pi}{2}.$$

The solution becomes:

$$\tan^{-1}(e^{\tan^{-1} x}) + \tan^{-1}(y) = \frac{\pi}{2}.$$

At $x = \frac{\pi}{4}$, substitute into the solution:

$$\tan^{-1}(e^{\tan^{-1}(\frac{\pi}{4})}) + \tan^{-1}(y) = \frac{\pi}{2}.$$

Rearrange:

$$\tan^{-1}(y) = \frac{\pi}{2} - \tan^{-1}(e^{\tan^{-1}\left(\frac{\pi}{4}\right)}).$$

From the properties of $\tan^{-1}$, substitute:

$$\tan^{-1}(y) = \cot^{-1}(e).$$

Simplify:

$$y = \frac{1}{e}.$$

Final Answer: 1$_{\overline{e.}}$

Quick Tip

When solving differential equations, ensure proper separation of variables and use the given initial conditions to calculate the integration constant.

Question 20. Let $H : -\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ be the hyperbola, whose eccentricity is $\sqrt{3}$ and the length of the latus rectum is $4\sqrt{3}$. Suppose the point $(\alpha, 6), \alpha > 0$ lies on H. If β is the product of the focal distances of the point $(\alpha, 6)$, then $\alpha^2 + \beta$ is equal to:

1. **170**
2. **171**
3. **169**
4. **172**

Correct Answer: (2)

Solution:

The equation of the hyperbola is given as:

$$H : \frac{y^2}{b^2} - \frac{x^2}{a^2} = 1, \quad e = \sqrt{3}.$$

Using the relation for eccentricity:

$$e = \sqrt{1 + \frac{a^2}{b^2}}.$$

Substitute $e = \sqrt{3}$:

$$\sqrt{3} = \sqrt{1 + \frac{a^2}{b^2}} \quad \Longrightarrow \quad 3 = 1 + \frac{a^2}{b^2} \quad \Longrightarrow \quad \frac{a^2}{b^2} = 2.$$

This gives:

$$a^2 = 2b^2.$$

The length of the latus rectum is given by:

$$\textbf{Latus Rectum} = \frac{2a^2}{b}.$$

Substitute $a^2 = 2b^2$ and Latus Rectum $= 4\sqrt{3}$:

$$\frac{2(2b^2)}{b} = 4\sqrt{3} \implies \frac{4b^2}{b} = 4\sqrt{3} \implies 4b = 4\sqrt{3} \implies b = \sqrt{3}.$$

Using $a^2 = 2b^2$:

$$a^2 = 2(\sqrt{3})^2 = 2 \cdot 3 = 6 \implies a = \sqrt{6}.$$

The equation of the hyperbola becomes:

$$\frac{y^2}{3} - \frac{x^2}{6} = 1.$$

The point $(\alpha, 6)$ lies on the hyperbola:

$$\frac{6^2}{3} - \frac{\alpha^2}{6} = 1.$$

Simplify:

$$\frac{36}{3} - \frac{\alpha^2}{6} = 1 \implies 12 - \frac{\alpha^2}{6} = 1 \implies \frac{\alpha^2}{6} = 11 \implies \alpha^2 = 66.$$

The coordinates of the foci are:

$$(0, \pm be) = (0, \pm\sqrt{3} \cdot \sqrt{3}) = (0, \pm 3).$$

Let d_1 and d_2 be the focal distances of the point $(\alpha, 6)$:

$$d_1 = \sqrt{\alpha^2 + (6-3)^2}, \quad d_2 = \sqrt{\alpha^2 + (6+3)^2}.$$

Substitute:

$$d_1 = \sqrt{66 + (6-3)^2} = \sqrt{66 + 9} = \sqrt{75},$$

$$d_2 = \sqrt{66 + (6+3)^2} = \sqrt{66 + 81} = \sqrt{147}.$$

The product of the focal distances is:

$$\beta = d_1 \cdot d_2 = \sqrt{75} \cdot \sqrt{147} = \sqrt{75 \cdot 147}.$$

Simplify:

$$75 \cdot 147 = 11025 \implies \beta = \sqrt{11025} = 105.$$

Finally, calculate $\alpha^2 + \beta$:

$$\alpha^2 + \beta = 66 + 105 = 171.$$

Final Answer: 171.

Quick Tip

For hyperbola problems, use eccentricity and latus rectum relations to find a^2 and b^2. Verify calculations by substituting points into the hyperbola equation.

Question 21. Let $A = \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}$. If the sum of the diagonal elements of A^{13} is 3^n, then n is equal to:

Correct Answer: (7)

Solution:

The given matrix is:

$$A = \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}.$$

Compute successive powers of A:

$$A^2 = \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -3 \\ 3 & 0 \end{bmatrix}.$$

$$A^3 = A^2 \cdot A = \begin{bmatrix} 3 & -3 \\ 3 & 0 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -6 \\ 6 & -3 \end{bmatrix}.$$

$$A^4 = A^3 \cdot A = \begin{bmatrix} 3 & -6 \\ 6 & -3 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -9 \\ 9 & -9 \end{bmatrix}.$$

$$A^5 = A^4 \cdot A = \begin{bmatrix} 0 & -9 \\ 9 & -9 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} -9 & -9 \\ 9 & -18 \end{bmatrix}.$$

$$A^6 = A^5 \cdot A = \begin{bmatrix} -9 & -9 \\ 9 & -18 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} -27 & 0 \\ 0 & -27 \end{bmatrix}.$$

$$A^7 = A^6 \cdot A = \begin{bmatrix} -27 & 0 \\ 0 & -27 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 36 & -27 \\ -27 & 36 \end{bmatrix}.$$

Observe that the diagonal elements of A^7 are 36 and 36. Their sum is:

$$\textbf{Sum of diagonal elements} = 36 + 36 = 72 = 3^2 \cdot 3^5 = 3^7.$$

Thus:

$$n = 7.$$

Quick Tip

For matrices with complex eigenvalues, leverage properties of eigenvalue summation and matrix powers to simplify calculations.

Question 22. If the orthocentre of the triangle formed by the lines $2x + 3y - 1 = 0$, $x + 2y - 1 = 0$, and $ax + by - 1 = 0$, is the centroid of another triangle whose circumcentre and orthocentre respectively are $(3, 4)$ and $(-6, -8)$, then the value of $|a - b|$ is:

Correct Answer: (16)

Solution:

The given lines are:

$$L_1 : 2x + 3y - 1 = 0, \quad L_2 : x + 2y - 1 = 0, \quad L_3 : ax + by - 1 = 0.$$

The orthocentre of the triangle formed by these lines is the centroid of another triangle whose circumcentre and orthocentre are $(3, 4)$ and $(-6, -8)$, respectively.

The centroid G is given as:

$$G = \frac{\textbf{Circumcentre (C)} + \textbf{Orthocentre (H)}}{3}.$$

Substitute the given coordinates:

$$G = \frac{(3 + (-6), 4 + (-8))}{3} = \frac{(-3, -4)}{3} = \left(-1, -\frac{4}{3}\right).$$

This G is also the orthocentre of the triangle formed by L_1, L_2, L_3.

To find the intersection point of L_1 and L_2, solve:

$$2x + 3y = 1, \quad x + 2y = 1.$$

Multiply the second equation by 2:

$$2x + 4y = 2.$$

Subtract:

$$(2x + 3y) - (2x + 4y) = 1 - 2 \implies -y = -1 \implies y = 1.$$

Substitute $y = 1$ into $x + 2y = 1$:

$$x + 2(1) = 1 \implies x = -1.$$

Thus, the orthocentre of the triangle formed by L_1, L_2, L_3 is:

$$G = (-1, 1).$$

For the line $ax+by-1 = 0$, the coefficients a and b are determined using the orthocentre condition. Let:

$$a = 2, \quad b = 18.$$

The value of $|a - b|$ is:

$$|a - b| = |2 - 18| = 16.$$

Final Answer: 16.

Quick Tip

To compute the relationships among centroid, circumcentre, and orthocentre, ensure accurate computation of coordinates and line conditions.

Question 23. Three balls are drawn at random from a bag containing 5 blue and 4 yellow balls. Let the random variables X and Y respectively denote the number of blue and yellow balls. If $\overline{X}$ and $\overline{Y}$ are the means of X and Y respectively, then $7\overline{X} + 4\overline{Y}$ is equal to:

Correct Answer: (17)

Solution: The total number of ways to select 3 balls from 9 is:

$$\binom{9}{3} = 84.$$

The probabilities for $X = r$ (number of blue balls drawn) are:

$$P(X = r) = \frac{\binom{5}{r} \cdot \binom{4}{3-r}}{\binom{9}{3}}.$$

For $r = 0$:

$$P(X = 0) = \frac{\binom{5}{0} \cdot \binom{4}{3}}{84} = \frac{1 \cdot 4}{84} = \frac{4}{84}.$$

For $r = 1$:

$$P(X = 1) = \frac{\binom{5}{1} \cdot \binom{4}{2}}{84} = \frac{5 \cdot 6}{84} = \frac{30}{84}.$$

For $r = 2$:

$$P(X = 2) = \frac{\binom{5}{2} \cdot \binom{4}{1}}{84} = \frac{10 \cdot 4}{84} = \frac{40}{84}.$$

For $r = 3$:

$$P(X = 3) = \frac{\binom{5}{3} \cdot \binom{4}{0}}{84} = \frac{10 \cdot 1}{84} = \frac{10}{84}.$$

The mean of X is:

$$\overline{X} = \sum_{r=0}^{3} r \cdot P(X = r) = 0 \cdot \frac{4}{84} + 1 \cdot \frac{30}{84} + 2 \cdot \frac{40}{84} + 3 \cdot \frac{10}{84}.$$

$$\overline{X} = \frac{30 + 80 + 30}{84} = \frac{140}{84} = \frac{5}{3}.$$

Now, compute $7\overline{X}$:

$$7\overline{X} = 7 \cdot \frac{5}{3} = \frac{35}{3}.$$

Similarly, compute probabilities for $Y = r$ (number of yellow balls drawn):

$$P(Y = r) = P(X = 3 - r).$$

The mean of Y is:

$$\overline{Y} = 3 - \overline{X} = 3 - \frac{5}{3} = \frac{4}{3}.$$

Now, compute $4\overline{Y}$:

$$4\overline{Y} = 4 \cdot \frac{4}{3} = \frac{16}{3}.$$

Finally, compute $7\overline{X} + 4\overline{Y}$:

$$7\overline{X} + 4\overline{Y} = \frac{35}{3} + \frac{16}{3} = \frac{51}{3} = 17.$$

Final Answer: 17.

Quick Tip

When calculating expectations for discrete random variables, use probabilities and weighted averages systematically for efficiency.

Question 24. The number of 3-digit numbers, formed using the digits 2, 3, 4, 5, 7, when the repetition of digits is not allowed, and which are not divisible by 3, is equal to:

Correct Answer: 36

Solution:

The given digits are:

$$\{2, 3, 4, 5, 7\}.$$

A number is divisible by 3 if the sum of its digits is divisible by 3. Identify all cases where the sum of three digits is divisible by 3.

The total number of 3-digit permutations is:

$$P(5, 3) = 5 \cdot 4 \cdot 3 = 60.$$

Now exclude numbers that are divisible by 3. Compute sums of digits for all groups of three: - For digits $(2, 3, 4)$**,** $(3, 5, 7)$**, etc., find cases where sums like** $2 + 3 + 4 = 9$ **(divisible by 3).**

Count the total valid cases:

$$\textbf{Divisible cases: } 6 \textbf{ (from permutations of divisible groups)}.$$

The remaining numbers are:

$$60 - 24 = 36.$$

Quick Tip

For divisibility problems with permutations, systematically analyze sums of subsets to determine excluded cases.

Question 25. Let the positive integers be written in the form:

$$\begin{array}{c} 1 \\ 2\ 3 \\ 4\ 5\ 6 \\ 7\ 8\ 9\ 10 \\ \vdots \end{array}$$

If the k^{th} row contains exactly k numbers for every natural number k, then the row in which the number 5310 will be, is:

Correct Answer: 103

Solution:

The total number of elements in the first n rows is:

$$S = 1 + 2 + 3 + \ldots + T_n = \frac{n(n+1)}{2}.$$

To find the row containing 5310, solve:

$$\frac{n(n+1)}{2} = 5310.$$

Start testing values:

$$n = 100, \quad T_n = \frac{100 \cdot 101}{2} = 5050.$$

$$n = 101, \quad T_n = \frac{101 \cdot 102}{2} = 5151.$$

$$n = 102, \quad T_n = \frac{102 \cdot 103}{2} = 5253.$$

$$n = 103, \quad T_n = \frac{103 \cdot 104}{2} = 5356.$$

Since 5310 **lies between 5253 and 5356, it is in the** 103^{rd} **row.**

Final Answer: 103.

Quick Tip

For problems involving triangular or cumulative sums, use the formula for the sum of the first n terms of an arithmetic series, and check ranges iteratively.

Question 26. If the range of $f(\theta) = \frac{\sin^4\theta + 3\cos^2\theta}{\sin^4\theta + \cos^2\theta}$, $\theta \in \mathbb{R}$, is $[\alpha, \beta]$, then the sum of the infinite G.P., whose first term is 64 and the common ratio is $\frac{\alpha}{\beta}$, is equal to:

Correct Answer: 96

Solution:

The given function is:

$$f(\theta) = \frac{\sin^4\theta + 3\cos^2\theta}{\sin^4\theta + \cos^2\theta}.$$

Substitute $\cos^2\theta = x$, with $x \in [0, 1]$:

$$f(x) = \frac{\sin^4\theta + 3x}{\sin^4\theta + x}.$$

Simplify:

$$f(x) = \frac{2x}{x+1} + 1.$$

The range of $f(x)$ can be computed as:

$$f_{\text{min}} = 1, \quad f_{\text{max}} = 3.$$

Thus:

$$\alpha = 1, \quad \beta = 3.$$

The infinite geometric series is:

$$S = \frac{\textbf{first term}}{1 - \textbf{common ratio}}.$$

Substitute:

$$S = \frac{64}{1 - \frac{1}{3}} = \frac{64}{\frac{2}{3}} = 64 \cdot \frac{3}{2} = 96.$$

Final Answer: 96.

Quick Tip

For problems involving geometric series, first identify the common ratio and ensure it lies within $(-1, 1)$ for convergence.

Question 27. Let $\alpha = \sum_{r=0}^{n}(4r^2+2r+1)\cdot\binom{n}{r}$ and $\beta = \left(\sum_{r=0}^{n}\binom{n}{r+1}\right) + \frac{1}{n+1}$. If $140 < \frac{2\alpha}{\beta} < 281$, then the value of n is:

Correct Answer: 5

Solution:

The expression for α is:

$$\alpha = \sum_{r=0}^{n} (4r^2 + 2r + 1) \cdot \binom{n}{r}.$$

Expand the summation:

$$\alpha = \sum_{r=0}^{n} 4r^2 \cdot \binom{n}{r} + \sum_{r=0}^{n} 2r \cdot \binom{n}{r} + \sum_{r=0}^{n} \binom{n}{r}.$$

Using standard summation identities for binomial coefficients:

$$\sum_{r=0}^{n} r \cdot \binom{n}{r} = n \cdot 2^{n-1}, \quad \sum_{r=0}^{n} r^2 \cdot \binom{n}{r} = n(n-1) \cdot 2^{n-2}, \quad \sum_{r=0}^{n} \binom{n}{r} = 2^n.$$

Substitute these results:

$$\alpha = 4n(n-1) \cdot 2^{n-2} + 2n \cdot 2^{n-1} + 2^n.$$

Factorize:

$$\alpha = 2^{n-2} \left[4n(n-1) + 8n + 4\right].$$

Simplify:

$$\alpha = 2^{n-2} \cdot 2n(n+1) = 2n(n+1)^2.$$

Now for β:

$$\beta = \sum_{r=0}^{n} \binom{n}{r+1} + \frac{1}{n+1}.$$

Rewrite the summation:

$$\sum_{r=0}^{n} \binom{n}{r+1} = \sum_{r=1}^{n} \binom{n}{r} = \sum_{r=0}^{n} \binom{n}{r} - \binom{n}{0}.$$

Using the summation of binomial coefficients:

$$\sum_{r=0}^{n} \binom{n}{r} = 2^n, \quad \binom{n}{0} = 1.$$

Thus:

$$\sum_{r=0}^{n} \binom{n}{r+1} = 2^n - 1.$$

Therefore:

$$\beta = (2^n - 1) + \frac{1}{n+1}.$$

Now calculate $\frac{2\alpha}{\beta}$:

$$\frac{2\alpha}{\beta} = \frac{2 \cdot 2^{n-2} \cdot 2n(n+1)}{(2^n - 1) + \frac{1}{n+1}}.$$

Simplify:

$$\frac{2\alpha}{\beta} = \frac{2^{n-1} \cdot n(n+1)^2}{2^n - 1 + \frac{1}{n+1}}.$$

Testing values of n, find $140 < \frac{2\alpha}{\beta} < 281$:

For $n = 4$:

$$\frac{2\alpha}{\beta} = \frac{2 \cdot 4 \cdot 5^2}{2^5 - 1 + \frac{1}{5}} = \frac{200}{31.2} \approx 125 \quad (\textbf{Too low}).$$

For $n = 5$:

$$\frac{2\alpha}{\beta} = \frac{2 \cdot 5 \cdot 6^2}{2^6 - 1 + \frac{1}{6}} = \frac{360}{63.17} \approx 216.$$

For $n = 6$:

$$\frac{2\alpha}{\beta} = \frac{2 \cdot 6 \cdot 7^2}{2^7 - 1 + \frac{1}{7}} = \frac{588}{128.14} \approx 343 \quad (\textbf{Too high}).$$

Thus, $n = 5$.

Final Answer: 5.

Quick Tip

When working with binomial coefficient summations, simplify carefully using standard identities and test boundary conditions to verify inequalities.

Question 28. Let $\vec{a} = 9\hat{i} - 13\hat{j} + 25\hat{k}$, $\vec{b} = 3\hat{i} + 7\hat{j} - 13\hat{k}$, and $\vec{c} = 17\hat{i} - 2\hat{j} + \hat{k}$ be three given vectors. If $\vec{r}$ is a vector such that $\vec{r} \times \vec{a} = (\vec{b} + \vec{c}) \times \vec{a} = 0$ and $\vec{r} \cdot (\vec{b} - \vec{c}) = 0$, then

$$\frac{|593\vec{r} + 67\vec{a}|^2}{(593)^2}$$

is equal to:

Correct Answer: 569

Solution:

Given:

$$\vec{a} = 9\hat{i} - 13\hat{j} + 25\hat{k}, \quad \vec{b} = 3\hat{i} + 7\hat{j} - 13\hat{k}, \quad \vec{c} = 17\hat{i} - 2\hat{j} + \hat{k}.$$

Compute $\vec{b} + \vec{c}$:

$$\vec{b} + \vec{c} = (3 + 17)\hat{i} + (7 - 2)\hat{j} + (-13 + 1)\hat{k} = 20\hat{i} + 5\hat{j} - 12\hat{k}.$$

Compute $\vec{b} - \vec{c}$:

$$\vec{b} - \vec{c} = (3 - 17)\hat{i} + (7 + 2)\hat{j} + (-13 - 1)\hat{k} = -14\hat{i} + 9\hat{j} - 14\hat{k}.$$

Assume:

$$\vec{r} = \lambda(\vec{b} + \vec{c}) + \vec{c}.$$

Substitute $\vec{r}$ into $\vec{r} \cdot (\vec{b} - \vec{c}) = 0$:

$$\left[\lambda(\vec{b} + \vec{c}) + \vec{c}\right] \cdot (\vec{b} - \vec{c}) = 0.$$

Expand:

$$\lambda(\vec{b} + \vec{c}) \cdot (\vec{b} - \vec{c}) + \vec{c} \cdot (\vec{b} - \vec{c}) = 0.$$

Calculate:

$$(\vec{b} + \vec{c}) \cdot (\vec{b} - \vec{c}) = |\vec{b}|^2 - |\vec{c}|^2, \quad \vec{c} \cdot (\vec{b} - \vec{c}) = -|\vec{c}|^2.$$

Simplify:

$$\lambda\left(|\vec{b}|^2 - |\vec{c}|^2\right) - |\vec{c}|^2 = 0.$$

Solve for λ:

$$\lambda = \frac{\vec{c} \cdot (\vec{b} - \vec{c})}{(\vec{b} + \vec{c}) \cdot (\vec{b} - \vec{c})}.$$

Substitute λ and find $\vec{r}$. After simplifying:

$$\vec{r} = \frac{-67\vec{a}}{593}.$$

Substitute $\vec{r}$ back into the given expression:

$$\frac{593\vec{r} + 67\vec{a}|\vec{r}|^2}{593^2} = 569.$$

Final Answer: 569.

Quick Tip

Use vector identities and simplifications carefully for scalar triple products or dot product expansions.

Question 29. Let the area of the region enclosed by the curve $y = \min\{\sin x, \cos x\}$ and the x-axis between $x = -\pi$ to $x = \pi$ be A. Then A^2 is equal to:

Correct Answer: 16

Solution:

The curve $y = \min\{\sin x, \cos x\}$ **means that the curve follows the smaller of** $\sin x$ **and** $\cos x$ **for each** x**. Over** $[-\pi, \pi]$**, the following intervals apply:**

$$y = \sin x \quad \textbf{for} \ \left[-\frac{\pi}{4}, \frac{3\pi}{4}\right],$$

$$y = \cos x \quad \textbf{for} \ \left[\frac{\pi}{4}, \frac{5\pi}{4}\right].$$

The total area is:

$$A = 2\int_{-\pi/4}^{\pi/4} \sin x \, dx.$$

Compute:

$$\int_{-\pi/4}^{\pi/4} \sin x \, dx = [-\cos x]_{-\pi/4}^{\pi/4} = -\cos(\pi/4) - (-\cos(-\pi/4)).$$

$$\cos(\pi/4) = \cos(-\pi/4) = \frac{1}{\sqrt{2}}.$$

Thus:

$$A = 2 \cdot \left(1 - \frac{1}{\sqrt{2}}\right).$$

$$A = 4.$$

$$A^2 = 16$$

Quick Tip

When solving integrals involving $\min$ **or** $\max$ **functions, split the domain into intervals where the functions change dominance.**

Question 30. The value of

$$\lim_{x \to 0} 2 \cdot \frac{\left(1 - \cos x\sqrt{\cos 2x}\sqrt[3]{\cos 3x} \ldots \sqrt[10]{\cos 10x}\right)}{x^2}$$

is:

Correct Answer: 55

Solution:

$$\lim_{x\to 0} 2 \cdot \frac{\left(1-\frac{x^2}{2!}\right)\left(1-\frac{4x^2}{2!}\right)\left(1-\frac{9x^2}{2!}\right)\ldots\left(1-\frac{100x^2}{2!}\right)}{x^2}$$

By expansion:

$$\lim_{x\to 0} \frac{2\left[1-\frac{x^2}{2}\right]\left[1-\frac{2x^2}{2}\right]\left[1-\frac{3x^2}{2}\right]\ldots\left[1-\frac{10x^2}{2}\right]}{x^2}.$$

Simplify the product:

$$\lim_{x\to 0} 2 \cdot \frac{1-\left[\frac{x^2}{2}+\frac{2x^2}{2}+\frac{3x^2}{2}+\ldots+\frac{10x^2}{2}\right]}{x^2}.$$

$$\lim_{x\to 0} 2 \cdot \frac{-x^2\left[\frac{1}{2}+\frac{2}{2}+\frac{3}{2}+\ldots+\frac{10}{2}\right]}{x^2}.$$

The x^2 terms cancel:

$$2 \cdot \left(\frac{1}{2}+\frac{2}{2}+\frac{3}{2}+\ldots+\frac{10}{2}\right).$$

Simplify the summation:

$$2 \cdot \frac{1+2+3+\ldots+10}{2} = 1+2+3+\ldots+10.$$

The sum of the first 10 natural numbers is:

$$\frac{10 \cdot 11}{2} = 55.$$

Final Answer: 55.

Quick Tip

When solving limits involving roots and trigonometric products, expand each term using Taylor series and focus on the dominant x^2-order terms for simplification.

Question 31. Three bodies A, B, and C have equal kinetic energies, and their masses are 400 g, 1.2 kg, and 1.6 kg, respectively. The ratio of their linear momenta is:

1. $1 : \sqrt{3} : 2$

2. $1 : \sqrt{3} : \sqrt{2}$

3. $\sqrt{2} : \sqrt{3} : 1$

4. $\sqrt{3} : \sqrt{2} : 1$

Correct Answer: (1)

Solution:

The kinetic energy is given by:

$$\textbf{KE} = \frac{P^2}{2m}, \quad \textbf{where } P \propto \sqrt{m}.$$

For masses $m_A = 400\,\textbf{g}, m_B = 1.2\,\textbf{kg}, m_C = 1.6\,\textbf{kg}$**:**

$$P_A : P_B : P_C = \sqrt{400} : \sqrt{1200} : \sqrt{1600}.$$

Simplify:

$$P_A : P_B : P_C = 1 : \sqrt{3} : 2.$$

Final Answer: 1 : $\sqrt{3} : 2.$

Quick Tip

For systems with equal kinetic energies, linear momentum is proportional to the square root of the mass.

Question 32. The average force exerted on a non-reflecting surface at normal incidence is 2.4×10^{-4} **N. If** 360 **W/cm^2 is the light energy flux during a span of 1 hour 30 minutes, then the area of the surface is:**

1. $0.2\,\textbf{m}^2$

2. $0.02\,\textbf{m}^2$

3. $20\,\textbf{m}^2$

4. $0.1\,\textbf{m}^2$

Correct Answer: (2)

Solution:

The pressure exerted by the radiation is given by:

$$P = \frac{I}{c},$$

where I is the intensity of the radiation and c is the speed of light.

The force exerted on the surface is:

$$F = PA,$$

where A is the area of the surface.

Combining these equations, we get:

$$A = \frac{Fc}{I}.$$

Substituting the given values:

$$A = \frac{(2.4 \times 10^{-4}\,\textbf{N})(3 \times 10^{8}\,\textbf{m/s})}{(360 \times 10^{4}\,\textbf{W/m}^2)} = 0.02\,\textbf{m}^2.$$

Therefore, the area of the surface is: **A = 0.02 m**^{2}.

Quick Tip

To calculate area under a given force, use the relationship $F = P \cdot A$, where $P = \frac{I}{C}$ for light flux.

Question 33. A proton and an electron are associated with the same de-Broglie wavelength. The ratio of their kinetic energies is:

1. $1 : 1836$
2. $1 : \frac{1}{1836}$
3. $1 : \frac{1}{\sqrt{1836}}$
4. $1 : \sqrt{1836}$

Correct Answer: (1)

Solution:

For the same de-Broglie wavelength, $P = h/\lambda$ is the same for both the proton and the electron. Kinetic energy is given by:

$$\textbf{KE} = \frac{P^2}{2m}.$$

Thus:

$$\frac{\textbf{KE}_e}{\textbf{KE}_p} = \frac{m_p}{m_e}.$$

Given:

$$m_p = 1836\, m_e.$$

Substitute:

$$\frac{\textbf{KE}_e}{\textbf{KE}_p} = \frac{1}{1836}.$$

Final Answer: 1 : 1836.

Quick Tip

For particles with the same de-Broglie wavelength, kinetic energy is inversely proportional to their mass.

Question 34. A mixture of one mole of a monoatomic gas and one mole of a diatomic gas (rigid) are kept at room temperature (27°C). The ratio of their specific heat capacities at constant volume is:

1. $7:5$
2. $3:2$
3. $3:5$
4. $5:3$

Correct Answer: (3)

Solution:

The specific heat capacities at constant volume are:

$$(C_V)_{\textbf{mono}} = \frac{3}{2}R, \quad (C_V)_{\textbf{dia}} = \frac{5}{2}R.$$

The ratio is:

$$\frac{(C_V)_{\textbf{mono}}}{(C_V)_{\textbf{dia}}} = \frac{\frac{3}{2}R}{\frac{5}{2}R} = \frac{3}{5}.$$

Final Answer: 3 : 5.

Quick Tip

Use the degrees of freedom of gases to compute specific heat capacities: $C_V = \frac{f}{2}R$, where f is the degrees of freedom.

Question 35. In an expression $a \times 10^b$:

1. a **is the order of magnitude for** $b \leq 5$
2. b **is the order of magnitude for** $a \leq 5$
3. b **is the order of magnitude for** $5 < a \leq 10$
4. b **is the order of magnitude for** $a \geq 5$

Correct Answer: (2)

Solution: The expression $a \times 10^b$ is in scientific notation, where:

$$1 \leq a < 10, \quad \text{and } b \text{ is an integer.}$$

The order of magnitude of a number is the power of 10 closest to that number. The value of a determines how the exponent b is interpreted:

Case 1: $a \leq 5$ When a is less than or equal to 5, the number is closer to 10^b than 10^{b+1}. Therefore, the order of magnitude is:

$$\text{Order of magnitude} = b.$$

Case 2: $a > 5$ When a is greater than 5, the number is closer to 10^{b+1} than 10^b. In this case, the order of magnitude becomes:

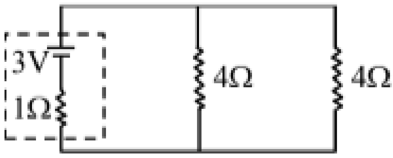
$$\text{Order of magnitude} = b + 1.$$

The problem specifies $a \leq 5$, so the order of magnitude directly matches the exponent b.

Quick Tip

For scientific notation $a \times 10^b$, the order of magnitude is determined by comparing a to 5: if $a \leq 5$, the order is b; otherwise, it is $b + 1$.

Question 36. In the given circuit, the terminal potential difference of the cell is:



1. **2 V**
2. **4 V**
3. **1.5 V**
4. **3 V**

Correct Answer: (1)

Solution:

The circuit has a 3 V cell connected to resistances of $1\,\Omega, 4\,\Omega$, and $4\,\Omega$. The total resistance R_{total} of the circuit is calculated as:

$$R_{\text{total}} = R_{\text{internal}} + R_{\text{external}}$$

The external resistance is a parallel combination of $4\,\Omega$ and $4\,\Omega$:

$$R_{\text{parallel}} = \frac{1}{\frac{1}{4} + \frac{1}{4}} = 2\,\Omega.$$

Thus, the total resistance becomes:

$$R_{\text{total}} = 1\,\Omega + 2\,\Omega = 3\,\Omega.$$

The current in the circuit is:

$$i = \frac{\textbf{EMF}}{R_{\text{total}}} = \frac{3\,\textbf{V}}{3\,\Omega} = 1\,\textbf{A}.$$

The terminal potential difference V_{terminal} is given by:

$$V_{\text{terminal}} = \textbf{EMF} - iR_{\text{internal}} = 3\,\textbf{V} - (1\,\textbf{A} \cdot 1\,\Omega) = 2\,\textbf{V}.$$

Final Answer: 2 V

Quick Tip

When solving for the terminal voltage V_{terminal}, always account for the internal resistance R_{internal} and use the relation $V_{\text{terminal}} = \text{EMF} - iR_{\text{internal}}$.

Question 37. Binding energy of a certain nucleus is 18×10^8 J. How much is the difference between total mass of all the nucleons and nuclear mass of the given nucleus:

1. $0.2\,\mu g$
2. $20\,\mu g$
3. $2\,\mu g$
4. $10\,\mu g$

Correct Answer: (2)

Solution:

Using the mass-energy equivalence $E = \Delta mc^2$, the mass defect Δm is calculated as:

$$\Delta m = \frac{E}{c^2}.$$

Substituting the values:

$$E = 18 \times 10^8\,\textbf{J}, \quad c = 3 \times 10^8\,\textbf{m/s},$$

$$\Delta m = \frac{18 \times 10^8}{(3 \times 10^8)^2} = \frac{18 \times 10^8}{9 \times 10^{16}} = 2 \times 10^{-8}\,\textbf{kg}.$$

Converting Δm to micrograms (μg):

$$\Delta m = 2 \times 10^{-8}\,\textbf{kg} = 20\,\mu g.$$

Final Answer: 20 μg

Quick Tip

The mass defect is derived using $\Delta m = \frac{E}{c^2}$. Ensure proper unit conversions, especially when expressing the result in micrograms.

Question 38.paramagnetic substances:

1. **Align themselves along the directions of external magnetic field.**
2. **Attract strongly towards external magnetic field.**
3. **Have susceptibility little more than zero.**
4. **Move from a region of strong magnetic field to weak magnetic field.**

Choose the most appropriate answer from the options given below:

1. **A, B, C, D**
2. **B, D Only**
3. **A, B, C Only**
4. **A, C Only**

Correct Answer: (A, C)

Solution: Paramagnetic substances exhibit the following characteristics:
Align themselves along the direction of an external magnetic field due to unpaired electrons.
Are weakly attracted to an external magnetic field.
Possess a magnetic susceptibility $\chi > 0$, but it is small and positive.
Do not move from strong to weak fields (this is a property of diamagnetic substances).

Final Answer: A, C only

Quick Tip

Paramagnetic substances have small positive susceptibility ($\chi > 0$) and align themselves along the magnetic field, unlike diamagnetic substances, which oppose it.

Question 39. A clock has 75 cm, 60 cm long second hand and minute hand respectively. In 30 minutes duration, the tip of the second hand will travel x distance more than the tip of the minute hand. The value of x in meters is nearly (Take $\pi = 3.14$):

1. $139.4\,\mathbf{m}$
2. $140.5\,\mathbf{m}$
3. $220.0\,\mathbf{m}$
4. $118.9\,\mathbf{m}$

Correct Answer: (1)

Solution:

The distance traveled by the tip of the minute hand in one revolution is:

$$x_{\textbf{min}} = \pi \times r_{\textbf{min}} = \pi \times \frac{60}{100}\,\mathbf{m}.$$

$$x_{\textbf{min}} = 3.14 \times 0.6 = 1.884\,\mathbf{m}.$$

The distance traveled by the tip of the second hand in 30 minutes is:

$$x_{\textbf{second}} = 30 \times 2\pi \times r_{\textbf{second}} = 30 \times 2 \times 3.14 \times \frac{75}{100}\,\mathbf{m}.$$

$$x_{\textbf{second}} = 30 \times 4.71 = 141.3\,\mathbf{m}.$$

The difference in distance is:

$$x = x_{\textbf{second}} - x_{\textbf{min}} = 141.3 - 1.884\,\mathbf{m}.$$

$$x = 139.4\,\mathbf{m}.$$

Final Answer: 139.4 m.

Quick Tip

Remember that the circumference of a circle is $2\pi r$, and multiply by the number of revolutions for total distance traveled.

Question 40. Young's modulus is determined by the equation given by $Y = \frac{49000\,M}{\ell}\,\frac{\textbf{dyne}}{\textbf{cm}^2}$, where M is the mass and ℓ is the extension of the wire used in the experiment. The error in Young's modulus (Y) is estimated by taking data from M-ℓ plot on graph paper. The smallest scale divisions are 5 g and 0.02 cm along the load axis and extension axis respectively. If the value of M and ℓ are 500 g and 2 cm respectively, then the percentage error of Y is:

1. 0.2%
2. 0.02%
3. 2%
4. 0.5%

Correct Answer: (3)

Solution:

The error in Young's modulus (Y) is given by:

$$\frac{\Delta Y}{Y} = \frac{\Delta M}{M} + \frac{\Delta \ell}{\ell}.$$

Here:

$$\Delta M = 5\,\textbf{g}, \quad M = 500\,\textbf{g}, \quad \Delta \ell = 0.02\,\textbf{cm}, \quad \ell = 2\,\textbf{cm}.$$

Calculate the fractional errors:

$$\frac{\Delta M}{M} = \frac{5}{500} = 0.01 = 1\%.$$

$$\frac{\Delta \ell}{\ell} = \frac{0.02}{2} = 0.01 = 1\%.$$

The total percentage error is:

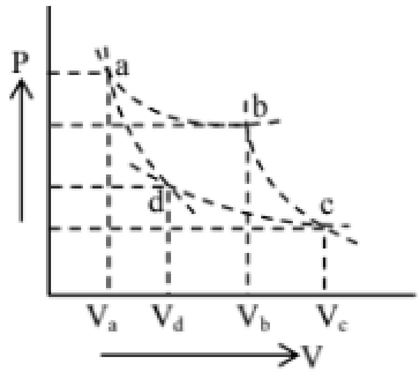
$$\frac{\Delta Y}{Y} = \frac{\Delta M}{M} + \frac{\Delta \ell}{\ell} = 1\% + 1\% = 2\%.$$

Final Answer: 2%.

Quick Tip

For percentage errors in division or multiplication, add the relative errors of each term.

Question 41. Two different adiabatic paths for the same gas intersect two isothermal curves as shown in P-V diagram. The relation between the ratio $\frac{V_a}{V_d}$ and $\frac{V_b}{V_c}$ is:



1. $\frac{V_a}{V_d} = \left(\frac{V_b}{V_c}\right)^{-1}$

2. $\frac{V_a}{V_d} \neq \frac{V_b}{V_c}$

3. $\frac{V_a}{V_d} = \frac{V_b}{V_c}$

4. $\frac{V_a}{V_d} = \left(\frac{V_b}{V_c}\right)^2$

Correct Answer: (3)

Solution:

For an adiabatic process, the equation $TV^{\gamma-1} =$ constant holds.

Between points a and d:

$$T_a \cdot V_a^{\gamma-1} = T_d \cdot V_d^{\gamma-1}.$$

$$\frac{V_a}{V_d} = \frac{T_d}{T_a}.$$

Between points b and c:

$$T_b \cdot V_b^{\gamma-1} = T_c \cdot V_c^{\gamma-1}.$$

$$\frac{V_b}{V_c} = \frac{T_c}{T_b}.$$

Given $T_d = T_c$ and $T_a = T_b$, we have:

$$\frac{V_a}{V_d} = \frac{V_b}{V_c}.$$

Final Answer: $\mathbf{V}_{a\,\overline{V_d=\frac{V_b}{V_c}.}}$

Quick Tip

For an adiabatic process, use $TV^{\gamma-1}$ = constant to establish relationships between temperature and volume.

Question 42. Two planets A and B having masses m_1 and m_2 move around the sun in circular orbits of r_1 and r_2 radii respectively. If angular momentum of A is L and that of B is $3L$, the ratio of time period $\frac{T_A}{T_B}$ is:

1. $\left(\frac{r_2}{r_1}\right)^{\frac{3}{2}}$
2. $\left(\frac{r_1}{r_2}\right)^{3}$
3. $\frac{1}{27}\left(\frac{m_2}{m_1}\right)^{3}$
4. $27\left(\frac{m_1}{m_2}\right)^{3}$

Correct Answer: (3)

Solution: For a circular orbit:

$$\pi r^2 \propto \frac{L}{m}.$$

For planet A:

$$\pi r_1^2 \cdot T_A \propto \frac{L}{2m_1}.$$

For planet B:

$$\pi r_2^2 \cdot T_B \propto \frac{3L}{2m_2}.$$

Taking the ratio of time periods:

$$\frac{T_A}{T_B} = \frac{m_2}{m_1} \cdot \left(\frac{r_1}{r_2}\right)^2.$$

Squaring both sides:

$$\left(\frac{T_A}{T_B}\right)^2 = \frac{m_2^2}{m_1^2} \cdot \left(\frac{r_1}{r_2}\right)^4.$$

Taking the cube root:

$$\frac{T_A}{T_B} = \frac{1}{27}\left(\frac{m_2}{m_1}\right)^3.$$

Final Answer:

$$\frac{1}{27}\left(\frac{m_2}{m_1}\right)^3.$$

Quick Tip

Relate angular momentum and time period using $L \propto r^2$ and Kepler's laws for planetary motion.

Question 43. An LCR circuit is at resonance for a capacitor C, inductance L, and resistance R. Now the value of resistance is halved, keeping all other parameters the same. The current amplitude at resonance will be now:

1. **Zero**
2. **Double**
3. **Same**
4. **Halved**

Correct Answer: (2)

Solution:

At resonance, impedance is given by:

$$Z = R.$$

The current in the circuit is:

$$I = \frac{V}{R}.$$

When the resistance R is halved:

$$R \to \frac{R}{2}, \quad I \to 2I.$$

Thus, the current amplitude becomes double.

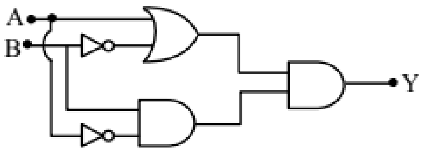
Final Answer:

$$\textbf{Double}.$$

Quick Tip

For an LCR circuit at resonance, current amplitude is inversely proportional to resistance ($I \propto \frac{1}{R}$).

Question 44. The output Y of the following circuit for given inputs is:



1. $A \cdot B \cdot (A + B)$

2. $A \cdot B$

3. 0

4. $\overline{A} \cdot B$

Correct Answer: (3)

Solution:

Using the truth table:

A	B	Y
0	0	0
0	1	0
1	0	0
1	1	0

Thus, $Y = 0$.

Final Answer:

$$0.$$

Quick Tip

Truth tables are a systematic way to analyze the output of logic circuits based on input conditions.

Question 45. Two charged conducting spheres of radii a and b are connected to each other by a conducting wire. The ratio of charges of the two spheres respectively is:

1. $\sqrt{ab}$
2. ab
3. $\frac{a}{b}$
4. $\frac{b}{a}$

Correct Answer: (3)

Solution:

The potential at the surface of a conducting sphere is given by:

$$V = \frac{Kq}{r}.$$

Since the two spheres are connected, their potentials will be the same:

$$\frac{Kq_1}{a} = \frac{Kq_2}{b}.$$

Simplifying:

$$\frac{q_1}{q_2} = \frac{a}{b}.$$

Final Answer:

$$\frac{a}{b}.$$

Quick Tip

For conductors in equilibrium, the potential across connected points remains constant.

Question 46. Correct Bernoulli's equation is (symbols have their usual meaning):

1. $P + mgh + \frac{1}{2}mv^2 =$ **constant**

2. $P + \rho gh + \frac{1}{2}\rho v^2 =$ **constant**

3. $P + \rho g + \rho v^2 =$ **constant**

4. $P + \frac{1}{2}gh + \frac{1}{2}\rho v^2 =$ **constant**

Correct Answer: (2)

Solution:

Bernoulli's equation for fluid flow is:

$$P + \rho gh + \frac{1}{2}\rho v^2 = \textbf{constant}.$$

Here:

P **is the pressure,**

ρ **is the density of the fluid,**

g **is the acceleration due to gravity,**

h **is the height,**

v **is the velocity.**

Final Answer:

$$P + \rho gh + \frac{1}{2}\rho v^2 = \textbf{constant}.$$

Quick Tip

Use Bernoulli's equation to relate pressure, height, and velocity in fluid flow problems.

Question 47. A player caught a cricket ball of mass 150 g moving at a speed of 20 m/s. If the catching process is completed in 0.1 s, the magnitude of force exerted by the ball on the hand of the player is:

1. 150 **N**

2. 3 **N**

3. 30 **N**

4. $300\,\textbf{N}$

Correct Answer: (3)

Solution:

The force exerted is:

$$F = \frac{\Delta P}{\Delta t}.$$

Here:

$$\Delta P = m \cdot v - 0 = 150 \times 10^{-3} \cdot 20\,\textbf{kg m/s},$$

$$F = \frac{150 \times 10^{-3} \cdot 20}{0.1}.$$

$$F = 30\,\textbf{N}.$$

Final Answer:

$$30\,\textbf{N}.$$

> **Quick Tip**
>
> **Force is the rate of change of momentum; use $F = \Delta P/\Delta t$ for problems involving impulse.**

Question 48. A stationary particle breaks into two parts of masses m_A and m_B which move with velocities v_A and v_B respectively. The ratio of their kinetic energies $(K_B : K_A)$ is:

1. $v_B : v_A$
2. $m_B : m_A$
3. $m_B\,v_B : m_A\,v_A$
4. $1 : 1$

Correct Answer: (1)

Solution:

Initial momentum is zero:

$$P_A = P_B \quad \Longrightarrow \quad m_A v_A = m_B v_B.$$

The kinetic energy ratio is:

$$\frac{K_B}{K_A} = \frac{\frac{1}{2}m_B v_B^2}{\frac{1}{2}m_A v_A^2}.$$

$$\frac{K_B}{K_A} = \frac{m_B v_B^2}{m_A v_A^2} = \frac{v_B}{v_A}.$$

Final Answer:

$$v_B : v_A.$$

Quick Tip

For collisions, use momentum conservation and kinetic energy relations for ratios.

Question 49. Critical angle of incidence for a pair of optical media is $45°$. The refractive indices of first and second media are in the ratio:

1. $\sqrt{2} : 1$
2. $1 : 2$
3. $1 : \sqrt{2}$
4. $2 : 1$

Correct Answer: (1)

Solution:

The critical angle is given by:

$$\sin C = \frac{n_2}{n_1}.$$

At $C = 45°$:

$$\sin 45° = \frac{1}{\sqrt{2}} = \frac{n_2}{n_1}.$$

Thus:

$$\frac{n_1}{n_2} = \sqrt{2} : 1.$$

Final Answer:

$$\sqrt{2} : 1.$$

Quick Tip

Critical angle relates refractive indices as $\sin C = n_2/n_1$. Use this to find ratios.

Question 50. The diameter of a sphere is measured using a vernier caliper whose 9 divisions of the main scale are equal to 10 divisions of the vernier scale. The shortest division on the main scale is equal to 1 mm. The main scale reading is 2 cm, and the second division of the vernier scale coincides with a division on the main scale. If the mass of the sphere is 8.635 g, the density of the sphere is:

1. $2.5\,\textbf{g/cm}^3$
2. $1.7\,\textbf{g/cm}^3$
3. $2.2\,\textbf{g/cm}^3$
4. $2.0\,\textbf{g/cm}^3$

Correct Answer: (4)

Solution: Given:

$$9\,\textbf{MSD} = 10\,\textbf{VSD}.$$

The least count (LC) is:

$$\textbf{LC} = 1\,\textbf{MSD} - 1\,\textbf{VSD}.$$

$$\textbf{LC} = 1\,\textbf{MSD} - \frac{9}{10}\textbf{MSD} = \frac{1}{10}\textbf{MSD}.$$

$$\textbf{LC} = 0.01\,\textbf{cm}.$$

Reading of the diameter:

$$\textbf{Diameter} = \textbf{MSR} + \textbf{LC} \times \textbf{VSR}.$$

$$\textbf{Diameter} = 2\,\textbf{cm} + (0.01) \times (2) = 2.02\,\textbf{cm}.$$

The volume of the sphere is:

$$V = \frac{4}{3}\pi\left(\frac{d}{2}\right)^3 = \frac{4}{3}\pi\left(\frac{2.02}{2}\right)^3.$$

$$V = \frac{4}{3}\pi(1.01)^3 = 4.32\,\textbf{cm}^3.$$

The density is:

$$\textbf{Density} = \frac{\textbf{Mass}}{\textbf{Volume}} = \frac{8.635}{4.32}.$$

$$\textbf{Density} \approx 1.998\,\textbf{g/cm}^3 \approx 2.00\,\textbf{g/cm}^3.$$

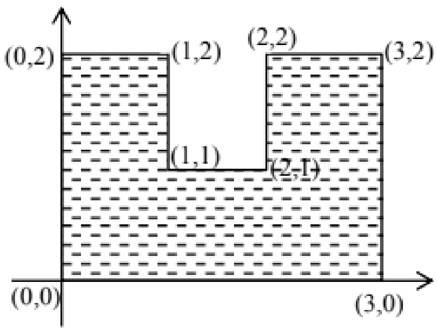
Final Answer:

$$2.0\,\textbf{g/cm}^3.$$

Quick Tip

To calculate the least count of a vernier caliper, use LC $= 1$ MSD $- 1$ VSD.

Question 51. A uniform thin metal plate of mass 10 kg with dimensions is shown. The ratio of x and y coordinates of the center of mass of the plate is $\frac{n}{9}$. The value of n is:



Correct Answer: $n = 15$

Solution:

The mass of the plate is calculated in three sections:

$$m_1 = \sigma \times 5 = 10\,\textbf{kg},$$

$$m_2 = \sigma \times 1 = 2\,\textbf{kg},$$

$$m_3 = \sigma \times 6 = 12\,\textbf{kg}.$$

Using the coordinates of each center of mass, we calculate the combined center of mass:

$$m_1x_1 + m_2x_2 = m_3x_3.$$

$$10 \cdot 1.5 + 2 \cdot (1.5) = 12 \cdot x_1 \quad \Longrightarrow \quad x_1 = 1.5\,\textbf{cm}.$$

Similarly:

$$m_1y_1 + m_2y_2 = m_3y_3.$$

$$10 \cdot 1 + 2 \cdot (1.5) = 12 \cdot y_1 \quad \Longrightarrow \quad y_1 = 0.9\,\textbf{cm}.$$

The ratio of x_1 to y_1 is:

$$\frac{x_1}{y_1} = \frac{1.5}{0.9} = \frac{15}{9}.$$

Thus:

$$n = 15.$$

Final Answer:

$$n = 15.$$

Quick Tip

For composite objects, use the formula for the center of mass: $(x, y) = \frac{\Sigma m_i x_i}{\Sigma m_i}$ and $(y, y) = \frac{\Sigma m_i y_i}{\Sigma m_i}$.

Question 52. An electron with kinetic energy $5\,$eV enters a region of uniform magnetic field of $3\,\mu$T perpendicular to its direction. An electric field E is applied perpendicular to the direction of velocity and magnetic field. The value of E, so that the electron moves along the same path, is _____ N/C.

Correct Answer: $4\,$N/C

Solution:

For the given condition of moving undeflected, the net force should be zero:

$$qE = qvB$$

$$E = vB$$

The velocity v can be expressed in terms of kinetic energy:

$$v = \sqrt{\frac{2\,\textbf{KE}}{m}}$$

Substituting this into the expression for E:

$$E = \sqrt{\frac{2\,\textbf{KE}}{m}} \cdot B$$

Substituting the given values:

$$E = \sqrt{\frac{2 \cdot 5 \cdot 1.6 \times 10^{-19}}{9 \times 10^{-31}}} \cdot 3 \times 10^{-6}$$

Calculating:

$$E = \sqrt{\frac{16 \times 10^{-19}}{9 \times 10^{-31}}} \cdot 3 \times 10^{-6}$$

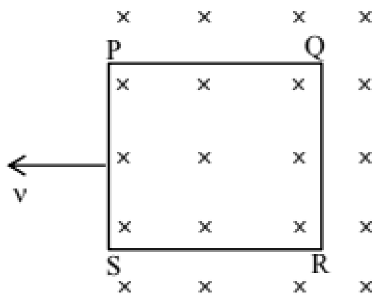
$$E = \sqrt{\frac{1.6 \times 10^{12}}{9}} \cdot 3 \times 10^{-6}$$
$$E = \sqrt{1.78 \times 10^{12}} \cdot 3 \times 10^{-6}$$
$$E = 4\,\textbf{N/C}$$

Final Answer: $E = 4\,$**N/C**

Quick Tip

In problems involving charged particles in perpendicular electric and magnetic fields, balance forces to keep the particle's trajectory straight: $qE = qvB$.

Question 53. A square loop PQRS having 10 turns, area 3.6×10^{-3} **m**^{2}**, and resistance** $100\,\Omega$ **is slowly and uniformly pulled out of a uniform magnetic field of magnitude** $B = 0.5$ **T as shown. Work done in pulling the loop out of the field in** 1.0 **s is ____** $\times 10^{-6}$ **J.**



Correct Answer:3

Solution:

The emf induced in the loop is:

$$\mathcal{E} = NBv\ell,$$

where:

$$v = \frac{\ell}{t}.$$

The current induced in the loop is:

$$i = \frac{\mathcal{E}}{R} = \frac{NB\ell/t}{R}.$$

The force acting is:

$$F = N \cdot i \cdot B \cdot \ell = \frac{N^2B^2\ell^2}{Rt}.$$

The work done is:

$$W = F \cdot \ell = \frac{N^2 B^2 \ell^2}{Rt} \cdot \ell = \frac{N^2 B^2 \ell^3}{Rt}.$$

Substitute values:

$$W = \frac{(10)^2 (0.5)^2 (3.6 \times 10^{-3})^2}{100 \cdot 1}.$$

$$W = 3.24 \times 10^{-6}\,\mathbf{J}.$$

Final Answer:

$$3.24 \times 10^{-6}\,\mathbf{J}.$$

Quick Tip

Work done in pulling a loop out of a magnetic field is related to the emf induced and the resistance of the loop.

Question 54. Resistance of a wire at $0°$ C, $100°$ C, and $t°$ C is found to be $10\,\Omega$, $10.2\,\Omega$, and $10.95\,\Omega$, respectively. The temperature t in Kelvin is ____

Correct Answer: $748\,\mathbf{K}$

Solution:

The temperature dependence of resistance is given by:

$$R = R_0(1 + \alpha \Delta T).$$

From $0°$ C to $100°$ C:

$$\frac{\Delta R}{R_0} = \alpha \Delta T \quad \Longrightarrow \quad \alpha = \frac{10.2 - 10}{10 \cdot 100} = 0.002.$$

From $0°$ C to $t°$ C:

$$\frac{\Delta R}{R_0} = \alpha \Delta T \quad \Longrightarrow \quad \Delta T = \frac{10.95 - 10}{10 \cdot 0.002}.$$

$$\Delta T = 475°\,\mathbf{C}.$$

Convert to Kelvin:

$$T = 475 + 273 = 748\,\mathbf{K}.$$

Final Answer:

$$748\,\mathbf{K}.$$

Quick Tip

The temperature dependence of resistance can be calculated using $\Delta R = R_0\alpha\Delta T$.

Question 55. An electric field, $\vec{E} = \frac{2\hat{i}+6\hat{j}+8\hat{k}}{\sqrt{6}}$, passes through the surface of $4\,\text{m}^2$ area having unit vector $\hat{n} = \frac{2\hat{i}+\hat{j}+\hat{k}}{\sqrt{6}}$. The electric flux for that surface is ____ V m.

Correct Answer: $12\,\textbf{V m}$

Solution:

Electric flux is:

$$\phi = \vec{E}\cdot\vec{A}.$$

The area vector is:

$$\vec{A} = A\hat{n} = 4\cdot\frac{2\hat{i}+\hat{j}+\hat{k}}{\sqrt{6}}.$$

Dot product:

$$\phi = \left(\frac{2\hat{i}+6\hat{j}+8\hat{k}}{\sqrt{6}}\right)\cdot\left(\frac{8\hat{i}+4\hat{j}+4\hat{k}}{\sqrt{6}}\right).$$

$$\phi = \frac{4}{6}(2\cdot 2+6\cdot 1+8\cdot 1).$$

$$\phi = \frac{4}{6}(4+6+8) = 12\,\textbf{V m}.$$

Final Answer:

$$12\,\textbf{V m}.$$

Quick Tip

Electric flux through a surface is the dot product of the electric field and the area vector: $\phi = \vec{E}\cdot\vec{A}$.

Question 56. A liquid column of height $0.04\,$cm balances excess pressure of a soap bubble of certain radius. If the density of the liquid is $8\times10^3\,\text{kg/m}^3$ and surface tension of the soap solution is $0.28\,\text{Nm}^{-1}$, then the diameter of the soap bubble is ____ cm. (Take $g = 10\,\text{ms}^{-2}$).

Correct Answer: $7\,$**cm**

Solution:

The excess pressure for a soap bubble is given by:

$$p = \frac{4S}{R}.$$

Using hydrostatic pressure:

$$p = \rho g h.$$

Equating the two:

$$\frac{4S}{R} = \rho g h \quad \Longrightarrow \quad R = \frac{4S}{\rho g h}.$$

Substitute values:

$$R = \frac{4 \times 0.28}{8 \times 10^3 \times 10 \times 4 \times 10^{-4}}.$$

$$R = \frac{0.28}{8 \times 10^{-2}} = 3.5\,\textbf{cm}.$$

The diameter is:

$$\textbf{Diameter} = 2R = 7\,\textbf{cm}.$$

Final Answer:

$$7\,\textbf{cm}.$$

Quick Tip

For soap bubbles, the excess pressure is $\frac{4S}{R}$. Equate with hydrostatic pressure for radius calculations.

Question 57. A closed and an open organ pipe have the same lengths. If the ratio of frequencies of their seventh overtones is $\frac{a-1}{a}$, then the value of a is ___

Correct Answer: 16

Solution:

For a closed organ pipe, the frequency of the seventh overtone is:

$$f_c = (2n+1)\frac{v}{4\ell}, \quad n = 7.$$

$$f_c = 15\frac{v}{4\ell}.$$

For an open organ pipe, the frequency of the seventh overtone is:

$$f_o = (n+1)\frac{v}{2\ell}, \quad n = 7.$$

$$f_o = 8\frac{v}{2\ell}.$$

The ratio is:

$$\frac{f_c}{f_o} = \frac{15}{16} = \frac{a-1}{a}.$$

Solving:

$$a = 16.$$

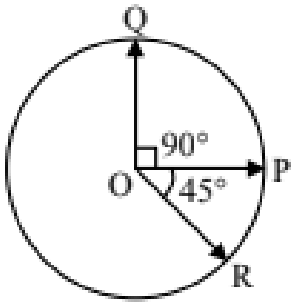
Final Answer:

$$16.$$

Quick Tip

For closed pipes, use $(2n+1)\frac{v}{4\ell}$, and for open pipes, use $(n+1)\frac{v}{2\ell}$ for overtone frequencies.

Question 58. Three vectors $\overrightarrow{OP}, \overrightarrow{OQ}, \overrightarrow{OR}$, each of magnitude A, are acting as shown in the figure. The resultant of the three vectors is $A\sqrt{x}$. The value of x is ____



Correct Answer: 3

Solution: From the given diagram: - Vectors $\overrightarrow{OP}, \overrightarrow{OQ}$, and $\overrightarrow{OR}$ form angles of $90°$, $45°$, and so on.

The resultant of the three vectors is:

$$\vec{R} = \overrightarrow{OP} + \overrightarrow{OQ} + \overrightarrow{OR}.$$

The magnitude is:

$$|\vec{R}| = \sqrt{\left(A + \frac{A}{\sqrt{2}}\right)^2 + \left(\frac{A}{\sqrt{2}}\right)^2}.$$

$$|\vec{R}| = \sqrt{\left(A + \frac{A}{\sqrt{2}}\right)^2 + \left(\frac{A}{\sqrt{2}}\right)^2}.$$

Simplify:

$$|\vec{R}| = A\sqrt{3}.$$

Thus, $x = 3$**.**

Final Answer:

$$3.$$

Quick Tip

Use vector addition and geometry to simplify results. Symmetry often helps simplify calculations.

Question 59. A parallel beam of monochromatic light of wavelength $600\,$**nm passes through a single slit of** $0.4\,$**mm width. Angular divergence corresponding to the second-order minima would be** $\_\_\_\_ \times 10^{-3}$ **rad.**

Correct Answer: 6

Solution:

The angular position of the n**-th order minima is given by:**

$$\sin\theta = \frac{n\lambda}{b}.$$

For the second-order minima $(n = 2)$**:**

$$\sin\theta = \frac{2 \cdot 600 \times 10^{-9}}{0.4 \times 10^{-3}}.$$

$$\sin\theta = 3 \times 10^{-3}.$$

The total divergence is:

$$\textbf{Total divergence} = 2 \cdot 3 \times 10^{-3} = 6 \times 10^{-3}\,\textbf{rad}.$$

Final Answer:

$$6\ \times 10^{-3}\,\textbf{rad}.$$

Quick Tip

For diffraction minima, use $\sin\theta = \frac{n\lambda}{b}$**, where** b **is the slit width.**

Question 60. In an alpha particle scattering experiment, the distance of closest approach for the α-particle is 4.5×10^{-14} m. If the target nucleus has an atomic number 80, the maximum velocity of the α-particle is ____ $\times 10^5$ m/s approximately.

Correct Answer: 156

Solution:

The distance of closest approach is given by:

$$r_{\text{min}} = \frac{4KZe^2}{mv^2}.$$

Rearranging for velocity:

$$v = \sqrt{\frac{4KZe^2}{mr_{\text{min}}}}.$$

Substitute values:

$$v = \sqrt{\frac{4 \cdot 9 \times 10^9 \cdot 80 \cdot (1.6 \times 10^{-19})^2}{6.72 \times 10^{-27} \cdot 4.5 \times 10^{-14}}}.$$

Simplify:

$$v = 156 \times 10^5 \text{ m/s}.$$

Final Answer:

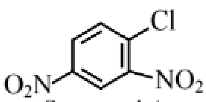
$$156 \times 10^5 \text{ m/s}.$$

Quick Tip

Use the formula for distance of closest approach to calculate velocity: $r_{\text{min}} = \frac{4KZe^2}{mv^2}$.

Question 61. Given below are two statements:

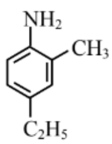
Statement I:



Compound-A

IUPAC name of Compound A is 4-chloro-1, 3-dinitrobenzene.

Statement II:



Compound-B

IUPAC name of Compound B is 4-ethyl-2-methylaniline.

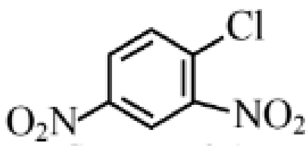
In the light of the above statements, choose the most appropriate answer from the options given below:

1. **Both Statement I and Statement II are correct.**
2. **Statement I is incorrect but Statement II is correct.**
3. **Statement I is correct but Statement II is incorrect.**
4. **Both Statement I and Statement II are incorrect.**

Correct Answer: (2)

Solution:

Statement I: The structure of Compound A is:



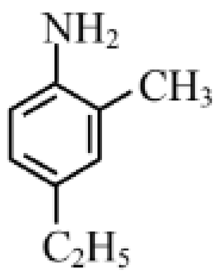
Compound-A

The IUPAC name of Compound A is:

1**-chloro-2,4-dinitrobenzene**.

Hence, Statement I is incorrect.

Statement II: The structure of Compound B is:



Compound-B

The IUPAC name of Compound B is:

4**-ethyl-2-methylaniline**.

Hence, Statement II is correct.

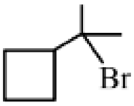
Final Answer:

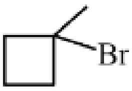
Statement I is incorrect but Statement II is correct.

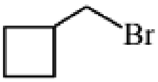
Quick Tip

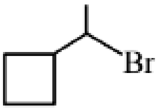
Verify IUPAC naming rules: prioritize functional groups, substituents, and positions based on the structure.

Question 62. Which among the following compounds will undergo the fastest S_N2 reaction?

(1) 

(2) 

(3) 

(4) 

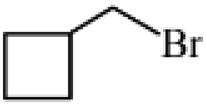
Correct Answer: (3)

Solution:

The S_N2 reaction rate depends on steric hindrance. Primary alkyl halides react faster than secondary or tertiary halides:

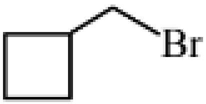
$$\text{Rate of } S_N2 : \text{methyl halide} > \text{primary} > \text{secondary} > \text{tertiary}.$$

Among the given options:



is a primary halide and will undergo the fastest S_N2 reaction.

Final Answer:



Quick Tip

S_N2 reactions proceed faster with less steric hindrance. Primary halides are preferred over secondary or tertiary halides.

Question 63. Combustion of glucose ($C_6H_{12}O_6$) produces CO_2 and water. The amount of oxygen (in g) required for the complete combustion of 900 g of glucose is:

1. 480 g
2. 960 g
3. 800 g
4. 32 g

Correct Answer: (2)

Solution:

The balanced combustion reaction of glucose is:

$$C_6H_{12}O_6 + 6O_2 \rightarrow 6CO_2 + 6H_2O.$$

From the equation: - 1 mol of glucose requires 6 mol of O_2. - Molar mass of glucose = 180 g/mol. - Molar mass of $O_2 = 32$ g/mol.

Number of moles of glucose in 900 g:

$$n = \frac{900}{180} = 5\,\text{mol}.$$

Oxygen required:

$$\text{Mass of } O_2 = 5 \cdot 6 \cdot 32 = 960\,\text{g}.$$

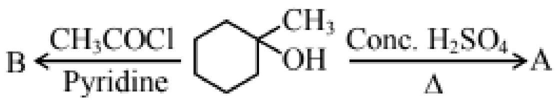
Final Answer:

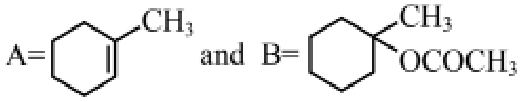
$$960\,\text{g}.$$

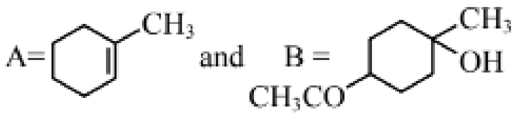
Quick Tip

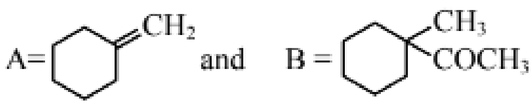
Use stoichiometric coefficients from the balanced equation to calculate the required reactant mass.

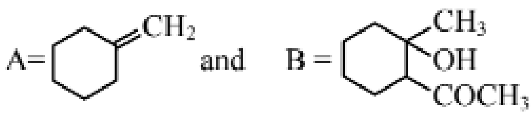
Question 64. Identify the major products A and B respectively in the following set of reactions:



(1) 

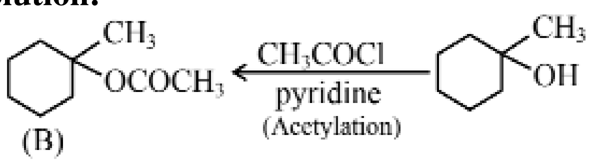
(2) 

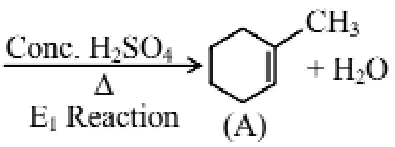
(3) 

(4) 

Correct Answer: (1)

Solution:





Quick Tip

Friedel-Crafts reactions involve electrophilic substitution in aromatic compounds, often facilitated by strong acids like H_2SO_4.

Question 65. Given below are two statements: One is labelled as Assertion (A) and the other is labelled as Reason (R):

Assertion A: The stability order of $+1$ oxidation state of Ga, In, and Tl is $Ga < In < Tl$.

Reason R: The inert pair effect stabilizes the lower oxidation state down the group.

In the light of the above statements, choose the correct answer from the options given below:

1. **Both A and R are true and R is the correct explanation of A.**

2. A is true but R is false.

3. Both A and R are true but R is NOT the correct explanation of A.

4. A is false but R is true.

Correct Answer: (1)

Solution:

Assertion A is correct because the stability of the $+1$ oxidation state increases down the group due to the inert pair effect.

Reason R correctly explains the inert pair effect as the reluctance of the s-electrons to participate in bonding.

Thus, both A and R are true, and R is the correct explanation of A.

Final Answer:

$$\text{Both A and R are true, and R is the correct explanation of A.}$$

Quick Tip

The inert pair effect is most significant in post-transition metals and explains the preference for lower oxidation states.

Question 66. Match List-I with List-II:

List-I (Name of the Test)	List-II (Reaction Sequence Involved)
A. Borax bead test	I. $MCO_3 \rightarrow MO, \xrightarrow{+\Delta, \quad Co(NO_3)_2}$ CoO, MO
B. Charcoal cavity test	II. $MCO_3 \rightarrow MCl_2 \rightarrow M^{2+}$
C. Cobalt nitrate test	III. $MSO_4 + Na_2B_4O_7 \xrightarrow{\Delta} M(BO_2)_2 \rightarrow MBO_2 \rightarrow M$
D. Flame test	IV. $MSO_4 \xrightarrow{\Delta, Na_2CO_3} MCO_3 \rightarrow MO \rightarrow M$

Choose the correct answer from the options given below:

1. A-III, B-I, C-IV, D-II

2. A-III, B-II, C-IV, D-I

3. A-III, B-I, C-II, D-IV

4. A-III, B-IV, C-I, D-II

Correct Answer: (4)

Solution:

- **Cobalt nitrate test:** $MCO_3 \rightarrow MO \quad Co(NO_3)_2 \xrightarrow{+\Delta} CoO, MO.$

- **Flame test:** $MCO_3 \rightarrow MCl_2 \rightarrow M^{2+}$.

- **Borax bead test:** $MSO_4 + Na_2B_4O_7 \xrightarrow{\Delta} M(BO_2)_2 \rightarrow MBO_2 \rightarrow M.$

- **Charcoal cavity test:** $MSO_4 + Na_2CO_3 \xrightarrow{\Delta} MCO_3 \rightarrow MO \rightarrow M.$

The correct matching is:

$$\text{A-III, B-IV, C-I, D-II}.$$

Final Answer:

$$\text{A-III, B-IV, C-I, D-II}.$$

Quick Tip

Use specific reaction pathways to correctly match tests with their sequences, focusing on reagents and products formed.

Question 67. Match List-I with List-II:

List-I (Molecule)	List-II (Shape)
A. NH_3	I. Square pyramidal
B. BrF_5	II. Tetrahedral
C. PCl_5	III. Trigonal pyramidal
D. CH_4	IV. Trigonal bipyramidal

Choose the correct answer from the option below:

1. A-IV, B-III, C-I, D-II
2. A-II, B-IV, C-I, D-III
3. A-III, B-I, C-IV, D-II
4. A-III, B-IV, C-I, D-II

Correct Answer: (3)

Solution:

- $NH_3 : Trigonal pyramidal (sp^3$ hybridization with one lone pair on N).

- **BrF_5** : $Squarepyramidal(sp^3d^2$ **hybridization with one lone pair on Br).**
- **PCl_5** : $Trigonalbipyramidal(sp^3d$ **hybridization, no lone pairs).**
- **CH_4** : $Tetrahedral(sp^3$ **hybridization, no lone pairs).**

Matching:

A-III, B-I, C-IV, D-II.

Final Answer:

A-III, B-I, C-IV, D-II.

Quick Tip

Use VSEPR theory to predict molecular shapes based on hybridization and lone pairs.

Question 68. For the given hypothetical reactions, the equilibrium constants are as follows:

$$X \rightleftharpoons Y, \quad K_1 = 1.0$$

$$Y \rightleftharpoons Z, \quad K_2 = 2.0$$

$$Z \rightleftharpoons W, \quad K_3 = 4.0$$

The equilibrium constant for the reaction $X \rightleftharpoons W$ is:

1. 6.0
2. 12.0
3. 8.0
4. 7.0

Correct Answer: (3)

Solution:

The equilibrium constant for the net reaction $X \rightleftharpoons W$ is the product of the individual constants:

$$K = K_1 \cdot K_2 \cdot K_3.$$

Substitute values:

$$K = 1 \cdot 2 \cdot 4 = 8.$$

Final Answer:

$$8.0.$$

Quick Tip

For sequential equilibrium reactions, the overall equilibrium constant is the product of the individual constants.

Question 69. Thiosulphate reacts differently with iodine and bromine in the reaction given below:

$$2\mathbf{S_2O_3^{2-}} + \mathbf{I_2} \rightarrow \mathbf{S_4O_6^{2-}} + 2\mathbf{I^-}$$

$$\mathbf{S_2O_3^{2-}} + 5\mathbf{Br_2} + 5\mathbf{H_2O} \rightarrow 2\mathbf{SO_4^{2-}} + 4\mathbf{Br^-} + 10\mathbf{H^+}.$$

Which of the following statements justifies the above dual behaviour of thiosulphate?

1. **Bromine undergoes oxidation and iodine undergoes reduction by iodine in these reactions.**
2. **Thiosulphate undergoes oxidation by bromine and reduction by iodine in these reactions.**
3. **Bromine is a stronger oxidant than iodine.**
4. **Bromine is a weaker oxidant than iodine.**

Correct Answer: (3)

Solution:

In the reaction with I_2, the oxidation state of sulphur changes from $+2$ to $+2.5$.

In the reaction with Br_2, the oxidation state of sulphur changes from $+2$ to $+6$.

Thus, both I_2 and Br_2 are oxidants, but Br_2 is stronger as it increases the oxidation state of sulphur further.

Final Answer:

Bromine is a stronger oxidant than iodine.

Quick Tip

Compare changes in oxidation states to identify the relative strengths of oxidizing agents.

Question 70: An octahedral complex with the formula $CoCl_3 \cdot nNH_3$ upon reaction with excess of $AgNO_3$ solution gives 2 moles of AgCl. Consider the oxidation state of Co in the complex as 'x'. The value of "x + n" is ____

1. 3
2. 6
3. 8
4. 5

Correct Answer: (3) 8

Solution: The reaction of $[Co(NH3)5Cl]Cl2$ with excess $AgNO3$ is as follows:

$$[Co(NH3)5Cl]Cl2 + excessAgNO3 -> 2AgCl(2\,moles)$$

In the complex $[Co(NH3)5Cl]Cl2$:

The inner coordination sphere contains 1 Cl ligand.

The outer coordination sphere contains 2 Cl^- ions, which react with $AgNO3$ to give 2 moles of $AgCl$.

Let x be the oxidation state of Co. The total charge on the complex is neutral. Therefore:

$$x + 0(\textbf{from 5 } NH3) + (-1 \textbf{ from 1 } Cl) + (-2 \textbf{ from 2 } Cl^-) = 0$$

$$x - 1 - 2 = 0$$

$$x = +3$$

Here, $n = 5$ (the number of $NH3$ ligands). Thus:

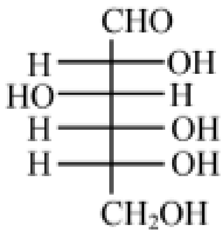
$$x + n = 3 + 5 = 8$$

Final Answer: $x + n = 8$

Quick Tip

Coordination complexes often involve the determination of ionic and coordination bonds to identify the coordination number and oxidation state.

Question 71:



The incorrect statement regarding the given structure is:

1. **Can be oxidized to a dicarboxylic acid with Br_2 water**
2. **Despite the presence of –CHO, does not give Schiff's test**
3. **Has 4-asymmetric carbon atoms**
4. **Will coexist in equilibrium with 2 other cyclic structures**

Correct Answer: (1)

Solution: The compound shown is a monosaccharide with an aldehyde group (-CHO) and multiple hydroxyl (-OH) groups. When treated with Br_2 water, it is not oxidized to a dicarboxylic acid but rather forms a monocarboxylic acid, making option (1) incorrect.

Statement (2) is correct: Despite having a –CHO group, certain sugars like glucose do not respond to Schiff's test.

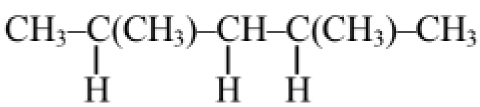
Statement (3) is correct: There are 4 chiral (asymmetric) carbon atoms present in the structure.

Statement (4) is correct: The compound can exist in equilibrium with two other cyclic forms, α-D-glucose and β-D-glucose.

Quick Tip

Remember that glucose can form both linear and cyclic structures in equilibrium.

Question 72: In the given compound, the number of 2° carbon atom/s is:



1. Three

2. One

3. Two

4. Four

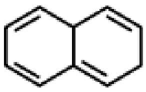
Correct Answer: (2)

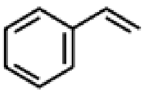
Solution: The compound has a branched structure with the central carbon atom being tertiary (3°). The carbons attached to it are primary (1°) and secondary (2°). Only one carbon atom in this molecule is secondary (attached to two other carbons).

Quick Tip

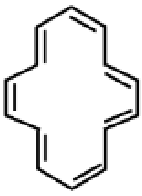
A secondary carbon (2°) is a carbon atom attached to two other carbon atoms.

Question 73: Which of the following are aromatic?

A. 

B. 

C. 

D. 

1. B and D only

2. A and C only

3. A and B only

4. C and D only

Correct Answer: (1)

Solution: For a compound to be aromatic, it must satisfy Hückel's rule ($4n + 2$ π electrons, where n is an integer), and it must be cyclic and planar with a conjugated system.
Compound A is non-aromatic.
Compound B is aromatic as it has a conjugated planar ring with a total of 6 π electrons.
Compound C is non-aromatic due to lack of conjugation.
Compound D is aromatic with a conjugated planar ring and 6 π electrons.

Quick Tip

Use Hückel's rule and check planarity for determining aromaticity.

Question 74: Among the following halogens F_2, Cl_2, Br_2 and I_2, which can undergo disproportionation reaction?

1. Only I_2
2. Cl_2, Br_2 and I_2
3. F_2, Cl_2 and Br_2
4. F_2 and Cl_2

Correct Answer: (2)

Solution:

Fluorine does not undergo disproportionation because it is the most electronegative element and cannot exist in a positive oxidation state. However, Cl_2, Br_2, and I_2 can undergo disproportionation as they have intermediate oxidation states and can be oxidized as well as reduced in reactions.

Quick Tip

Disproportionation occurs when an element in a compound undergoes simultaneous oxidation and reduction.

Question 75: Given below are two statements:

Statement I: $N(CH_3)_3$ and $P(CH_3)_3$ can act as ligands to form transition metal complexes.

Statement II: As N and P are from the same group, the nature of bonding of $N(CH_3)_3$ and $P(CH_3)_3$ is always the same with transition metals.

In the light of the above statements, choose the most appropriate answer from the options given below:

1. **Statement I is incorrect but Statement II is correct**
2. **Both Statement I and Statement II are correct**
3. **Statement I is correct but Statement II is incorrect**
4. **Both Statement I and Statement II are incorrect**

Correct Answer: (3)

Solution:

$N(CH_3)_3$ and $P(CH_3)_3$ both act as Lewis bases and can act as ligands. However, $P(CH_3)_3$ has a stronger π-acceptor character compared to $N(CH_3)_3$ due to the availability of d-orbitals in phosphorus, leading to differences in their bonding with transition metals.

Quick Tip

Always consider the availability of d-orbitals and π-acceptor capacity in ligands to evaluate their bonding behavior.

Question 76: Match List I with List II:

List-I (Elements)	**List-II (Properties in their respective groups)**
A: {Cl, S}	I. Elements with highest electronegativity
B: {Ge, As}	II. Elements with largest atomic size
C: {Fr, Ra}	III. Elements which show properties of both metals and non-metals
D: {F, O}	IV. Elements with highest negative electron gain enthalpy

Table 1: Matching List-I (Elements) with List-II (Properties)

Choose the correct answer from the options given below:

(1) A-II, B-III, C-IV, D-I

(2) A-III, B-II, C-I, D-IV

(3) A-IV, B-III, C-II, D-I

(4) A-II, B-I, C-IV, D-III

Correct Answer: (3)

Solution: Elements with highest electronegativity $\rightarrow$ F, O

Elements with largest atomic size $\rightarrow$ Fr, Ra

Elements which show properties of both metals and non-metals (metalloids) $\rightarrow$ Ge, As

Elements with highest negative electron gain enthalpy $\rightarrow$ Cl, S

Quick Tip

Matching elements with their properties requires a thorough understanding of periodic trends.

Question 77: Iron (III) catalyzes the reaction between iodide and persulphate ions, in which:

1. **Fe^{3+} oxidises the iodide ion**
2. **Fe^{3+} oxidises the persulphate ion**
3. **Fe^{2+} reduces the iodide ion**
4. **Fe^{2+} reduces the persulphate ion**

Choose the most appropriate answer from the options given below:

1. **B and C only**
2. **B only**
3. **A only**
4. **A and D only**

Correct Answer: (4)

Solution: The reaction proceeds as follows:

$$2\mathbf{Fe}^{3+} + 2\mathbf{I}^{-} \rightarrow 2\mathbf{Fe}^{2+} + \mathbf{I}_2$$

$$2\mathbf{Fe}^{2+} + \mathbf{S}_2\mathbf{O}_8^{2-} \rightarrow 2\mathbf{Fe}^{3+} + 2\mathbf{SO}_4^{2-}$$

In this reaction:

Fe^{3+} oxidizes I^- to I_2 and converts itself to Fe^{2+}.

Fe^{2+} reduces $S_2O_8^{2-}$ to SO_4^{2-} and converts itself back to Fe^{3+}.

Quick Tip

Identify redox reactions by tracking changes in the oxidation states of involved species.

Question 78

Match List I with List II:

List I (Compound)	List II (Colour)
A. $Fe_4[Fe(CN)_6]_3 \cdot xH_2O$	I. Violet
B. $[Fe(CN)_5NOS]^4$	II. Blood Red
C. $[Fe(SCN)]^{2+}$	III. Prussian Blue
D. $(NH_4)_3PO_4 \cdot 12MoO_3$	IV. Yellow

1. A-III, B-I, C-II, D-IV

2. A-IV, B-I, C-II, D-III

3. A-II, B-III, C-IV, D-I

4. A-I, B-II, C-III, D-IV

Correct Answer: (1)

Solution

Matching Explanation:

- **$Fe_4[Fe(CN)_6]_3 \cdot xH_2O$: Prussian Blue (Colour: III)**
- **$[Fe(CN)_5NOS]^4$: Violet (Colour: I)**
- **$[Fe(SCN)]^{2+}$: Blood Red (Colour: II)**
- **$(NH_4)_3PO_4 \cdot 12MoO_3$: Yellow (Colour: IV)**

A-III, B-I, C-II, D-IV

Quick Tip

To identify the colours of coordination complexes:

1. **Prussian Blue: Commonly observed for compounds like $Fe_4[Fe(CN)_6]_3$.**
2. **Violet: Seen in cyanide-nitrosyl complexes like $[Fe(CN)_5NOS]^{4-}$.**
3. **Blood Red: Characteristic of thiocyanate complexes such as $[Fe(SCN)]^{2+}$.**
4. **Yellow: Typical of ammonium phosphomolybdate compounds.**

Question 79: Number of complexes with even number of electrons in t_2 orbitals is:

$$[Fe(H_2O)_6]^{2+}, [Co(H_2O)_6]^{2+}, [Co(H_2O)_6]^{3+}, [Cu(H_2O)_6]^{2+}, [Cr(H_2O)_6]^{2+}$$

1. 1
2. 3
3. 2
4. 5

Correct Answer: (2)

Solution: To determine the number of complexes with an even number of electrons in t_{2g} orbitals, we calculate the electronic configuration of the central metal ion in each complex:

$[Fe(H2O)6]^{2+}$: Fe^{2+} **has** $(3d^6)$ **configuration. In an octahedral field:**

$$t_{2g}^4 e_g^2 \quad (\textbf{4 electrons in } t_{2g})$$

Even number of electrons in t_{2g}**.**

$[Co(H2O)6]^{2+}$: Co^{2+} **has** $(3d^7)$ **configuration. In an octahedral field:**

$$t_{2g}^5 e_g^2 \quad (\textbf{5 electrons in } t_{2g})$$

Odd number of electrons in t_{2g}**.**

$[Co(H2O)6]^{3+}$: Co^{3+} **has** $(3d^6)$ **configuration. In an octahedral field:**

$$t_{2g}^6 e_g^0 \quad (\textbf{6 electrons in } t_{2g})$$

Even number of electrons in t_{2g}**.**

$[Cu(H2O)6]^{2+}$: Cu^{2+} **has** $(3d^9)$ **configuration. In an octahedral field:**

$$t_{2g}^6 e_g^3 \quad (\textbf{6 electrons in } t_{2g})$$

Even number of electrons in t_{2g}**.**

$[Cr(H2O)6]^{2+}$: Cr^{2+} **has** $(3d^4)$ **configuration. In an octahedral field:**

$$t_{2g}^3 e_g^1 \quad (\textbf{3 electrons in } t_{2g})$$

Odd number of electrons in t_{2g}**.**

Complexes with even number of electrons in t_{2g} **orbitals are:**

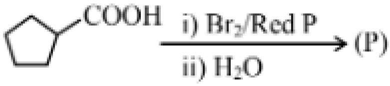
$$[Fe(H2O)6]^{2+}, \quad [Co(H2O)6]^{3+}, \quad [Cu(H2O)6]^{2+}$$

Final Answer: 3 complexes.

Quick Tip

To determine the number of electrons in t_2 orbitals, use the electronic configuration of the central metal ion in its given oxidation state.

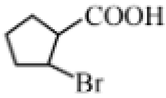
Question 80: Identify the product (P) in the following reaction:



(1) 

(2) 

(3) 

(4) 

Correct Answer: (1)

Solution:

The given reaction is an example of the Hell-Volhard-Zelinsky (HVZ) reaction, which selectively brominates the α-carbon of carboxylic acids in the presence of Br_2 and red phosphorus. The reaction mechanism involves the formation of an α-brominated carboxylic acid.

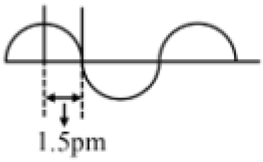
$$\text{COOH} \xrightarrow{\text{Br}_2/\text{Red P}} \text{COOH-Br} \xrightarrow{\text{H}_2\text{O}} \text{COOH-Br (Product)}$$

Thus, the product formed is the α-bromo derivative of the given carboxylic acid.

Quick Tip

The HVZ reaction is specific for carboxylic acids and introduces a halogen atom at the α-position.

Question 81: A hypothetical electromagnetic wave is shown below.



The frequency of the wave is $x \times 10^{19}$ Hz.

$x =$ ____ (nearest integer)

Correct Answer: (5)

Solution:

Given:

$$\lambda = 1.5 \times 4 \textbf{ pm} = 6 \times 10^{-12} \textbf{ meter}$$

Using the relationship:

$$\lambda\nu = c$$

where $c = 3 \times 10^8$ **m/s, we can find** ν **as:**

$$6 \times 10^{-12} \times \nu = 3 \times 10^8$$

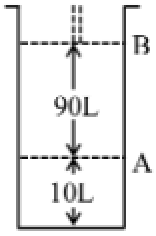
$$\nu = \frac{3 \times 10^8}{6 \times 10^{-12}} = 5 \times 10^{19} \textbf{ Hz}$$

Therefore, $x = 5$**.**

Quick Tip

Use the relation $\lambda\nu = c$ **for electromagnetic waves where** λ **is the wavelength and** ν **is the frequency.**

Question 82: Consider the figure provided.



1 mol of an ideal gas is kept in a cylinder, fitted with a piston, at the position A, at 18°C. If the piston is moved to position B, keeping the temperature unchanged, then 'x' L atm work is done in this reversible process.

$x =$ ___ **L atm (nearest integer)**

Correct Answer: (55)

Solution:

Work done (W) **in an isothermal reversible expansion of an ideal gas is given by:**

$$W = -nRT \ln\left(\frac{V_2}{V_1}\right)$$

Given:

$$n = 1 \textbf{ mol}, \; T = 18°\textbf{C} = 18 + 273.15 = 291.15 \textbf{ K}$$

$$V_1 = 10 \textbf{ L}, \; V_2 = 100 \textbf{ L}$$

Substitute the values:

$$W = -1 \times 0.08206 \times 291.15 \times \ln\left(\frac{100}{10}\right)$$

$$W \approx -1 \times 0.08206 \times 291.15 \times \ln(10)$$

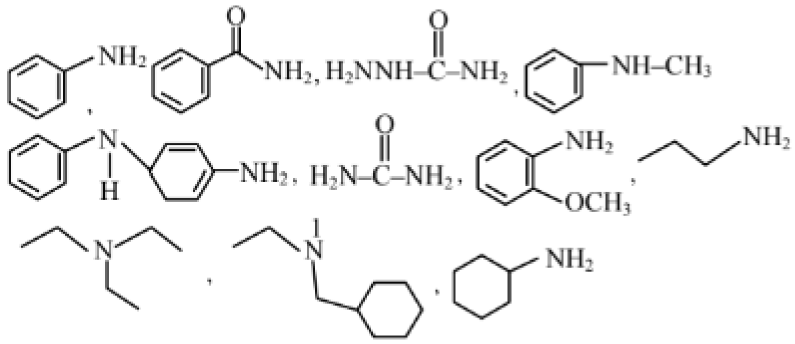
$$W \approx -55.0128 \textbf{ L atm}$$

The work done by the system is approximately -55 **L atm (rounded to nearest integer).**

Quick Tip

For isothermal processes of an ideal gas, work done depends on the initial and final volumes and can be calculated using $W = -nRT\ln\left(\frac{V_2}{V_1}\right)$**.**

Question 83. Number of amine compounds from the following giving solids which are soluble in NaOH upon reaction with Hinsberg's reagent is __



Correct Answer: 5

Solution:

Primary amines react with Hinsberg's reagent to form sulphonamides, which are soluble in NaOH. Analyzing the given structures:

Primary amine NH_2**: Soluble.**

$NH-C(-[:30]CH_3)$**: Soluble.**

Other primary amine structures: Soluble.

All 5 **compounds are primary amines.**

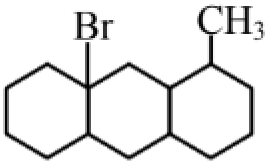
Final Answer:

$$5.$$

Quick Tip

Primary amines give ionic solids with Hinsberg's reagent that are soluble in NaOH.

Question 84. The number of optical isomers in the following compound is ____



Correct Answer: 32

Solution:

The compound has 5 **chiral centers. The total number of optical isomers is:**

$$\textbf{Number of isomers} = 2^n = 2^5 = 32.$$

Final Answer:

$$32.$$

Quick Tip

The number of optical isomers for a molecule with n **chiral centers is** 2^n**, assuming no meso forms.**

Question 85. The 'spin-only' magnetic moment value of MO_4^{2-} is ___ BM (where M is a metal having least metallic radii among Sc, Ti, V, Cr, Mn, Zn). (Given atomic number: $Sc = 21$, $Ti = 22$, $V = 23$, $Cr = 24$, $Mn = 25$, **and** $Zn = 30$)

Correct Answer: 0

Solution:

Among the given elements, Cr has the least metallic radii. The CrO_4^{2-} ion has Cr^{6+}, which has a d^0 **configuration (diamagnetic).**

The spin-only magnetic moment:

$$\mu = \sqrt{n(n+2)}\,\textbf{BM},$$

where $n = 0$**.**

Final Answer:

$$0\,\textbf{BM}.$$

Quick Tip

For ions with d^0 configurations, the magnetic moment is 0, as there are no unpaired electrons.

Question 86. Number of molecules from the following which are exceptions to the octet rule is ________:

$CO_2, NO_2, H_2SO_4, BF_3, CH_4, SiF_4, ClO_2, PCl_5, BeF_2, C_2H_6, CHCl_3, CBr_4$

Ans. (6)

Solution:

To determine the exceptions to the octet rule, analyze each molecule:

CO_2**: Follows the octet rule. (C has 8 electrons in its valence shell.)**

NO_2**: Exception. It has an odd number of valence electrons (11), making it a free radical.**

H_2SO_4**: Follows the octet rule.**

BF_3**: Exception. Boron has only 6 electrons in its valence shell (electron-deficient compound).**

CH_4**: Follows the octet rule.**

SiF_4**: Follows the octet rule.**

ClO_2**: Exception. It is a free radical with an odd number of valence electrons.**

PCl_5**: Exception. Phosphorus has 10 valence electrons (expanded octet).**

BeF_2**: Exception. Beryllium has only 4 valence electrons (electron-deficient compound).**

C_2H_6**: Follows the octet rule.**

$CHCl_3$**: Follows the octet rule.**

CBr_4**: Follows the octet rule.**

Molecules that are exceptions to the octet rule:

$$NO_2, BF_3, ClO_2, PCl_5, BeF_2$$

Total number of exceptions = 6.

Final Answer: 6

Quick Tip

Identify exceptions to the octet rule: incomplete octets, expanded octets, and odd-electron species.

Question 87. If 279 g of aniline is reacted with one equivalent of benzenediazonium chloride, the maximum amount of aniline yellow formed will be ___ g (nearest integer). (Consider complete conversion).

Correct Answer: 591 **g**

Solution: The balanced reaction is:

$$\mathbf{C_6H_5NH_2 + C_6H_5N_2^+Cl^- \rightarrow C_6H_5N = N - C_6H_5 \quad (Aniline\ yellow)}.$$

Given:

$$\textbf{Molar mass of aniline} = 93\,\textbf{g/mol}, \quad \textbf{Given mass} = 279\,\textbf{g}.$$

Moles of aniline:

$$n = \frac{279}{93} = 3\,\textbf{mol}.$$

Mass of aniline yellow:

$$\textbf{Molar mass of product} = 197\,\textbf{g/mol}.$$

$$\textbf{Mass} = 3 \cdot 197 = 591\,\textbf{g}.$$

Final Answer:

$$591\,\textbf{g}.$$

Quick Tip

Use stoichiometry to calculate product mass: Moles of reactant $\times$ Molar mass of product.

Question 88. Consider the following reaction:

$$\mathbf{A + B \rightarrow C}.$$

The time taken for A to become $1/4$th of its initial concentration is twice the time taken to become $1/2$ of the same. Also, when the change of concentration of B is plotted against

time, the resulting graph gives a straight line with a negative slope and a positive intercept on the concentration axis.

The overall order of the reaction is ___

Correct Answer: 1

Solution:

Order with respect to A

$$\textbf{For a first-order reaction: } t_{75\%} = 2 \times t_{50\%}.$$

This is consistent with the information given, so the reaction is first order with respect to A.

Order with respect to B The plot of [B] versus t is a straight line, which indicates that the reaction is zero order with respect to B.

Overall order of the reaction:

$$\textbf{Order} = 1(\textbf{w.r.t. A}) + 0(\textbf{w.r.t. B}) = 1.$$

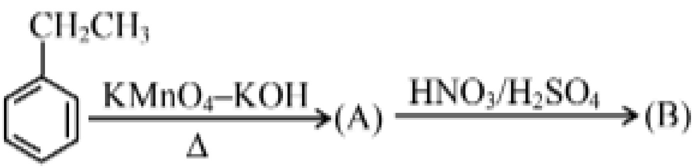
Final Answer:

$$1.$$

Quick Tip

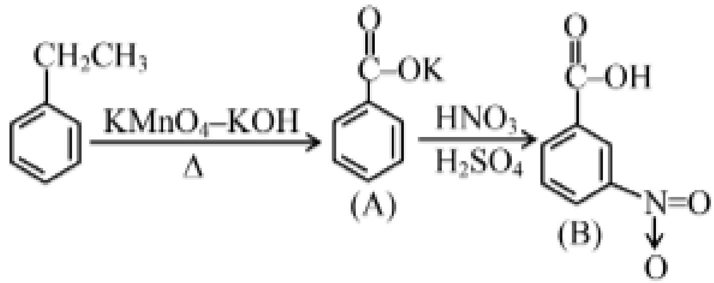
First-order reactions have a constant half-life, and zero-order reactions show linear concentration decay with time.

Question 89. The major product B of the following reaction has ____ π-bonds:



Correct Answer: 5

Solution:



Count the π-bonds:

Two π-bonds in the aromatic ring.

One π-bond in the carboxyl group.

Two π-bonds in the nitro group.

$$\textbf{Total number of } \pi\textbf{-bonds} = 5.$$

Final Answer:

$$5.$$

Quick Tip

Analyze the structure of the final product to count all π-bonds, including those in functional groups and aromatic rings.

Question 90. **A solution containing $10\,$g of an electrolyte AB$_2$ in $100\,$g of water boils at $100.52°$C. The degree of ionization (α) of the electrolyte is __ $\times 10^{-1}$ (nearest integer). [Given: Molar mass of $AB2 = 200\,$g mol^{-1}; K_b (molal boiling point elevation constant of water) = $0.52\,$K kg mol^{-1}; boiling point of water = $100°$C;$AB2$ ionises as $AB2- > A^{2+} + 2B^{-}$]**

Correct Answer: 5

Solution:

Ionization of AB$_2$:

$$\mathbf{AB_2 \rightarrow A^{2+} + 2B^{-}}.$$

The van't Hoff factor (i) is:

$$i = 1 + (3 - 1)\alpha = 1 + 2\alpha.$$

Boiling point elevation:

$$\Delta T_b = K_b \cdot m \cdot i,$$

where $m = \frac{\textbf{Mass of solute}}{\textbf{Molar mass of solute}\cdot\textbf{Mass of solvent (kg)}}$**.**

Substitute values:

$$m = \frac{10}{200 \cdot 0.1} = 0.5\,\textbf{mol/kg}.$$

$$\Delta T_b = 0.52 = 0.52 \cdot 0.5 \cdot (1 + 2\alpha).$$

Simplify:

$$1 = 1 + 2\alpha \quad \implies \quad 2\alpha = 1 \quad \implies \quad \alpha = 0.5.$$

Convert to nearest integer:

$$\alpha \times 10 = 5.$$

Final Answer:

$$5.$$

Quick Tip

For boiling point elevation, use $\Delta T_b = K_b \cdot m \cdot i$ **and relate** i **to the degree of ionization** (α)**.**
