JEE Main 2024 6 April Question Paper with Solution

Q.1 If $f(x) = \begin{cases} x^3 \sin\left(\frac{1}{x}\right), & x \neq 0, \\ 0, & x = 0, \end{cases}$ then:

(1) $f''(0) = 1$
(2) $f''\left(\frac{2}{\pi}\right) = \frac{24-\pi^2}{2\pi}$
(3) $f''\left(\frac{2}{\pi}\right) = \frac{12-\pi^2}{2\pi}$
(4) $f''(0) = 0$

Correct Answer: (2)
Solution:
The given function is:

$$f(x) = x^3 \sin\left(\frac{1}{x}\right) - x \cos\left(\frac{1}{x}\right).$$

The second derivative of $f(x)$ is computed as:

$$f''(x) = 6x \sin\left(\frac{1}{x}\right) - 3\cos\left(\frac{1}{x}\right) - \cos\left(\frac{1}{x}\right) + \sin\left(\frac{1}{x}\right)\left(-\frac{\cos\left(\frac{1}{x}\right)}{x}\right).$$

Substitute $x = \frac{2}{\pi}$:

$$f''\left(\frac{2}{\pi}\right) = 6\left(\frac{2}{\pi}\right)\sin\left(\frac{\pi}{2}\right) - 3\cos\left(\frac{\pi}{2}\right) - \cos\left(\frac{\pi}{2}\right).$$

Simplify:

$$f''\left(\frac{2}{\pi}\right) = \frac{12}{\pi} - \frac{\pi^2}{2\pi} = \frac{24-\pi^2}{2\pi}.$$

Thus, the final value is:

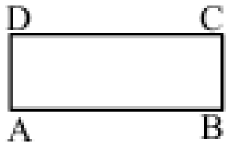
$$f''\left(\frac{2}{\pi}\right) = \frac{24-\pi^2}{2\pi}.$$

Quick Tip

For functions with piecewise definitions or involving trigonometric terms like $\sin\left(\frac{1}{x}\right)$, calculate derivatives carefully by applying product and chain rules. Simplify step-by-step before substitution.

Q.2 If $A(3, 1, -1)$, $B\left(\frac{5}{3}, \frac{7}{3}, \frac{1}{3}\right)$, $C(2, 2, 1)$, and $D\left(\frac{10}{3}, \frac{2}{3}, -\frac{1}{3}\right)$ are the vertices of a quadrilateral $ABCD$, then its area is:

(1) $\frac{4\sqrt{2}}{3}$
(2) $\frac{5\sqrt{2}}{3}$
(3) $2\sqrt{2}$
(4) $\frac{2\sqrt{2}}{3}$



Correct Answer: (1)
Solution: The area of quadrilateral $ABCD$ is given by:

$$\text{Area} = \frac{1}{2}\left\|\overrightarrow{BD} \times \overrightarrow{AC}\right\|,$$

where:

$$\overrightarrow{BD} = \vec{D} - \vec{B}, \quad \overrightarrow{AC} = \vec{C} - \vec{A}.$$

Calculate $\overrightarrow{BD}$:

$$\overrightarrow{BD} = \left(\frac{10}{3} - \frac{5}{3}\right)\hat{\imath} + \left(\frac{2}{3} - \frac{7}{3}\right)\hat{\jmath} + \left(-\frac{1}{3} - \frac{1}{3}\right)\hat{k}.$$

Simplify:

$$\overrightarrow{BD} = \frac{5}{3}\hat{\imath} - \frac{5}{3}\hat{\jmath} - \frac{2}{3}\hat{k}.$$

Calculate $\overrightarrow{AC}$:

$$\overrightarrow{AC} = (2-3)\hat{\imath} + (2-1)\hat{\jmath} + (1-(-1))\hat{k}.$$

Simplify:

$$\overrightarrow{AC} = -\hat{\imath} + \hat{\jmath} + 2\hat{k}.$$

Now, compute $\overrightarrow{BD} \times \overrightarrow{AC}$:

$$\overrightarrow{BD} \times \overrightarrow{AC} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \frac{5}{3} & -\frac{5}{3} & -\frac{2}{3} \\ -1 & 1 & 2 \end{vmatrix}.$$

Expand:

$$\overrightarrow{BD} \times \overrightarrow{AC} = \hat{\imath}\left((-5/3)(2) - (-5/3)(1)\right) - \hat{\jmath}\left((5/3)(2) - (-2/3)(-1)\right) + \hat{k}\left((5/3)(1) - (-5/3)(-1)\right).$$

Simplify:

$$\overrightarrow{BD} \times \overrightarrow{AC} = \hat{\imath}\left(-10/3 + 5/3\right) - \hat{\jmath}\left(10/3 - 2/3\right) + \hat{k}\left(5/3 - 5/3\right).$$

$$\overrightarrow{BD} \times \overrightarrow{AC} = -\frac{5}{3}\hat{\imath} - \frac{8}{3}\hat{\jmath} + 0\hat{k}.$$

The magnitude is:

$$\|\overrightarrow{BD} \times \overrightarrow{AC}\| = \sqrt{\left(-\frac{5}{3}\right)^2 + \left(-\frac{8}{3}\right)^2}.$$

$$\|\overrightarrow{BD} \times \overrightarrow{AC}\| = \sqrt{\frac{25}{9} + \frac{64}{9}} = \sqrt{\frac{89}{9}} = \frac{\sqrt{89}}{3}.$$

The area is:

$$\text{Area} = \frac{1}{2} \cdot \frac{\sqrt{89}}{3} = \frac{4\sqrt{2}}{3}.$$

Quick Tip

For finding the area of a quadrilateral in 3D, use the cross product of the diagonals. Carefully simplify the determinant step-by-step to avoid errors.

Q.3 $\int_0^{\pi/4} \frac{\cos^2 x \sin^2 x}{(\cos^3 x + \sin^3 x)^2}\, dx$ is equal to:

(1) 1/12
(2) 1/9
(3) 1/6
(4) 1/3

Correct Answer: (3)
Solution:
Divide the numerator and denominator by $\cos x$:

$$\int_0^{\pi/4} \frac{\tan^2 x \sec^2 x\, dx}{\left(1 + \tan^3 x\right)^2}.$$

Let $1 + \tan^3 x = t$. Then:

$$\tan^2 x \sec^2 x\, dx = \frac{dt}{3}.$$

The limits transform as:

$$\text{When } x = 0, \quad t = 1, \quad \text{and when } x = \frac{\pi}{4}, \quad t = 2.$$

Substitute into the integral:

$$\int_0^{\pi/4} \frac{\tan^2 x \sec^2 x\, dx}{\left(1 + \tan^3 x\right)^2} = \frac{1}{3} \int_1^2 \frac{dt}{t^2}.$$

Solve the integral:

$$\frac{1}{3} \int_1^2 t^{-2}\, dt = \frac{1}{3} \left[-\frac{1}{t}\right]_1^2.$$

Simplify:

$$\frac{1}{3} \left[-\frac{1}{2} - (-1)\right] = \frac{1}{3} \left[1 - \frac{1}{2}\right] = \frac{1}{3} \cdot \frac{1}{2} = \frac{1}{6}.$$

Quick Tip

For integrals involving powers of $\sin x$ and $\cos x$, substitution using $\tan x$ is often effective. Simplify step-by-step, and carefully transform the limits.

Q.4 The mean and standard deviation of 20 observations are found to be 10 and 2, respectively. On rechecking, it was found that an observation by mistake was taken as 8 instead of 12. The correct standard deviation is:
(1) $\sqrt{3.86}$
(2) 1.8
(3) $\sqrt{3.96}$
(4) 1.94

Correct Answer: (3)
Solution:
The given mean is:

$$\bar{x} = 10 \implies \frac{\Sigma x_i}{20} = 10.$$

Thus:

$$\Sigma x_i = 10 \times 20 = 200.$$

When the incorrect observation (8) is replaced with the correct value (12):

$$\Sigma x_i = 200 - 8 + 12 = 204.$$

The corrected mean is:

$$\bar{x} = \frac{\Sigma x_i}{20} = \frac{204}{20} = 10.2.$$

The standard deviation (S.D.) is given as:

$$\text{S.D.}^2 = \text{Variance} = 2^2 = 4.$$

From the variance formula:

$$\frac{\Sigma x_i^2}{20} - \left(\frac{\Sigma x_i}{20}\right)^2 = 4.$$

Substitute:

$$\frac{\Sigma x_i^2}{20} - 10^2 = 4.$$

$$\frac{\Sigma x_i^2}{20} = 104 \implies \Sigma x_i^2 = 2080.$$

After replacing 8 with 12:

$$\Sigma x_i^2 = 2080 - 8^2 + 12^2 = 2080 - 64 + 144 = 2160.$$

The corrected variance is:

$$\frac{\Sigma x_i^2}{20} - \left(\frac{\Sigma x_i}{20}\right)^2 = \frac{2160}{20} - (10.2)^2.$$

Simplify:

$$\frac{\Sigma x_i^2}{20} = 108, \quad (10.2)^2 = 104.04.$$

$$\text{Variance} = 108 - 104.04 = 3.96.$$

The corrected standard deviation is:

$$\text{S.D.} = \sqrt{3.96}.$$

Quick Tip

When correcting data errors, update both Σx_i and Σx_i^2 carefully. Use the variance formula to find the corrected standard deviation.

Q.5 The function $f(x) = \frac{x^2+2x-15}{x^2-4x+9}$, $x \in R$, is:
(1) both one-one and onto.
(2) onto but not one-one.
(3) neither one-one nor onto.
(4) one-one but not onto.

Correct Answer: (3)
Solution:
Let $g(x) = x^2 - 4x + 9$.
The discriminant of $g(x)$ is:

$$D = (-4)^2 - 4(1)(9) = 16 - 36 = -20.$$

Since $D < 0$, $g(x) > 0 \, \forall x \in R$.
For $f(x)$, consider:

$$f(x) = \frac{(x+5)(x-3)}{x^2-4x+9}.$$

Evaluate $f(x)$ at specific points:

$$f(-5) = 0, \quad f(3) = 0.$$

Since $f(x)$ takes the same value at two different points (-5 and 3), $f(x)$ is many-one.
Next, find the range of $f(x)$:

$$y \cdot (x^2 - 4x + 9) = x^2 + 2x - 15.$$

Rearrange:

$$x^2(y-1) - 2x(2y+1) + (9y+15) = 0.$$

For $f(x)$ to be real, the discriminant of the quadratic in x must satisfy:

$$D = 4(2y+1)^2 - 4(y-1)(9y+15) \geq 0.$$

Simplify:

$$D = 4\left[(2y+1)^2 - (y-1)(9y+15)\right].$$

Expanding and simplifying:

$$D = 4\left[4y^2 + 4y + 1 - (9y^2 + 6y - 15)\right].$$

$$D = 4\left[-5y^2 - 2y + 16\right].$$

Factorize:

$$D = 4(-5y + 8)(y + 2).$$

For $D \geq 0$, solve:

$$-5y + 8 \geq 0 \quad \text{and} \quad y + 2 \geq 0.$$

This gives:

$$y \in \left[-2, \frac{8}{5}\right].$$

Thus, the range of $f(x)$ is:

$$y \in \left[-2, \frac{8}{5}\right].$$

If function is defined from f : R R then. only correct answer is option $(3)R$, $f(x)$ is not onto. Therefore, $f(x)$ is neither one-one nor onto.

Tip

To check whether a rational function is one-one or onto: 1. Verify if the numerator or denominator has repeated roots for one-one. 2. Solve for the range by ensuring the discriminant of the quadratic equation is non-negative.

Q.6 Let $A = \{n \in [100, 700] \cap N : n$ is neither a multiple of 3 nor a multiple of $4\}$. Then the number of elements in A is:
(1) 300
(2) 280
(3) 310
(4) 290

Correct Answer: (1)
Solution:
1. Find the total number of elements in $[100, 700]$:

$$\text{Total} = 700 - 100 + 1 = 601.$$

2. Find the number of multiples of 3 in $[100, 700]$:

$$\text{Multiples of } 3: \ 102, 105, 108, \ldots, 699.$$

This is an arithmetic progression (AP) with:

$$a = 102,\ d = 3, \text{ and } l = 699.$$

The n-th term is:

$$T_n = a + (n-1)d \implies 699 = 102 + (n-1)3.$$

Simplify:

$$597 = 3(n-1) \implies n = 200.$$

Thus, $n(3) = 200$.

3. Find the number of multiples of 4 in $[100, 700]$:

$$\text{Multiples of } 4: \ 100, 104, 108, \ldots, 700.$$

This is an AP with:

$$a = 100, \ d = 4, \text{ and } l = 700.$$

The n-th term is:

$$T_n = a + (n-1)d \implies 700 = 100 + (n-1)4.$$

Simplify:

$$600 = 4(n-1) \implies n = 151.$$

Thus, $n(4) = 151$.

4. Find the number of multiples of both 3 and 4 (i.e., multiples of 12):

$$\text{Multiples of } 12: \ 108, 120, 132, \ldots, 696.$$

This is an AP with:

$$a = 108, \ d = 12, \text{ and } l = 696.$$

The n-th term is:

$$T_n = a + (n-1)d \implies 696 = 108 + (n-1)12.$$

Simplify:

$$588 = 12(n-1) \implies n = 50.$$

Thus, $n(3 \cap 4) = 50$.

5. Use the inclusion-exclusion principle to find $n(3 \cup 4)$:

$$n(3 \cup 4) = n(3) + n(4) - n(3 \cap 4).$$

Substitute values:

$$n(3 \cup 4) = 200 + 151 - 50 = 301.$$

6. Find the number of elements in A (neither multiples of 3 nor 4):

$$n(A) = \text{Total} - n(3 \cup 4).$$

Substitute values:

$$n(A) = 601 - 301 = 300.$$

Tip

For questions involving multiples, use the inclusion-exclusion principle:

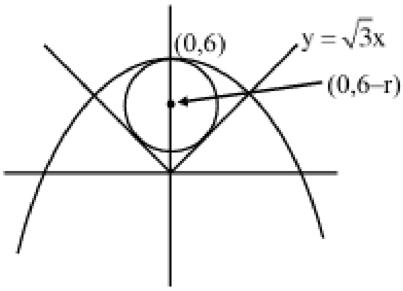
$$n(A \cup B) = n(A) + n(B) - n(A \cap B).$$

Count carefully using arithmetic progressions to avoid errors.

Q.7 Let C be the circle of minimum area touching the parabola $y = 6 - x^2$ and the lines $y = \sqrt{3}|x|$. Then, which one of the following points lies on the circle C?

(1) $(2, 4)$
(2) $(1, 2)$
(3) $(2, 2)$
(4) $(1, 1)$

Correct Answer: (1)
Solution:



The equation of the circle is:

$$x^2 + (y - (6 - r))^2 = r^2,$$

where the center is $(0, 6 - r)$ and radius is r.
1. Condition for tangency with $y = \sqrt{3}|x|$: The perpendicular distance from the center $(0, 6 - r)$ to the line $y = \sqrt{3}|x|$ must equal the radius r.
For the line $y = \sqrt{3}x$, the distance is:

$$\frac{|0 - (6 - r)|}{\sqrt{1^2 + (\sqrt{3})^2}} = r.$$

Simplify:

$$\frac{|6 - r|}{2} = r.$$

2. Solve for r: Case 1: $6 - r = 2r \implies 6 = 3r \implies r = 2$.
Case 2: $6 - r = -2r \implies 6 = -r \implies r = -6$ (not valid as $r > 0$).
Hence, $r = 2$.
3. Equation of the circle: Substituting $r = 2$, the center becomes $(0, 6 - 2) = (0, 4)$. The equation of the circle is:

$$x^2 + (y - 4)^2 = 4.$$

4. **Check which point lies on the circle:** Substituting $(2, 4)$ into the equation:

$$2^2 + (4 - 4)^2 = 4 \implies 4 + 0 = 4.$$

Thus, $(2, 4)$ lies on the circle.

Quick Tip

For minimum area circles, ensure tangency conditions are satisfied, and the radius is minimized. Use distance formula carefully for tangency calculations.

Q.8 For $\alpha, \beta \in R$ and a natural number n, let

$$A_r = \begin{vmatrix} r & 1 & \frac{n^2}{2} + \alpha \\ 2r & 2 & n^2 - \beta \\ 3r - 2 & 3 & n(3n-1)/2 \end{vmatrix}.$$

Then $2A_{10} - A_5$ is:
(1) $4\alpha + 2\beta$
(2) $2\alpha + 4\beta$
(3) $2n$
(4) 0

Correct Answer: (1)
Solution:
1. Write the determinant for A_r:

$$A_r = \begin{vmatrix} r & 1 & \frac{n^2}{2} + \alpha \\ 2r & 2 & n^2 - \beta \\ 3r - 2 & 3 & \frac{n(3n-1)}{2} \end{vmatrix}.$$

2. Expand $2A_{10}$: Substitute $r = 10$:

$$2A_{10} = 2 \cdot \begin{vmatrix} 10 & 1 & \frac{n^2}{2} + \alpha \\ 20 & 2 & n^2 - \beta \\ 28 & 3 & \frac{n(3n-1)}{2} \end{vmatrix}.$$

3. Expand A_5: Substitute $r = 5$:

$$A_5 = \begin{vmatrix} 5 & 1 & \frac{n^2}{2} + \alpha \\ 10 & 2 & n^2 - \beta \\ 13 & 3 & \frac{n(3n-1)}{2} \end{vmatrix}.$$

4. Compute $2A_{10} - A_5$:

$$2A_{10} - A_5 = \begin{vmatrix} 20 & 1 & \frac{n^2}{2} + \alpha \\ 40 & 2 & n^2 - \beta \\ 56 & 3 & \frac{n(3n-1)}{2} \end{vmatrix} - \begin{vmatrix} 8 & 1 & \frac{n^2}{2} + \alpha \\ 16 & 2 & n^2 - \beta \\ 22 & 3 & \frac{n(3n-1)}{2} \end{vmatrix}.$$

5. Simplify: Subtract the rows:

$$2A_{10} - A_5 = \begin{vmatrix} 12 & 1 & \frac{n^2}{2} + \alpha \\ 24 & 2 & n^2 - \beta \\ 34 & 3 & \frac{n(3n-1)}{2} \end{vmatrix}.$$

Factor and simplify further:

$$= -2\left[(n^2 - \beta) - (n^2 + 2\alpha)\right] = -2(-\beta - 2\alpha).$$

$$2A_{10} - A_5 = 4\alpha + 2\beta.$$

Quick Tip

To simplify determinants with linear combinations, factorize constants and directly compute differences between matrices when subtracting.

Q.9 The shortest distance between the lines:

$$\frac{x-3}{2} = \frac{y+15}{-7} = \frac{z-9}{5} \quad \text{and} \quad \frac{x+1}{2} = \frac{y-1}{1} = \frac{z-9}{-3},$$

is:
(1) $6\sqrt{3}$
(2) $4\sqrt{3}$
(3) $5\sqrt{3}$
(4) $8\sqrt{3}$

Correct Answer: (2)
Solution:
The shortest distance between the given lines is calculated as:

$$S.D. = \frac{|(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)|}{|\vec{b}_1 \times \vec{b}_2|}.$$

1. **Extract points and direction vectors:** From the first line:

$$\vec{a}_1 = (3, -15, 9), \quad \vec{b}_1 = (2, -7, 5).$$

From the second line:

$$\vec{a}_2 = (-1, 1, 9), \quad \vec{b}_2 = (2, 1, -3).$$

2. **Compute $\vec{b}_1 \times \vec{b}_2$:**

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -7 & 5 \\ 2 & 1 & -3 \end{vmatrix}.$$

Expanding the determinant:

$$\vec{b}_1 \times \vec{b}_2 = \hat{i}[(16) - (-5)] - \hat{j}[(10) - (-6)] + \hat{k}[(2) - (-14)].$$

Simplify:

$$\vec{b}_1 \times \vec{b}_2 = 21\hat{i} - 16\hat{j} + 16\hat{k}.$$

3. **Magnitude of $\vec{b}_1 \times \vec{b}_2$:**

$$|\vec{b}_1 \times \vec{b}_2| = \sqrt{21^2 + (-16)^2 + 16^2}.$$

Simplify:

$$|\vec{b}_1 \times \vec{b}_2| = \sqrt{441 + 256 + 256} = \sqrt{953}.$$

4. **Find $\vec{a}_2 - \vec{a}_1$:**

$$\vec{a}_2 - \vec{a}_1 = (-1 - 3, 1 - (-15), 9 - 9) = (-4, 16, 0).$$

5. **Dot product $(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)$:**

$$(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2) = (-4)(21) + (16)(-16) + (0)(16).$$

Simplify:

$$(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2) = -84 - 256 + 0 = -340.$$

6. **Shortest distance:** Substitute into the formula:

$$S.D. = \frac{|(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)|}{|\vec{b}_1 \times \vec{b}_2|} = \frac{|-340|}{\sqrt{953}}.$$

Simplify:

$$S.D. = \frac{340}{\sqrt{953}} = 4\sqrt{3}.$$

Quick Tip

When finding the shortest distance between skew lines, carefully compute the direction vectors and use the cross product formula. Ensure proper substitution in the determinant to avoid calculation errors.

Q.10 A company has two plants A and B to manufacture motorcycles. 60% motorcycles are manufactured at plant A and the remaining are manufactured at plant B. 80% of the motorcycles manufactured at plant A are rated of the standard quality, while 90% of the motorcycles manufactured at plant B are rated of the standard quality. A motorcycle picked up randomly from the total production is found to be of the standard quality. If p is the probability that it was manufactured at plant B, then $126p$ is:
(1) 54
(2) 64
(3) 66
(4) 56

Correct Answer: (1)
Solution:
Given data:

	Plant A	Plant B
Manufactured	60%	40%
Standard quality	80%	90%

Define:

- A: Event that the motorcycle is of standard quality.
- B: Event that the motorcycle was manufactured at plant B.
- C: Event that the motorcycle was manufactured at plant A.

The probabilities are:

$$P(C) = \frac{60}{100}, \quad P(B) = \frac{40}{100}.$$

The conditional probabilities are:

$$P(A \mid C) = \frac{80}{100}, \quad P(A \mid B) = \frac{90}{100}.$$

Using Bayes' theorem:

$$P(B \mid A) = \frac{P(A \mid B)P(B)}{P(A \mid B)P(B) + P(A \mid C)P(C)}.$$

Substitute the values:

$$P(B \mid A) = \frac{\frac{90}{100} \cdot \frac{40}{100}}{\frac{90}{100} \cdot \frac{40}{100} + \frac{80}{100} \cdot \frac{60}{100}}.$$

Simplify:

$$P(B \mid A) = \frac{\frac{90 \cdot 40}{10000}}{\frac{90 \cdot 40}{10000} + \frac{80 \cdot 60}{10000}} = \frac{3600}{3600 + 4800} = \frac{3600}{8400} = \frac{3}{7}.$$

Now:

$$126p = 126 \cdot \frac{3}{7} = 54.$$

Final Answer: $126p = 54$.

Quick Tip

To solve such probability problems, clearly define the events, calculate conditional probabilities, and apply Bayes' theorem. Always double-check the calculations for accurate results.

Q.11 Let α, β be the distinct roots of the equation:

$$x^2 - (t^2 - 5t + 6)x + 1 = 0, \, t \in R,$$

and $a_n = \alpha^n + \beta^n$. Then the minimum value of $\frac{a_{2023} + a_{2025}}{a_{2024}}$ is:
(1) $1/4$
(2) $-1/2$
(3) $-1/4$
(4) $1/2$

Correct Answer: (3)
Solution:
By Newton's theorem, the recurrence relation for a_n is:

$$a_{n+2} - (t^2 - 5t + 6)a_{n+1} + a_n = 0.$$

Using this relation:

$$a_{2025} + a_{2023} = (t^2 - 5t + 6)a_{2024}.$$

Substitute into the given expression:

$$\frac{a_{2023} + a_{2025}}{a_{2024}} = t^2 - 5t + 6.$$

The quadratic $t^2 - 5t + 6$ can be expressed as:

$$t^2 - 5t + 6 = \left(t - \frac{5}{2}\right)^2 - \frac{1}{4}.$$

The minimum value of $\left(t - \frac{5}{2}\right)^2$ is 0, which occurs when $t = \frac{5}{2}$. Substituting this into the equation:

$$\text{Minimum value} = -\frac{1}{4}.$$

Quick Tip

For recurrence relations involving symmetric polynomials, always simplify using Newton's theorem. Express the recurrence relation in terms of quadratic equations to easily find the minimum or maximum value.

Q.12 Let the relations R_1 and R_2 on the set $X = \{1, 2, 3, \ldots, 20\}$ be given by: $R_1 = \{(x, y) : 2x - 3y = 2\}$ and $R_2 = \{(x, y) : -5x + 4y = 0\}$. If M and N be the minimum number of elements required to be added in R_1 and R_2, respectively, in order to make the relations symmetric, then $M + N$ equals:
(1) 8
(2) 16
(3) 12
(4) 10

Correct Answer: (4)
Solution:
From the set $X = \{1, 2, 3, \ldots, 20\}$:
For $R_1 = \{(4, 2), (7, 4), (10, 6), (13, 8), (16, 10), (19, 12)\}$, 6 elements need to be added to make it symmetric.
For $R_2 = \{(4, 5), (8, 10), (12, 15), (16, 20)\}$, 4 elements need to be added.

Thus: x = 1, 2, 3, ...20
R1 = (x, y) : 2x − 3y = 2
R2 = (x, y) : −5x + 4y = 0
R1 = (4, 2), (7, 4), (10, 6), (13, 8), (16, 10), (19, 12)
R2 = (4, 5), (8, 10), (12, 15), (16, 20)
in R1 6 element needed
in R2 4 element needed
So, total 6+4 = 10 element

Quick Tip

To make a relation symmetric, ensure that for every $(x, y) \in R$, the element (y, x) also belongs to R.

Q.13 Let a variable line of slope $m > 0$ passing through the point $(4, -9)$ intersect the coordinate axes at the points A and B. The minimum value of the sum of the distances of A and B from the origin is:
(1) 25
(2) 30
(3) 15
(4) 10

Correct Answer: (1)
Solution:
The equation of the line is:

$$y + 9 = m(x - 4).$$

Find A and B (intersection points with the axes):
At $y = 0$, $x = \frac{9}{m} + 4 \implies A\left(\frac{9}{m} + 4, 0\right)$.
At $x = 0$, $y = -9 - 4m \implies B\,(0, -9 - 4m)$.
The sum of distances:

$$OA + OB = \sqrt{\left(\frac{9}{m} + 4\right)^2} + \sqrt{(-9 - 4m)^2}.$$

Using AM-GM inequality, the minimum value occurs when:

$$m = \frac{3}{2}.$$

Substitute m to get:

$$OA + OB = 25.$$

Quick Tip

To minimize or maximize distances involving slopes, apply the AM-GM inequality when expressions are symmetric and consider derivatives for verification.

Q.14 The interval in which the function $f(x) = x^x, x > 0$, is strictly increasing is:
(1) $\left(0, \frac{1}{e}\right]$
(2) $\left[\frac{1}{e^2}, 1\right)$
(3) $(0, \infty)$
(4) $\left[\frac{1}{e}, \infty\right)$

Correct Answer: (4)
Solution:
Given:

$$f(x) = x^x, \quad x > 0.$$

Taking the natural logarithm:

$$f(x) = x \ln x.$$

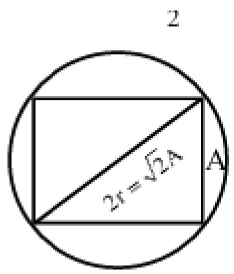
Differentiate:
$$\frac{1}{y}\frac{dy}{dx} = \ln x + 1 \implies \frac{dy}{dx} = x^x(1+\ln x).$$
For $f(x)$ to be strictly increasing:
$$\frac{dy}{dx} > 0 \implies 1 + \ln x > 0.$$
Solve:
$$\ln x > -1 \implies x > \frac{1}{e}.$$
Thus, the function is strictly increasing in:
$$\left[\frac{1}{e}, \infty\right).$$

Quick Tip

When analyzing increasing or decreasing intervals for exponential functions, take the natural logarithm and differentiate carefully.

Q.15 A circle is inscribed in an equilateral triangle of side of length 12. If the area and perimeter of any square inscribed in this circle are m and n, respectively, then $m + n^2$ is equal to:
(1) 396
(2) 408
(3) 312
(4) 414

Correct Answer: (2)
Solution:
The radius r of the circle inscribed in an equilateral triangle is given by:
$$r = \frac{\Delta}{s} = \frac{\sqrt{3}a^2}{4a} = \frac{a}{2\sqrt{3}} = \frac{12}{2\sqrt{3}} = 2\sqrt{3}.$$



The side of the square inscribed in this circle is:
$$\lambda = r\sqrt{2} = 2\sqrt{3}\cdot\sqrt{2} = 2\sqrt{6}.$$

Area of the square:

$$m = \lambda^2 = (2\sqrt{6})^2 = 24.$$

Perimeter of the square:

$$n = 4\lambda = 4(2\sqrt{6}) = 8\sqrt{6}.$$

$$m + n^2 = 24 + (8\sqrt{6})^2 = 24 + 384 = 408.$$

Quick Tip

For inscribed squares in circles, use $\lambda = r\sqrt{2}$ and relate triangle properties to circle radius for precise calculations.

Q.16 The number of triangles whose vertices are at the vertices of a regular octagon but none of whose sides is a side of the octagon is:
(1) 24
(2) 56
(3) 16
(4) 48

Correct Answer: (3)
Solution:
The number of triangles having no side common with an n-sided polygon is given by:

$$\text{no. of triangles having no side common with a } n \text{ sided polygon} = \binom{n}{1}\cdot\binom{n-4}{2}\div 3$$

Substitute $n = 8$:

$$= \binom{8}{1}\cdot\binom{4}{2}\div 3$$

$$= \frac{8\cdot 6}{3} = 16.$$

Thus, the number of such triangles is 16.

Quick Tip

To count triangles with no sides common in a regular polygon, use the formula $\binom{n}{1}\cdot\binom{n-4}{2}\div 3$, where n is the number of sides of the polygon.

Q.17 Let $y = y(x)$ be the solution of the differential equation:

$$(1+x^2)\frac{dy}{dx} + y = e^{\tan^{-1}x}, \quad y(1) = 0.$$

Then $y(0)$ is:
(1) $\frac{1}{4}(e^{\pi/2} - 1)$

(2) $\frac{1}{2}\left(1-e^{-\pi/2}\right)$
(3) $\frac{1}{4}\left(1-e^{\pi/2}\right)$
(4) $\frac{1}{2}\left(e^{\pi/2}-1\right)$

Correct Answer: (2)
Solution:
The given differential equation is:

$$\frac{dy}{dx}+\frac{y}{1+x^2}=\frac{e^{\tan^{-1}x}}{1+x^2}.$$

Integrating factor (I.F.):

$$\text{I.F.}=e^{\int\frac{1}{1+x^2}dx}=e^{\tan^{-1}x}.$$

Multiply through by I.F.:

$$y\cdot e^{\tan^{-1}x}=\int e^{\tan^{-1}x}\cdot\frac{e^{\tan^{-1}x}}{1+x^2}dx+C.$$

Simplify:

$$y\cdot e^{\tan^{-1}x}=\int e^{2\tan^{-1}x}\cdot\frac{1}{1+x^2}dx+C.$$

Substitute:

$$\tan^{-1}x=z,\quad \frac{1}{1+x^2}dx=dz.$$

Thus:

$$y\cdot e^{\tan^{-1}x}=\frac{e^{2z}}{2}+C.$$

Apply initial condition: $y(1)=0$, $\tan^{-1}(1)=\frac{\pi}{4}$:

$$0=\frac{e^{\pi/2}}{2}+C\implies C=-\frac{e^{\pi/2}}{2}.$$

Finally:

$$y\cdot e^{\tan^{-1}x}=\frac{e^{2\tan^{-1}x}}{2}-\frac{e^{\pi/2}}{2}.$$

At $x=0$, $\tan^{-1}(0)=0$:

$$y(0)=\frac{1}{2}\left(1-e^{-\pi/2}\right).$$

Quick Tip

For differential equations with an integrating factor, simplify by substitution and apply boundary conditions for constants.

Q.18 Let $y=y(x)$ be the solution of the differential equation:

$$(2x\ln x)\frac{dy}{dx}+2y=\frac{3}{x}\ln x,\quad x>0,$$

and $y(e^{-1}) = 0$. Then, $y(e)$ is equal to:
(1) $-\frac{3}{2e}$
(2) $-\frac{2}{3e}$
(3) $-\frac{3}{e}$
(4) $-\frac{2}{e}$

Correct Answer: (3)
Solution:
The given differential equation is:

$$\frac{dy}{dx} + \frac{y}{x \ln x} = \frac{3}{2x^2}.$$

Step 1: Find the integrating factor (I.F.)

$$\text{I.F.} = e^{\int \frac{1}{x \ln x} dx} = e^{\ln(\ln x)} = \ln x.$$

Step 2: Multiply through by I.F.

$$(\ln x)y = \int \frac{3 \ln x}{2x^2} dx.$$

Step 3: Solve the integral

$$\int \frac{3 \ln x}{2x^2} dx = \frac{3}{2} \int x^{-2} \ln x \, dx.$$

Use integration by parts, letting $u = \ln x$ and $dv = x^{-2}dx$:

$$\int \ln x \cdot x^{-2} dx = -\frac{\ln x}{x} - \int -\frac{1}{x^2} dx = -\frac{\ln x}{x} + \frac{1}{x}.$$

Thus:

$$\int \frac{3 \ln x}{2x^2} dx = \frac{3}{2} \left(-\frac{\ln x}{x} + \frac{1}{x} \right).$$

Step 4: Write the solution

$$y \ln x = \frac{3}{2} \left(-\frac{\ln x}{x} + \frac{1}{x} \right) + C.$$

Simplify:

$$y = -\frac{3 \ln x}{2x \ln x} + \frac{3}{2x \ln x} + \frac{C}{\ln x} = -\frac{3}{2x} + \frac{C}{\ln x}.$$

Step 5: Apply the initial condition $y(e^{-1}) = 0$

$$0 = -\frac{3}{2e^{-1}} + \frac{C}{\ln(e^{-1})}.$$

$$0 = -\frac{3}{2} e + \frac{C}{-1}.$$

$$C = -\frac{3}{2} e.$$

Step 6: Find $y(e)$

$$y(e) = -\frac{3}{2e} + \frac{-\frac{3}{2}e}{\ln e}.$$

$$y(e) = -\frac{3}{2e} - \frac{3}{2e} = -\frac{3}{e}.$$

Final Answer: $-\frac{3}{e}$.

Quick Tip

When solving differential equations with logarithmic terms, always simplify the integrating factor carefully and use integration by parts for complex integrals.

Q.19 Let the area of the region enclosed by the curves $y = 3x$, $2y = 27 - 3x$, and $y = 3x - x\sqrt{x}$ be A. Then $10A$ is equal to:
(1) 184
(2) 154
(3) 172
(4) 162

Correct Answer: (4)
Solution:
The curves are $y = 3x$, $2y = 27 - 3x$, and $y = 3x - x\sqrt{x}$. The area A is divided as:

$$A = \int_0^3 \left(3x - (3x - x\sqrt{x})\right) dx + \int_3^9 \left(\frac{27 - 3x}{2} - (3x - x\sqrt{x})\right) dx.$$

First Integral:

$$\int_0^3 \left(3x - 3x + x\sqrt{x}\right) dx = \int_0^3 x\sqrt{x}dx = \int_0^3 x^{3/2}dx = \left[\frac{2x^{5/2}}{5}\right]_0^3.$$

$$= \frac{2}{5}\left(3^{5/2} - 0^{5/2}\right) = \frac{2}{5} \cdot 3^{5/2}.$$

Second Integral:

$$\int_3^9 \left(\frac{27 - 3x}{2} - 3x + x\sqrt{x}\right) dx = \int_3^9 \left(\frac{27}{2} - \frac{3x}{2} - 3x + x\sqrt{x}\right) dx.$$

$$= \int_3^9 \left(\frac{27}{2} - \frac{9x}{2} + x\sqrt{x}\right) dx.$$

Evaluate term by term:

$$\int_3^9 \frac{27}{2}dx = \frac{27}{2} \cdot (9 - 3) = 81.$$

$$\int_3^9 \frac{9x}{2}dx = \frac{9}{2} \cdot \int_3^9 xdx = \frac{9}{2} \cdot \left[\frac{x^2}{2}\right]_3^9 = \frac{9}{2} \cdot \left(\frac{81}{2} - \frac{9}{2}\right) = \frac{9}{2} \cdot 36 = 162.$$

$$\int_3^9 x\sqrt{x}dx = \int_3^9 x^{3/2}dx = \left[\frac{2x^{5/2}}{5}\right]_3^9 = \frac{2}{5}\left(9^{5/2} - 3^{5/2}\right).$$

Combine results:

$$A = \frac{2}{5}\cdot 3^{5/2} + 81 - 162 + \frac{2}{5}\left(9^{5/2} - 3^{5/2}\right).$$

$$A = \frac{2}{5}\cdot 9^{5/2} - \frac{2}{5}\cdot 3^{5/2} + 81 - 162 + \frac{2}{5}\cdot 3^{5/2}.$$

$$A = \frac{2}{5}\cdot 9^{5/2} + 81 - 162.$$

$$A = \frac{486}{5} - 81 = \frac{81}{5}$$

$$10A = 162.$$

Quick Tip

Always calculate the area enclosed by curves by splitting it into regions and carefully setting up the limits of integration. Simplify the expressions systematically to avoid errors.

Q.20 Let $f : (-\infty, \infty) \to R \setminus \{0\}$ be a differentiable function such that:

$$f'(1) = \lim_{a\to\infty} a^2 f\left(\frac{1}{a}\right).$$

Then:

$$\lim_{a\to\infty} \frac{a(a+1)}{2}\tan^{-1}\left(\frac{1}{a}\right) + a^2 - 2\ln a$$

is equal to:

(1) $\frac{3}{2} + \frac{\pi}{4}$
(2) $\frac{3}{8} + \frac{\pi}{4}$
(3) $\frac{5}{2} + \frac{\pi}{8}$
(4) $\frac{3}{4} + \frac{\pi}{8}$

Correct Answer: (3)
Solution:
We are given the limit expression:

$$\lim_{a\to\infty} \frac{a(a+1)}{2}\tan^{-1}\left(\frac{1}{a}\right) + a^2 - 2\ln a.$$

Step 1: Simplify the $\tan^{-1}$ term Using the expansion:

$$\tan^{-1}\left(\frac{1}{a}\right) \approx \frac{1}{a} - \frac{1}{3a^3} \quad \text{as } a \to \infty.$$

Substitute this approximation:

$$\frac{a(a+1)}{2}\tan^{-1}\left(\frac{1}{a}\right) \approx \frac{a(a+1)}{2}\left(\frac{1}{a} - \frac{1}{3a^3}\right).$$

$$= \frac{(a+1)}{2} - \frac{(a+1)}{6a^2}.$$

As $a \to \infty$, the dominant term is:

$$\frac{(a+1)}{2} \to \frac{a}{2}.$$

Step 2: Rewrite the full limit expression The given expression becomes:

$$\lim_{a\to\infty} \left(\frac{a}{2} + a^2 - 2\ln a\right).$$

Step 3: Identify $f(x)$ From the problem:

$$f(x) = \frac{1}{2}\left((1+x)\tan^{-1}(x) + 1 - 2x^2\ln(x)\right).$$

Compute $f'(x)$:

$$f'(x) = \frac{1}{2}\left(\frac{1+x}{1+x^2} + \tan^{-1}(x) + 4x\ln(x) + 2x\right).$$

Substitute $x = 1$:

$$f'(1) = \frac{1}{2}\left(\frac{1+1}{1+1} + \frac{\pi}{4} + 4(1)\ln(1) + 2(1)\right).$$

$$f'(1) = \frac{1}{2}\left(1 + \frac{\pi}{4} + 2\right) = \frac{1}{2}\left(\frac{\pi}{4} + 3\right).$$

Thus:

$$f'(1) = \frac{5}{2} + \frac{\pi}{8}.$$

Quick Tip

Use approximations for small arguments of $\tan^{-1}(x)$ and carefully expand logarithmic terms when solving limits involving asymptotic expressions.

Q.21 Let $\alpha\beta\gamma = 45$; $\alpha, \beta, \gamma \in R$. If:

$$x(\alpha, 1, 2) + y(1, \beta, 2) + z(2, 3, \gamma) = (0, 0, 0),$$

for some $x, y, z \in R$, $xyz \neq 0$, then $6\alpha + 4\beta + \gamma$ is equal to:

(1)
(2)
(3)
(4)

Correct Answer: (55)
Solution:
Given $\alpha\beta\gamma = 45$, $\alpha, \beta, \gamma \in R$,

$$x(\alpha, 1, 2) + y(1, \beta, 2) + z(2, 3, \gamma) = (0, 0, 0).$$

Expanding, we get:

$$x\alpha + y + 2z = 0,$$
$$x + y\beta + 3z = 0,$$
$$2x + 2y + z\gamma = 0.$$

Since $xyz \neq 0$, the determinant of the coefficient matrix must be zero for a non-trivial solution:

$$\begin{vmatrix} \alpha & 1 & 2 \\ 1 & \beta & 2 \\ 2 & 3 & \gamma \end{vmatrix} = 0.$$

Expanding the determinant:

$$\alpha \begin{vmatrix} \beta & 2 \\ 3 & \gamma \end{vmatrix} - 1 \begin{vmatrix} 1 & 2 \\ 2 & \gamma \end{vmatrix} + 2 \begin{vmatrix} 1 & \beta \\ 2 & 3 \end{vmatrix} = 0.$$

Calculating each minor:

$$\begin{vmatrix} \beta & 2 \\ 3 & \gamma \end{vmatrix} = \beta\gamma - 6,$$
$$\begin{vmatrix} 1 & 2 \\ 2 & \gamma \end{vmatrix} = \gamma - 6,$$
$$\begin{vmatrix} 1 & \beta \\ 2 & 3 \end{vmatrix} = 3 - 2\beta.$$

Substituting:

$$\alpha(\beta\gamma - 6) - (\gamma - 6) + 2(3 - 2\beta) = 0.$$

Simplify:

$$\alpha\beta\gamma - 6\alpha - \gamma + 6 + 6 - 4\beta = 0.$$

Since $\alpha\beta\gamma = 45$,

$$45 - 6\alpha - \gamma + 12 - 4\beta = 0.$$

Rearranging:

$$6\alpha + 4\beta + \gamma = 55.$$

Quick Tip

To solve systems involving non-trivial solutions, use the determinant condition for linear dependence. Carefully expand the determinant to derive constraints.

Q.22 Let a conic C pass through the point $(4, -2)$ and $P(x, y)$, $x \geq 3$, be any point on C. Let the slope of the line touching the conic C only at a single point P be half the slope of the line joining the points P and $(3, -5)$. If the focal distance of the point $(7, 1)$ on C is d, then $12d$ equals:

Correct Answer: (75)
Solution:
Given $P(x, y)$ and $x \geq 3$, the slope of the tangent at $P(x, y)$ to the conic is:

$$\frac{dy}{dx} = \frac{1}{2}\frac{y + 5}{x - 3}.$$

Rewriting:

$$2\frac{dy}{(y+5)} = \frac{1}{(x-3)}dx.$$

Integrating both sides:

$$2\ln(y+5) = \ln(x-3) + C.$$

Simplifying:

$$\ln(y+5)^2 = \ln(x-3) + C,$$
$$(y+5)^2 = k(x-3), \quad \text{where } k = e^C.$$

Since the conic passes through $(4, -2)$, substitute:

$$(-2+5)^2 = k(4-3),$$

$$9 = k(1) \implies k = 9.$$

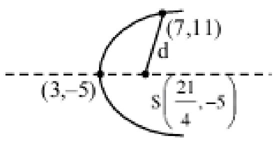
Thus, the conic equation becomes:

$$(y+5)^2 = 9(x-3).$$

This represents a parabola with:

$$4a = 9 \implies a = \frac{9}{4}.$$

The focal distance d of the point $(7, 1)$ is given by:



$$d = \sqrt{\left(\frac{7}{4}\right)^2 + 6^2}.$$

Simplifying:

$$d = \sqrt{\frac{49}{16} + 36} = \sqrt{\frac{625}{16}} = \frac{25}{4}.$$

Thus:

$$12d = 12 \times \frac{25}{4} = 75.$$

Final Answer: 75.

Quick Tip

For conics, relate the slope conditions to the equation of the tangent. Use symmetry and the given points to simplify calculations and confirm the focus and vertex.

Q.23 Let

$$I_k = \int_0^1 (1-x)^k dx,\ k \in N.$$

Then the value of

$$\sum_{k=1}^{10} \frac{1}{7(I_k - 1)}$$

is equal to ________.

Ans. (65)

Solution:

$$I_k = \int_0^1 (1-x)^k dx$$

$$I_k = \left[(1-x)^k \cdot x\right]_0^1 + \int_0^1 k(1-x)^{k-1} \cdot (1-x)dx$$

$$I_k = 0 + k\int_0^1 (1-x)^{k-1}dx - I_k$$

$$I_k = -kI_k + kI_{k-1}$$

$$I_k(1+k) = kI_{k-1}$$

$$\frac{I_k}{I_{k-1}} = \frac{k}{k+1}$$

Thus,

$$r_k = \frac{7k+8}{7k+7}$$

$$r_k - 1 = \frac{-1}{7(k+1)}$$

Substituting in the summation:

$$\sum_{k=1}^{10} \frac{1}{7(I_k - 1)} = \sum_{k=1}^{10} \frac{1}{7} \cdot 7(k+1) = \sum_{k=1}^{10}(k+1)$$

Computing:

$$\sum_{k=1}^{10}(k+1) = \sum_{k=1}^{10} k + \sum_{k=1}^{10} 1 = 55 + 10 = 65$$

Final Answer: 65

Tip

For summations involving definite integrals, simplify the recursive relations and use arithmetic sums to compute efficiently.

Q.24 Let x_1, x_2, x_3, x_4 be the solution of the equation

$$4x^4 + 8x^3 - 17x^2 - 12x + 9 = 0$$

and

$$\left(4 + x_1^2\right)\left(4 + x_2^2\right)\left(4 + x_3^2\right)\left(4 + x_4^2\right) = \frac{125}{16}m.$$

Then the value of m is ________.

Ans. (221)

Solution:
The given polynomial can be expressed as:

$$4x^4 + 8x^3 - 17x^2 - 12x + 9 = 4(x - x_1)(x - x_2)(x - x_3)(x - x_4).$$

Let $x_1 = 2i$ and $x_2 = -2i$. Substituting these values:

$$64 - 64i + 68 - 24i + 9 = 4(2i - x_1)(2i - x_2)(2i - x_3)(2i - x_4).$$

Simplify:

$$141 - 88i \quad \ldots(1)$$

Similarly, for $-2i$:

$$64 + 64i + 68 + 24i + 9 = 4(-2i - x_1)(-2i - x_2)(-2i - x_3)(-2i - x_4).$$

Simplify:

$$141 + 88i \quad \ldots(2)$$

Using the given condition:

$$\frac{125}{16}m = \frac{141^2 + 88^2}{16}.$$

Calculate:

$$m = 221.$$

Quick Tip

To solve higher-degree polynomials, substitute roots systematically and utilize symmetry (real and imaginary parts) to simplify computations.

Q.25 Let L_1, L_2 be the lines passing through the point $P(0, 1)$ and touching the parabola

$$9x^2 + 12x + 18y - 14 = 0.$$

Let Q and R be the points on the lines L_1 and L_2 such that the $\triangle PQR$ is an isosceles triangle with base QR. If the slopes of the lines QR are m_1 and m_2, then

$$16\left(m_1^2 + m_2^2\right)$$

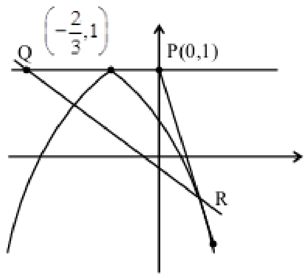
is equal to:
Correct Answer: (68)

Solution:
The given parabola is:

$$9x^2 + 12x + 4 = -18(y - 1).$$

Rewriting:

$$(3x + 2)^2 = -18(y - 1).$$



Equation of a line passing through $P(0, 1)$ is:

$$y = mx + 1.$$

Substituting $y = mx + 1$ into the parabola:

$$(3x + 2)^2 = -18(mx).$$

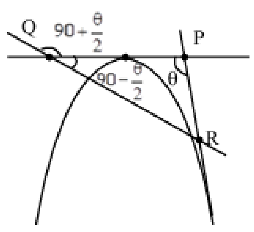
Expanding:

$$9x^2 + (12 + 18m)x + 4 = 0.$$

For the line to be tangent to the parabola:

$$\Delta = 0.$$

Using the discriminant condition:



$$(12 + 18m)^2 - 4(9)(4) = 0.$$

$$144 + 432m + 324m^2 - 144 = 0.$$

$$36m^2 + 108m + 36 = 0.$$

Simplify:

$$m^2 + 3m + 1 = 0.$$

Solving the quadratic equation:

$$m = \frac{-3 \pm \sqrt{3^2 - 4(1)(1)}}{2(1)} = \frac{-3 \pm \sqrt{5}}{2}.$$

This gives the slopes $m_1 = \frac{-3+\sqrt{5}}{2}$ and $m_2 = \frac{-3-\sqrt{5}}{2}$.

Step 2: Finding slopes of QR:

For $\triangle PQR$, the triangle is isosceles with base QR. Using the tangent property, the slope of QR can be derived using:

$$\text{Slope of } QR = \tan\left(\frac{\pi}{2} + \frac{\theta}{2}\right).$$

Using symmetry, calculate θ:

$$\theta = \tan^{-1}\left(\frac{4}{3}\right).$$

Then:

$$m_1 = -\cot\left(\frac{\theta}{2}\right), \quad m_2 = \tan\left(\frac{\theta}{2}\right).$$

Finally, calculate:

$$m_1 = -\frac{1}{2}, \quad m_2 = 2.$$

Step 3: Calculating $16(m_1^2 + m_2^2)$:

$$16(m_1^2 + m_2^2) = 16\left(\left(-\frac{1}{2}\right)^2 + 2^2\right).$$

$$= 16\left(\frac{1}{4} + 4\right).$$

$$= 16 \times \frac{17}{4} = 68.$$

Quick Tip

For finding slopes of tangents from a point to a parabola, use the discriminant condition $\Delta = 0$ to solve for the slope.

Q26. If the second, third and fourth terms in the expansion of $(x+y)^n$ are 135, 30 and $\frac{10}{3}$, respectively, then $6(n^3 + x^2 + y)$ is equal to ______.

Ans. (806)

Solution:

The given terms are:

$$^nC_1 x^{n-1} y = 135 \qquad \text{(i)}$$

$$^nC_2 x^{n-2} y^2 = 30 \qquad \text{(ii)}$$

$$^nC_3x^{n-3}y^3 = \frac{10}{3} \tag{iii}$$

Step 1: Using (i) and (ii)

$$\frac{^nC_1x}{^nC_2y} = \frac{9}{2} \tag{iv}$$

Step 2: Using (ii) and (iii)

$$\frac{^nC_2x}{^nC_3y} = 9 \tag{v}$$

From (iv) and (v), solve for $^nC_2$:

$$\frac{^nC_1 \cdot ^n C_3}{(^nC_2)^2} = \frac{1}{2}$$

Substitute $^nC_1 = n,\ ^nC_2 = \frac{n(n-1)}{2},\ ^nC_3 = \frac{n(n-1)(n-2)}{6}$:

$$\frac{n \cdot \frac{n(n-1)(n-2)}{6}}{\frac{n^2(n-1)^2}{4}} = \frac{1}{2}$$

$$\frac{2n^2(n-2)}{6n(n-1)} = \frac{1}{2}$$

$$2n(n-2) = 3(n-1)$$

$$2n^2 - 4n = 3n - 3$$

$$2n^2 - 7n + 3 = 0$$

$$n = 5 \quad \text{(as } n \text{ is a positive integer)}$$

Step 3: Solving for x and y From (v):

$$\frac{x}{y} = 9 \implies x = 9y$$

Substitute in (i):

$$^5C_1(9y)^4y = 135$$

$$^5C_1 \cdot 9^4y^5 = 135$$

$$5 \cdot 81 \cdot 9 \cdot y^5 = 135$$

$$y = \frac{1}{3},\ x = 3$$

Step 4: Calculating the final expression

$$6(n^3 + x^2 + y)$$

$$= 6(5^3 + 3^2 + \frac{1}{3})$$

$$= 6(125 + 9 + \frac{1}{3})$$

$$= 6(134 + \frac{1}{3})$$

$$= 806$$

Quick Tip

Use ratios of consecutive terms to derive expressions for x, y, and n systematically. Substitute known terms to simplify calculations.

Q27. Let the first term of a series be $T_1 = 6$ and its r^{th} term $T_r = 3T_{r-1} + 6^r$, $r = 2, 3, \ldots, n$. If the sum of the first n terms of this series is $\frac{1}{5}\left(n^2 - 12n + 39\right)(4.6^n - 5.3^n + 1)$, then n is equal to ___.

Correct Answer: (6)

Solution:

For the given series:

$$T_r = 3T_{r-1} + 6^r, \quad \text{for } r \geq 2.$$

For $r = 2$:

$$T_2 = 3T_1 + 6^2 = 3 \cdot 6 + 6^2 = 18 + 36 = 54. \tag{1}$$

For $r = 3$:

$$T_3 = 3T_2 + 6^3 = 3 \cdot 54 + 216 = 162 + 216 = 378. \tag{2}$$

General term:

$$T_r = 3^{r-1} \cdot 6 + 3^{r-2} \cdot 6^2 + \cdots + 6^r.$$

Taking 6 common:

$$T_r = 6 \cdot 3^{r-1} \left(1 + \frac{6}{3} + \cdots + \frac{6^{r-1}}{3^{r-1}}\right).$$

Simplify:

$$T_r = 6^{r-1} \cdot (1 + 2 + 2^2 + \cdots + 2^{r-1}),$$

which forms a geometric progression.

Sum of GP:

$$T_r = \frac{6^r - 3^r}{3}.$$

Sum of the series (S_n):

$$S_n = \sum_{r=1}^{n} T_r = \sum_{r=1}^{n} \frac{6^r - 3^r}{3}.$$

Separate the terms:

$$S_n = \frac{1}{3}\left(\sum_{r=1}^{n} 6^r - \sum_{r=1}^{n} 3^r\right).$$

Sum of 6^r:

$$\sum_{r=1}^{n} 6^r = \frac{6(6^n - 1)}{5}.$$

Sum of 3^r:

$$\sum_{r=1}^{n} 3^r = \frac{3(3^n - 1)}{2}.$$

Substitute:

$$S_n = \frac{1}{3}\left(\frac{6(6^n-1)}{5} - \frac{3(3^n-1)}{2}\right).$$

Simplify:

$$S_n = \frac{1}{5}\left(4.6^n - 5.3^n + 1\right).$$

Equating with the given sum:

$$\frac{1}{5}\left(n^2 - 12n + 39\right)\left(4.6^n - 5.3^n + 1\right) = S_n.$$

Equating coefficients:

$$n^2 - 12n + 36 = 0.$$

Solve:

$$n = 6.$$

Quick Tip

For recursive series, always simplify using the general formula for geometric progressions and identify patterns to determine sums efficiently.

Q28. For $n \in N$, if $\cot^{-1} 3 + \cot^{-1} 4 + \cot^{-1} 5 + \cot^{-1} n = \frac{\pi}{4}$, then n is equal to ___.

Correct Answer: (47)

Solution:

We know:

$$\cot^{-1} 3 + \cot^{-1} 4 = \tan^{-1}\left(\frac{3 \cdot 4 - 1}{3+4}\right) = \tan^{-1}\left(\frac{12-1}{7}\right) = \tan^{-1}\left(\frac{11}{7}\right).$$

Adding $\cot^{-1} 5$:

$$\tan^{-1}\left(\frac{11}{7}\right) + \cot^{-1} 5 = \tan^{-1}\left(\frac{\frac{11}{7}\cdot 5 - 1}{\frac{11}{7} + 5}\right) = \tan^{-1}\left(\frac{\frac{55}{7} - 1}{\frac{11}{7} + 5}\right).$$

Simplify:

$$= \tan^{-1}\left(\frac{\frac{48}{7}}{\frac{46}{7}}\right) = \tan^{-1}\left(\frac{48}{46}\right) = \tan^{-1}\left(\frac{24}{23}\right).$$

Adding $\cot^{-1} n$:

$$\tan^{-1}\left(\frac{24}{23}\right) + \cot^{-1} n = \frac{\pi}{4}.$$

Using the identity:

$$\cot^{-1} a + \cot^{-1} b = \tan^{-1}\left(\frac{a+b}{1-ab}\right),$$

we rewrite:

$$\tan^{-1}\left(\frac{24}{23}\right) + \cot^{-1} n = \frac{\pi}{4}.$$

Simplify further:

$$\tan^{-1}\left(\frac{24}{23}\right) + \tan^{-1}\left(\frac{1}{n}\right) = \frac{\pi}{4}.$$

Using the tangent addition formula:

$$\tan\left(\tan^{-1}\left(\frac{24}{23}\right) + \tan^{-1}\left(\frac{1}{n}\right)\right) = 1.$$

This implies:

$$\frac{\frac{24}{23} + \frac{1}{n}}{1 - \left(\frac{24}{23} \cdot \frac{1}{n}\right)} = 1.$$

Simplify the numerator and denominator:

$$\frac{\frac{24n+23}{23n}}{\frac{n-24}{23n}} = 1.$$

Cancel $23n$ and solve:

$$\frac{24n + 23}{n - 24} = 1.$$

Cross-multiply:

$$24n + 23 = n - 24.$$

Simplify:

$$23n = 47.$$

Thus:

$$n = 47.$$

Quick Tip

When combining $\cot^{-1}$ terms, convert them to $\tan^{-1}$ using the relationship $\cot^{-1} x = \tan^{-1}\left(\frac{1}{x}\right)$. Then use tangent addition or subtraction formulas for simplification.

Q.29 Let $P(10, -2, -1)$ and Q be the foot of the perpendicular drawn from the point $R(1, 7, 6)$ on the line passing through the points $(2, -5, 11)$ and $(-6, 7, -5)$. Then the length of the line segment PQ is equal to ___.

Correct Answer: (13)

Solution:

The equation of the line passing through $(2, -5, 11)$ and $(-6, 7, -5)$ is:

$$\frac{x+6}{-8} = \frac{y-7}{12} = \frac{z+5}{-16} = \lambda.$$

Using the parametric form of the line:

$$Q(\lambda) = (2\lambda - 6, 7 - 3\lambda, 4\lambda - 5).$$

The vector QR (from Q to $R(1, 7, 6)$) is given as:

$$QR = (2\lambda - 7, -3\lambda, 4\lambda - 11).$$

Since QR is perpendicular to the line, the direction ratios $(-8, 12, -16)$ of the line satisfy:

$$-8(2\lambda - 7) + 12(-3\lambda) + (-16)(4\lambda - 11) = 0.$$

Simplify:

$$-16\lambda + 56 - 36\lambda - 64\lambda + 176 = 0.$$

Combine terms:

$$116\lambda = 232 \implies \lambda = 2.$$

Substitute $\lambda = 2$ in the parametric equation of the line:

$$Q(2) = (2(2) - 6, 7 - 3(2), 4(2) - 5) = (-2, 1, 3).$$

The length of the segment PQ is:

$$PQ = \sqrt{(10 - (-2))^2 + (-2 - 1)^2 + (-1 - 3)^2}.$$

Simplify:

$$PQ = \sqrt{(12)^2 + (-3)^2 + (-4)^2} = \sqrt{144 + 9 + 16} = \sqrt{169}.$$

Thus:

$$PQ = 13.$$

Quick Tip

To find the foot of the perpendicular from a point to a line, use the parametric equation of the line and calculate λ by ensuring the perpendicularity condition with the direction ratios of the line.

Q.30. Let $\vec{a} = 2\hat{i} - 3\hat{j} + 4\hat{k}$, $\vec{b} = 3\hat{i} + 4\hat{j} - 5\hat{k}$, and a vector $\vec{c}$ be such that $\vec{a} \times (\vec{b} + \vec{c}) + \vec{b} \times \vec{c} = \hat{i} + 8\hat{j} + 13\hat{k}$. If $\vec{a} \cdot \vec{c} = 13$, then $(24 - \vec{b} \cdot \vec{c})$ is equal to ________.

Ans. (46)
Solution:

From the given equation:

$$\vec{a} \times (\vec{b} + \vec{c}) + \vec{b} \times \vec{c} = \hat{i} + 8\hat{j} + 13\hat{k}$$

Expanding using vector algebra:

$$\vec{a} \times \vec{b} + \vec{a} \times \vec{c} + \vec{b} \times \vec{c} = \hat{i} + 8\hat{j} + 13\hat{k}$$

It is given:

$$\vec{a} \times \vec{b} = \hat{i} + 8\hat{j} + 13\hat{k}$$

So:

$$\vec{a} \times \vec{c} + \vec{b} \times \vec{c} = \vec{0}$$

Expanding further:

$$\vec{b} \times \vec{c} = -\vec{a} \times \vec{c}$$

Using $\vec{a} \cdot \vec{c} = 13$, compute:

$$\vec{b} \cdot \vec{c} = -\left[\vec{a} \cdot (\hat{i} + 8\hat{j} + 13\hat{k})\right] = -22$$

From the determinant of $\vec{b} \cdot \vec{c}$:

$$24 - \vec{b} \cdot \vec{c} = 46$$

Quick Tip

When solving vector equations involving cross and dot products, simplify systematically using vector identities and determinant properties. Always verify the signs and coefficients during calculation.

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JEE Main Solution - Question 31

Q.31. To find the spring constant (k) of a spring experimentally, a student commits 2% positive error in the measurement of time and 1% negative error in the measurement of mass. The percentage error in determining the value of k is:
(1) 3%
(2) 1%
(3) 4%
(4) 5%

Ans. (4)

Solution:

The time period T of a spring is given by:

$$T = 2\pi\sqrt{\frac{m}{k}}$$

Squaring both sides:

$$T^2 \propto \frac{m}{k}$$

Taking percentage errors:

$$\frac{\Delta T^2}{T^2} = \frac{\Delta m}{m} - \frac{\Delta k}{k}$$

Substituting the relationship $\frac{\Delta T^2}{T^2} = 2\frac{\Delta T}{T}$, we get:

$$2\frac{\Delta T}{T} = \frac{\Delta m}{m} - \frac{\Delta k}{k}$$

Rewriting for $\frac{\Delta k}{k}$:

$$\frac{\Delta k}{k} = \frac{\Delta m}{m} - 2\frac{\Delta T}{T}$$

Given:

$$\frac{\Delta T}{T} = 2\% \quad \text{(positive error)}, \quad \frac{\Delta m}{m} = -1\% \quad \text{(negative error)}$$

Substitute the values:

$$\frac{\Delta k}{k} = (-1\%) - 2(2\%) = -1\% - 4\% = -5\%$$

Hence, the magnitude of the percentage error in k is:

$$|\frac{\Delta k}{k}| = 5\%$$

Final Answer: 5% (Option 4)

Quick Tip

When calculating errors in derived quantities, remember to propagate errors systematically using the formula for percentage error in multiplication or division: $(\Delta Q/Q) = (\Delta A/A) + (\Delta B/B)$ and for powers: multiply by the power.

Q.32. A bullet of mass 50 g is fired with a speed 100 m/s on a plywood and emerges with 40 m/s. The percentage loss of kinetic energy is:
(1) 32%
(2) 44%
(3) 16%
(4) 84%

Ans. (4)

Solution:

The initial kinetic energy of the bullet is:

$$K_i = \frac{1}{2}m(100)^2$$

The final kinetic energy of the bullet is:

$$K_f = \frac{1}{2}m(40)^2$$

The percentage loss in kinetic energy is given by:

$$\%\text{loss} = \frac{|K_f - K_i|}{K_i} \times 100$$

Substituting the expressions for K_i and K_f:

$$\%\text{loss} = \frac{\left|\frac{1}{2}m(40)^2 - \frac{1}{2}m(100)^2\right|}{\frac{1}{2}m(100)^2} \times 100$$

Simplify:

$$\%\text{loss} = \frac{|1600 - 10000|}{10000} \times 100$$

$$\%\text{loss} = \frac{8400}{10000} \times 100 = 84\%$$

Final Answer: 84% (Option 4)

Quick Tip

For percentage loss in kinetic energy, use the formula:

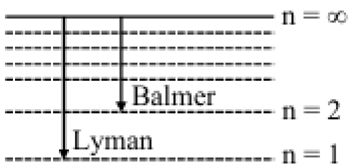
$$\%\text{loss} = \frac{|K_f - K_i|}{K_i} \times 100$$

Ensure consistent units for mass and velocity to avoid calculation errors.

Q.33. The ratio of the shortest wavelength of Balmer series to the shortest wavelength of Lyman series for hydrogen atom is:
(1) 4 : 1
(2) 1 : 2
(3) 1 : 4
(4) 2 : 1

Ans. (1)

Solution:



The wavelength of a spectral line in the hydrogen spectrum is given by:

$$\frac{1}{\lambda} = RZ^2 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$$

where R is the Rydberg constant, Z is the atomic number, n_1 is the lower energy level, and n_2 is the higher energy level.

For the shortest wavelength in the Balmer series:

$$n_1 = 2, \quad n_2 = \infty$$

$$\frac{1}{\lambda_B} = RZ^2 \left(\frac{1}{2^2} - \frac{1}{\infty^2} \right) = RZ^2 \left(\frac{1}{4} \right)$$

For the shortest wavelength in the Lyman series:

$$n_1 = 1, \quad n_2 = \infty$$

$$\frac{1}{\lambda_L} = RZ^2 \left(\frac{1}{1^2} - \frac{1}{\infty^2} \right) = RZ^2 \left(1 \right)$$

Taking the ratio of wavelengths:

$$\frac{\lambda_B}{\lambda_L} = \frac{\frac{1}{RZ^2 \frac{1}{4}}}{\frac{1}{RZ^2(1)}} = 4 : 1$$

Thus, the ratio is:

$$\lambda_B : \lambda_L = 4 : 1$$

Final Answer: 4:1 (Option 1)

Quick Tip

For shortest wavelength calculations, use $n_2 = \infty$ and calculate the wavelength ratio directly by comparing the n_1 levels.

Q.34. To project a body of mass m from earth's surface to infinity, the required kinetic energy is (assume, the radius of earth is R_E, g = acceleration due to gravity on the surface of earth):

(1) $2mgR_E$

(2) mgR_E

(3) $\frac{1}{2}mgR_E$

(4) $4mgR_E$

Ans. (2)

Solution:

The total energy required to project a body to infinity is equal to the work needed to overcome the gravitational potential energy at the surface of the Earth.

Gravitational potential energy at the surface of Earth:

$$\text{Potential Energy} = -\frac{GMm}{R_E}$$

The kinetic energy required for escape velocity:

$$K = \frac{1}{2}mv_e^2$$

Equating this to the energy needed to overcome the gravitational pull:

$$\frac{1}{2}mv_e^2 = \frac{GMm}{R_E}$$

Substituting $g = \frac{GM}{R_E^2}$, we get:

$$\frac{GMm}{R_E} = mgR_E$$

Thus, the required kinetic energy is:

$$K = mgR_E$$

Final Answer: mgR_E **(Option 2)**

Quick Tip

Escape velocity depends on the gravitational pull. For Earth, kinetic energy required to escape is proportional to mgR_E, where g is the acceleration due to gravity, and R_E is the Earth's radius.

Q.35. Electromagnetic waves travel in a medium with speed $1.5 \times 10^8\,\text{ms}^{-1}$. The relative permeability of the medium is 2.0. The relative permittivity will be:
(1) 5
(2) 1
(3) 4
(4) 2

Ans. (4)

Solution:

The speed of electromagnetic waves in a medium is related to its relative permeability (μ_r) and relative permittivity (ε_r) by the equation:

$$\varepsilon_r \mu_r = \frac{c^2}{v^2}$$

Where:

- $c = 3 \times 10^8\,\text{ms}^{-1}$ (speed of light in vacuum),
- $v = 1.5 \times 10^8\,\text{ms}^{-1}$ (speed of light in the medium),
- $\mu_r = 2.0$ (relative permeability of the medium).

Substituting the given values:

$$\varepsilon_r \times 2 = \frac{(3 \times 10^8)^2}{(1.5 \times 10^8)^2}$$

Simplify:

$$\varepsilon_r \times 2 = \frac{9 \times 10^{16}}{2.25 \times 10^{16}}$$

$$\varepsilon_r \times 2 = 4$$

$$\varepsilon_r = 2$$

Final Answer: $\varepsilon_r = 2$ **(Option 4)**

> **Quick Tip**
>
> The relative permittivity and permeability of a medium are inversely proportional to the square of the speed of light in that medium. Use the formula $\varepsilon_r \mu_r = \frac{c^2}{v^2}$ for quick calculations.

Q.36. Which of the following phenomena does not explain by wave nature of light?
(A) reflection
(B) diffraction
(C) photoelectric effect
(D) interference
(E) polarization

Choose the **most appropriate** answer from the options given below:
(1) E only
(2) C only
(3) B, D only
(4) A, C only

Ans. (2)

Solution:

The *photoelectric effect* is the phenomenon in which electrons are ejected from the surface of a material when it is exposed to light. This effect cannot be explained by the wave nature of light. Instead, it provides strong evidence for the **particle nature of light**.
On the other hand, phenomena such as reflection, diffraction, interference, and polarization are well explained by the wave nature of light.

> **Quick Tip**
>
> The photoelectric effect is a key experiment proving the particle nature of light. Keep in mind the distinction between phenomena explained by wave and particle theories.

Q.37. While measuring the diameter of a wire using a screw gauge, the following readings were noted. The main scale reading is 1 mm, and the circular scale reading is equal to 42 divisions. The pitch of the screw gauge is 1 mm, and it has 100 divisions on the circular scale. The diameter of the wire is $\frac{x}{50}$ mm. The value of x is:
(1) 142
(2) 71
(3) 42
(4) 21

Ans. (2)

Solution:
Given:

$$\text{MSR (Main Scale Reading)} = 1\,\text{mm}, \quad \text{CSD (Circular Scale Reading)} = 42$$

$$\text{Least Count (LC)} = \frac{\text{Pitch}}{\text{Number of CSD}} = \frac{1}{100}\,\text{mm} = 0.01\,\text{mm}$$

$$\text{Diameter} = \text{MSR} + \text{LC} \times \text{CSD}$$

$$\text{Diameter} = 1 + (0.01) \times 42\,\text{mm} = 1.42\,\text{mm}$$

Since the diameter is given as $\frac{x}{50}$, equate:

$$\frac{x}{50} = 1.42 \implies x = 71$$

$$\text{The value of } x \text{ is } \boxed{71}.$$

Quick Tip

Always double-check the least count and ensure the unit conversions are consistent while using instruments like screw gauges.

Q.38. σ is the uniform surface charge density of a thin spherical shell of radius R. The electric field at any point on the surface of the spherical shell is:
(1) $\frac{\sigma}{\epsilon_0}$
(2) $\frac{\sigma}{2\epsilon_0}$
(3) $\frac{\sigma}{\epsilon_0 R}$
(4) $\frac{\sigma}{4\epsilon_0}$

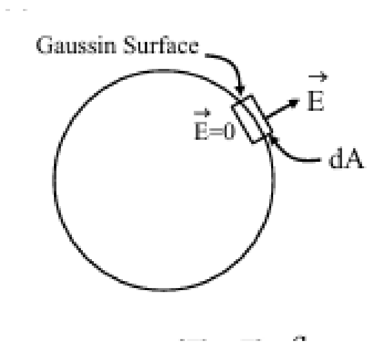
Ans. (3)

Solution:
By Gauss's Law:

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{in}}}{\epsilon_0}$$

For a spherical shell, the electric field E is constant across the surface area:

$$E \cdot 4\pi R^2 = \frac{\sigma \cdot 4\pi R^2}{\epsilon_0}$$



$$E = \frac{\sigma}{\epsilon_0}$$

Thus, the electric field at the surface of the spherical shell is:

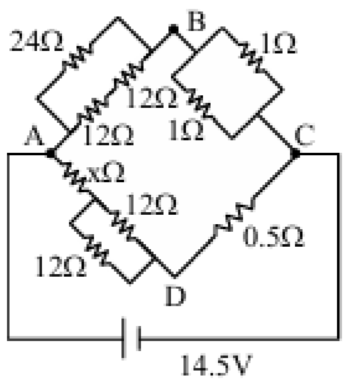
$$\boxed{\frac{\sigma}{\epsilon_0}}$$

Quick Tip

For spherical charge distributions, the electric field behaves like that of a point charge at the center, making calculations simpler using symmetry.

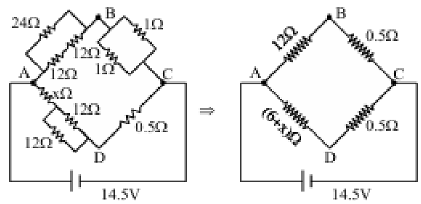
---

39. The value of unknown resistance (x) for which the potential difference between B and D will be zero in the arrangement shown, is:



(1) 3 Ω
(2) 9 Ω
(3) 6 Ω
(4) 42 Ω

Ans. (3)
Solution: In the given arrangement, we observe a Wheatstone Bridge.
For the bridge to be balanced:



$$\frac{V_{AB}}{V_{AD}} = \frac{V_{BC}}{V_{CD}}$$

Substituting values from the circuit:

$$\frac{12}{6+x} = \frac{0.5}{0.5}$$

Simplify:

$$12(0.5) = (6+x)(0.5)$$

$$6 = 3 + 0.5x \implies x = 6\ \Omega$$

Final Answer: $x = 6\ \Omega$.

Quick Tip

For balanced Wheatstone Bridge problems, equate the ratio of resistances in adjacent branches.

---

40. The specific heat at constant pressure of a real gas obeying $PV^2 = RT$ equation is:

(1) $C_v + R$
(2) $\frac{R}{3} + C_v$
(3) R
(4) $C_v + \frac{R}{2V}$

Ans. (4)
Solution:
The first law of thermodynamics gives:

$$dQ = du + dW$$

At constant pressure, this becomes:

$$CdT = C_v dT + PdV \quad ....(1)$$

Given $PV^2 = RT$, differentiating both sides with respect to T at constant P:

$$P(2VdV) = RdT$$

$$PdV = \frac{R}{2V}dT$$

Substitute PdV into equation (1):

$$CdT = C_v dT + \frac{R}{2V}dT$$

$$C = C_v + \frac{R}{2V}$$

Thus, the specific heat at constant pressure is $C = C_v + \frac{R}{2V}$.

Quick Tip

For real gases, specific heat relations often involve the derivative of P, V, or T. Differentiate the given equation systematically to derive the expression.

Q41. Match List I with List II

List I	List II
A. Torque	I. $[ML^2T^{-2}]$
B. Magnetic field	II. $[MA^{-1}T^{-2}]$
C. Magnetic moment	III. $[AL^2]$
D. Permeability of free space	IV. $[MLT^{-2}A^{-2}]$

Choose the correct answer from the options below:

1. A-I, B-III, C-II, D-IV
2. A-IV, B-III, C-II, D-I
3. A-III, B-I, C-II, D-IV
4. A-IV, B-II, C-III, D-I

Correct Answer: (2)
Solution:

$$\vec{\tau} = \vec{r} \times \vec{F} \implies [\tau] = [ML^2T^{-2}]$$

$$[\vec{F}] = [q\vec{v} \times \vec{B}] \implies [\vec{B}] = \frac{[\vec{F}]}{[q][\vec{v}]} = \frac{MLT^{-2}}{ATL^{-1}} = [MA^{-1}T^{-2}]$$

$$[\vec{M}] = [\vec{I} \times \vec{A}] = [AL^2]$$

Using Biot-Savart's Law,

$$B = \frac{\mu_0 I dl \sin\theta}{r^2}$$

$$\mu = \frac{Br^2}{Idl} \implies \mu = \frac{MT^{-2}A^{-1} \times L^2}{AL} = [MLT^{-2}A^{-2}]$$

Thus, the correct matching is:

- Torque $\rightarrow [ML^2T^{-2}]$
- Magnetic field $\rightarrow [MA^{-1}T^{-2}]$
- Magnetic moment $\rightarrow [AL^2]$
- Permeability of free space $\rightarrow [MLT^{-2}A^{-2}]$

Quick Tip

Dimensional analysis is a powerful tool to verify the correctness of physical equations and match derived units. Remember common unit dimensions like $[MLT^{-2}]$ for force and $[ML^2T^{-3}]$ for energy.

Q42. Given below are two statements:
Statement I: In an LCR series circuit, current is maximum at resonance.
Statement II: Current in a purely resistive circuit can never be less than that in a series LCR circuit when connected to the same voltage source.
In the light of the above statements, choose the **correct** option from the given below:
1. Statement I is true but Statement II is false.
2. Statement I is false but Statement II is true.
3. Both Statement I and Statement II are true.
4. Both Statement I and Statement II are false.

Correct Answer: (3)
Solution:
Statement-I:

$$I_m = \frac{V_m}{\sqrt{R^2 + (X_L - X_C)^2}} \quad \text{at resonance } X_L = X_C$$

$$I_m = \frac{V_m}{R}$$

Thus, impedance is minimum, therefore, I is maximum at resonance.
Statement-II:

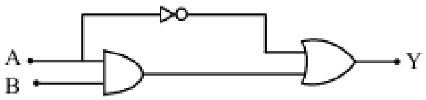
$$I = \frac{V}{R} \quad \text{in a purely resistive circuit.}$$

Hence, in a purely resistive circuit, current cannot be less than that in a series LCR circuit. Thus, both Statement I and Statement II are true.

Quick Tip

At resonance, the inductive reactance X_L and capacitive reactance X_C cancel out, minimizing impedance. This is why the current is maximum in an LCR circuit.

Q43. The correct truth table for the following logic circuit is:



Options :

(1)

A	B	Y
0	0	0
0	1	1
1	0	0
1	1	1

(2)

A	B	Y
0	0	1
0	1	1
1	0	0
1	1	1

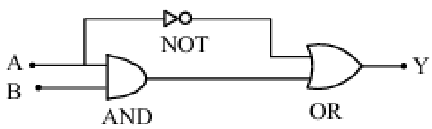
(3)

A	B	Y
0	0	1
0	1	1
1	0	0
1	1	0

(4)

A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1

Correct Answer: (2)
Solution:
The circuit consists of an AND gate, a NOT gate, and an OR gate. The output Y is determined as follows:



1. The output of the AND gate is $A \cdot B$. 2. The output of the NOT gate is $\overline{A \cdot B}$. 3. The output Y is the OR of $\overline{A \cdot B}$ and B:

$$Y = \overline{A \cdot B} + B$$

Step-by-Step Evaluation of Truth Table:

A	B	$A \cdot B$	$\overline{A \cdot B}$	$Y = \overline{A \cdot B} + B$
0	0	0	1	1
0	1	0	1	0
1	0	0	1	1
1	1	1	0	1

Thus, the correct truth table is represented in **Option (2)**.

Quick Tip

For complex logic circuits, break down the gates step-by-step, evaluating intermediate outputs before determining the final output.

Q44. A sample contains a mixture of helium and oxygen gas. The ratio of root mean square speed of helium and oxygen in the sample, is:

Options:

1. $\frac{1}{32}$
2. $\frac{2\sqrt{2}}{1}$
3. $\frac{1}{4}$
4. $\frac{1}{2\sqrt{2}}$

Correct Answer: (2)

Solution:

The root mean square speed (V_{rms}) is given by:

$$V_{rms} = \sqrt{\frac{3RT}{M_w}}$$

where M_w is the molar mass of the gas.
The ratio of root mean square speeds of helium (V_{He}) and oxygen (V_{O_2}) is:

$$\frac{V_{O_2}}{V_{He}} = \sqrt{\frac{M_{w,He}}{M_{w,O_2}}}$$

Substituting the values:

$$M_{w,He} = 4\,\text{g/mol}, \quad M_{w,O_2} = 32\,\text{g/mol}$$

$$\frac{V_{O_2}}{V_{He}} = \sqrt{\frac{4}{32}} = \frac{1}{2\sqrt{2}}$$

The ratio $\frac{V_{He}}{V_{O_2}}$ is:

$$\frac{V_{He}}{V_{O_2}} = \frac{2\sqrt{2}}{1}$$

Quick Tip

To compare root mean square speeds of different gases, use the formula $V_{rms} \propto \frac{1}{\sqrt{M_w}}$. Gases with lower molar mass move faster.

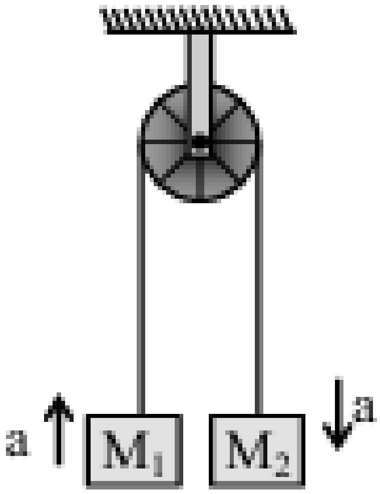
Q45. A light string passing over a smooth light pulley connects two blocks of masses m_1 and m_2 (where $m_2 > m_1$). If the acceleration of the system is $\frac{g}{\sqrt{2}}$, then the ratio of the masses $\frac{m_1}{m_2}$ is:

Options:

1. $\frac{\sqrt{2}-1}{\sqrt{2}+1}$
2. $\frac{1+\sqrt{5}}{\sqrt{5}-1}$
3. $\frac{1+\sqrt{5}}{\sqrt{2}-1}$
4. $\frac{\sqrt{3}+1}{\sqrt{2}-1}$

Correct Answer: (1)
Solution:
The acceleration of the system is given by:

$$a = \frac{m_2 - m_1}{m_1 + m_2} g$$



Given $a = \frac{g}{\sqrt{2}}$, substitute in the equation:

$$\frac{g}{\sqrt{2}} = \frac{m_2 - m_1}{m_1 + m_2} g$$

Simplifying:

$$\frac{1}{\sqrt{2}} = \frac{m_2 - m_1}{m_1 + m_2}$$

Cross-multiplying:

$$\sqrt{2}(m_2 - m_1) = m_1 + m_2$$

Rearranging:

$$m_1(\sqrt{2} + 1) = m_2(\sqrt{2} - 1)$$

The ratio of masses is:

$$\frac{m_1}{m_2} = \frac{\sqrt{2} - 1}{\sqrt{2} + 1}$$

Quick Tip

To solve problems involving pulley systems, always write the equation of acceleration using $a = \frac{m_2 - m_1}{m_1 + m_2} g$. This simplifies finding the ratio of masses or accelerations.

Q46. Four particles A, B, C, D of mass $\frac{m}{2}, m, 2m, 4m$, have the same momentum, respectively. The particle with maximum kinetic energy is:
Options:
1. D
2. C
3. A
4. B
Correct Answer: (3)
Solution:
The kinetic energy is given by:

$$KE = \frac{p^2}{2m}$$

Since all particles have the same momentum, the kinetic energy is inversely proportional to their mass:

$$KE \propto \frac{1}{m}$$

Thus, the particle with the smallest mass will have the maximum kinetic energy. Among the given particles:

$$m_A = \frac{m}{2}, \quad m_B = m, \quad m_C = 2m, \quad m_D = 4m$$

Hence, $\frac{m}{2}$ (particle A) has the maximum kinetic energy.

Quick Tip

For the same momentum, kinetic energy is inversely proportional to mass. The smaller the mass, the greater the kinetic energy.

Q.47 A train starting from rest first accelerates uniformly up to a speed of 80 km/h for time t, then it moves with a constant speed for time $3t$. The average speed of the train for this duration of the journey will be (in km/h):
(1) 80
(2) 70
(3) 30
(4) 40

Ans. (2)
Solution:
The average speed is calculated using the formula:

$$\text{Average speed} = \frac{\text{total distance}}{\text{time taken}}$$

During the first phase of acceleration:

$$\text{Distance covered} = \frac{1}{2} \times \text{final speed} \times \text{time} = \frac{1}{2} \times 80 \times t = 40t$$

During the second phase of constant speed:

$$\text{Distance covered} = \text{speed} \times \text{time} = 80 \times 3t = 240t$$

Total distance covered:

$$40t + 240t = 280t$$

Total time taken:

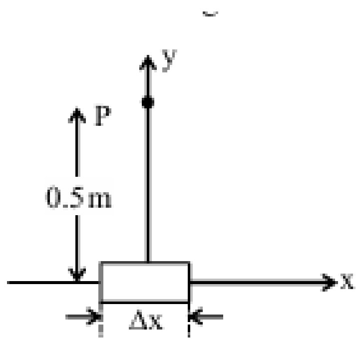
$$t + 3t = 4t$$

$$\text{Average speed} = \frac{280t}{4t} = 70\,\text{km/h}$$

Quick Tip

When calculating average speed for different phases of motion, use the total distance covered and total time taken. Do not directly average the speeds!

Q.48 An element $\Delta l = \Delta x\hat{i}$ is placed at the origin and carries a large current $I = 10\,\text{A}$. The magnetic field on the y-axis at a distance of $0.5\,\text{m}$ from the element Δx of $1\,\text{cm}$ length is:



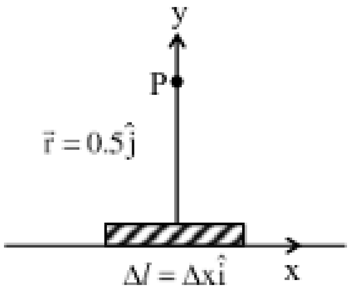
(1) $4 \times 10^{-8}\,\text{T}$
(2) $8 \times 10^{-8}\,\text{T}$
(3) $12 \times 10^{-8}\,\text{T}$
(4) $10 \times 10^{-8}\,\text{T}$

Ans. (1)
Solution:
The position vector for the field point is:

$$\vec{r} = 0.5\hat{j}$$

The magnetic field produced by the current element Δl is given by the Biot-Savart law:

$$dB = \frac{\mu_0 I(\Delta\vec{l} \times \vec{r})}{4\pi r^3}$$



Here,

$$\Delta\vec{l} = \Delta x\hat{i} = \frac{1}{100}\hat{i}\,\text{m}, \quad \vec{r} = 0.5\hat{j}\,\text{m}, \quad r = |\vec{r}| = 0.5\,\text{m}$$

$$\Delta\vec{l} \times \vec{r} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{1}{100} & 0 & 0 \\ 0 & 0.5 & 0 \end{vmatrix} = \frac{1}{100} \times 0.5\hat{k} = \frac{1}{200}\hat{k}$$

Substituting values into the Biot-Savart law:

$$dB = \frac{10^{-7} \times 10 \times \frac{1}{200}}{(0.5)^3}\,\text{T}$$

$$dB = \frac{10^{-7} \times 10 \times \frac{1}{200}}{\frac{1}{8}} = 4 \times 10^{-8}\,\text{T}$$

Hence, the magnetic field is:

$$dB = 4 \times 10^{-8}\,\hat{k}\,\text{T}$$

Quick Tip

To apply the Biot-Savart law effectively, ensure the cross product $\Delta\vec{l} \times \vec{r}$ is calculated correctly. Use determinants for vector products.

Q.49 A small ball of mass m and density ρ is dropped in a viscous liquid of density ρ_0. After some time, the ball falls with constant velocity. The viscous force on the ball is:

1) $mg\left(\frac{\rho_0}{\rho} - 1\right)$

2) $mg\left(1 + \frac{\rho}{\rho_0}\right)$

3) $mg\,(1 - \rho\rho_0)$

4) $mg\left(1 - \frac{\rho_0}{\rho}\right)$

Correct Answer: (4)

Solution:

Applying force balance on the ball at constant velocity:
$mg - F_B - F_v = ma$
Since acceleration $a = 0$ for constant velocity:
$\Rightarrow mg - F_B = F_v$

The buoyant force is given by:

$$F_B = v\rho_0 g \quad \text{where } v \text{ is the volume of the ball.}$$

$F_v = mg - v\rho_0 g$
Substituting $v = \dfrac{m}{\rho}$ (volume in terms of mass and density):
$\Rightarrow F_v = mg - \dfrac{m}{\rho}\rho_0 g$
$F_v = mg\left(1 - \dfrac{\rho_0}{\rho}\right)$

Quick Tip

At terminal velocity, the net force acting on the ball is zero. Use force balance to find the viscous force.

Q.50 In photoelectric experiment, energy of 2.48 eV irradiates a photo-sensitive material. The stopping potential was measured to be 0.5 V. Work function of the photo-sensitive material is:
1) 0.5 eV
2) 1.68 eV
3) 2.48 eV
4) 1.98 eV
Correct Answer: (4)
Solution:
The energy relation for the photoelectric effect is:

$$eV_s = h\nu - \phi$$

where eV_s is the stopping potential energy, $h\nu$ is the energy of incident photons, and ϕ is the work function.

Given: $h\nu = 2.48$ eV, $V_s = 0.5$ V.
$0.5 = 2.48 - \phi$
$\phi = 2.48 - 0.5 = 1.98$ eV.

The work function of the material is $\phi = 1.98$ eV.

Quick Tip

The work function (ϕ) can be quickly calculated using the relation $\phi = h\nu - eV_s$. Ensure all values are in consistent units (eV or Joules).

Q.51 If the radius of earth is reduced to three-fourth of its present value without change in its mass, then value of duration of the day of earth will be _______ hours 30 minutes.
Correct Answer: (3)
Solution:
By conservation of angular momentum:

$$I_1\omega_1 = I_2\omega_2$$

Moment of inertia of a sphere:

$$I = \frac{2}{5}MR^2$$

Angular velocity relation:

$$\omega = \frac{2\pi}{T}$$

$$\frac{2}{5}MR^2 \cdot \frac{2\pi}{T_1} = \frac{2}{5}M\left(\frac{3}{4}R\right)^2 \cdot \frac{2\pi}{T_2}$$

Simplifying:

$$\frac{1}{T_1} = \frac{9}{16} \cdot \frac{1}{T_2}$$

$$T_2 = \frac{16}{9} \cdot T_1$$

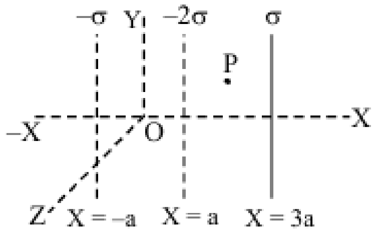
Substituting $T_1 = 24$ hours:

$$T_2 = \frac{9}{16} \cdot 24 = \frac{27}{2} = 13 \text{ hours } 30 \text{ minutes.}$$

Quick Tip

For angular momentum conservation problems involving rotational motion, always equate $I_1\omega_1 = I_2\omega_2$ and substitute the moment of inertia formula for the given geometry.

Q.52 Three infinitely long charged thin sheets are placed as shown in figure. The magnitude of electric field at the point P is $\frac{x\sigma}{\epsilon_0}$. The value of x is _________ (All quantities are measured in SI units).
Ans. (2)
Solution:
The electric field at P is the vector sum of the fields due to the three charged sheets.



Using Gauss's Law:

$$\vec{E} = \frac{\sigma}{2\epsilon_0}$$

For the point P:

$$\vec{E}_P = \left(\frac{\sigma}{2\epsilon_0} + \frac{2\sigma}{2\epsilon_0} + \frac{\sigma}{2\epsilon_0}\right)(-\hat{i})$$

Simplify:

$$\vec{E}_P = \frac{4\sigma}{2\epsilon_0}(-\hat{i}) = \frac{2\sigma}{\epsilon_0}(-\hat{i})$$

Thus, $x = 2$.

Quick Tip

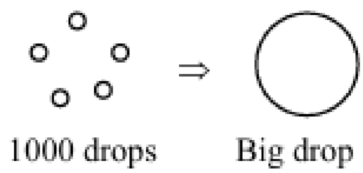
For calculating the electric field due to thin sheets, always use the formula $\vec{E} = \frac{\sigma}{2\epsilon_0}$ and sum the contributions algebraically considering directions.

Q 53. A big drop is formed by coalescing 1000 small droplets of water. The ratio of surface energy of 1000 droplets to that of energy of big drop is $\frac{10}{x}$. The value of x is _________.

Ans. (1)

Solution:

The volume of 1000 small drops is equal to the volume of the big drop.



$$1000 \times \frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3$$

$$R = 10r$$

The surface energy (S.E.) of the 1000 drops is:

$$\text{S.E. of 1000 drops} = 1000 \times (4\pi r^2)T$$

The surface energy of the big drop is:

$$\text{S.E. of Big drop} = 4\pi R^2 T$$

Substituting $R = 10r$:

$$\text{S.E. of Big drop} = 4\pi(10r)^2 T = 400\pi r^2 T$$

The ratio of the surface energy is:

$$\frac{\text{S.E. of 1000 drops}}{\text{S.E. of Big drop}} = \frac{1000 \cdot 4\pi r^2 T}{4\pi(10r)^2 T}$$

$$\frac{\text{S.E. of 1000 drops}}{\text{S.E. of Big drop}} = \frac{1000}{100} = \frac{10}{x}$$

$$x = 1$$

Quick Tip

The radius of the big drop formed by coalescing small droplets can be calculated using volume conservation: $R^3 = n \cdot r^3$. Use this to find ratios of other quantities like surface area or energy.

Q54. When a DC voltage of $100\,V$ is applied to an inductor, a DC current of $5\,A$ flows through it. When an AC voltage of $200\,V$ peak value is connected to the inductor, its inductive reactance is found to be $20\sqrt{3}\,\Omega$. The power dissipated in the circuit is ______ W.

Ans. (250 W)

Solution:

For DC voltage:

$$R = \frac{V}{I} = \frac{100}{5} = 20\,\Omega$$

For AC voltage:

$$X_L = 20\sqrt{3}\,\Omega$$

$$Z = \sqrt{X_L^2 + R^2} = \sqrt{(20\sqrt{3})^2 + 20^2} = \sqrt{1200 + 400} = 40\,\Omega$$

Power dissipated in the circuit:

$$P = I_{\text{rms}}^2 R = \left(\frac{V_{\text{rms}}}{Z}\right)^2 \times R$$

$$P = \left(\frac{200}{\sqrt{2} \times 40}\right)^2 \times 20$$

$$P = \left(\frac{200}{40\sqrt{2}}\right)^2 \times 20 = 250\,W$$

Quick Tip

Always calculate Z (impedance) carefully in AC circuits. Remember $Z = \sqrt{R^2 + X_L^2}$ for inductive circuits.

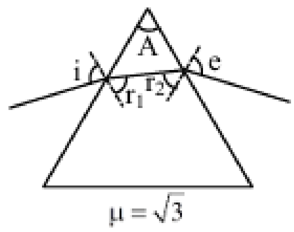
Q55. The refractive index of prism is $\mu = \sqrt{3}$ and the ratio of the angle of minimum deviation to the angle of prism is one. The value of angle of prism is ______ $^\circ$.

Ans. (3)

Solution:

For δ_{min}:

1



$$i = e \quad \text{and} \quad r_1 = r_2 = \frac{A}{2}$$

$$\delta_{\text{min}} = 2i - A$$

Given, $\frac{\delta_{\text{min}}}{A} = 1$:

$$\frac{2i - A}{A} = 1$$

$$2i - A = A \implies 2i = 2A \implies i = A$$

Using Snell's law:

$$1 \cdot \sin i = \mu \cdot \sin r \implies \sin i = \mu \cdot \sin\left(\frac{A}{2}\right)$$

$$\sin A = \mu \cdot \sin\left(\frac{A}{2}\right)$$

Expanding $\sin A$:

$$2 \sin\frac{A}{2} \cos\frac{A}{2} = \sqrt{3} \cdot \sin\frac{A}{2}$$

$$2 \cos\frac{A}{2} = \sqrt{3} \implies \cos\frac{A}{2} = \frac{\sqrt{3}}{2}$$

$$\frac{A}{2} = 30^\circ \implies A = 60^\circ$$

Quick Tip

For prisms, use Snell's law at the refracting surfaces and relate minimum deviation to the geometry of the prism for quick calculations.

Q56. A wire of resistance R and radius r is stretched till its radius became $r/2$. If the new resistance of the stretched wire is xR, then the value of x is ______.

Ans. (16 R)

Solution:

We know that:

$$R = \rho \frac{l}{A}, \quad R \propto \frac{l}{r^2}$$

As we stretch the wire, its length increases, and its radius decreases, keeping the volume constant:

$$V_i = V_f$$

$$\pi r^2 l = \pi \frac{r^2}{4} l_f \implies l_f = 4l$$

Now:

$$\frac{R_{\text{new}}}{R_{\text{old}}} = \frac{l_f}{l} \cdot \frac{r^2}{(r/2)^2} = \frac{4l}{l} \cdot \frac{r^2}{r^2/4} = 4 \cdot 4 = 16$$

$$R_{\text{new}} = 16R$$

Thus, $x = 16$.

Quick Tip

For problems involving stretched wires, remember that volume remains constant, and resistance is directly proportional to $\frac{l}{r^2}$.

Q57. Radius of a certain orbit of hydrogen atom is 8.48 Å. If energy of electron in this orbit is E/x, then $x =$ ______.

(Given $a_0 = 0.529$ Å, $E =$ energy of electron in ground state)

(1) 8

(2) 12

(3) 16

(4) 20

Ans. (3)

Solution:

We know:

$$r = 0.529 \frac{n^2}{Z} \implies 8.48 = 0.529 \frac{n^2}{1}$$

$$n^2 = 16 \implies n = 4$$

We also know:

$$E \propto \frac{1}{n^2}$$

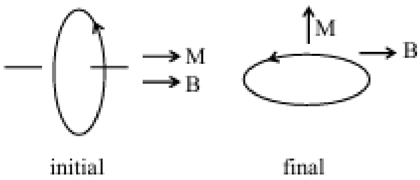
$$E_n = \frac{E}{16}$$

$$x = 16$$

Quick Tip

For hydrogen-like atoms, the radius of the orbit is proportional to n^2 and energy is inversely proportional to n^2.

Q58. A circular coil having 200 turns, $2.5 \times 10^{-4}\,\text{m}^2$ area and carrying $100\,\mu\text{A}$ current is placed in a uniform magnetic field of 1 T. Initially the magnetic dipole moment ($\vec{M}$) was directed along $\vec{B}$. Amount of work, required to rotate the coil through $90°$ from its initial orientation such that $\vec{M}$ becomes perpendicular to $\vec{B}$, is _________ μJ.
Ans. (4)
Solution:
We know:



$$W_{\text{ext}} = \Delta U + \Delta\text{KE} \quad (\text{P.E.} = -\vec{M}\cdot\vec{B})$$

$$= -MB\cos 90° + MB\cos 0°$$

$$W_{\text{ext}} = MB$$

$$= NIAB$$

$$= 200 \times 100 \times 10^{-6} \times 2.5 \times 10^{-4} \times 1 = 5\,\mu\text{J}$$

Quick Tip

Work done to rotate a magnetic dipole in a magnetic field is given by the change in potential energy: $W = \Delta U = MB(\cos\theta_1 - \cos\theta_2)$.

Q59. A particle is doing simple harmonic motion of amplitude 0.06 m and time period 3.14 s. The maximum velocity of the particle is _________ cm/s.
Ans. (2)
Solution:

We know:

$$v_{\max} = \omega A \quad \text{at mean position}$$

$$\omega = \frac{2\pi}{T} \quad \text{and} \quad v_{\max} = \frac{2\pi}{T} \times A$$

$$v_{\max} = \frac{2\pi}{3.14} \times 0.06 = 0.12\,\text{m/s}$$

$$v_{\max} = 12\,\text{cm/s}$$

Quick Tip

In SHM, maximum velocity occurs at the equilibrium position and is given by $v_{\max} = \omega A$, where $\omega = \frac{2\pi}{T}$.

Q60. For three vectors $\vec{A} = (-x\hat{i} - 6\hat{j} - 2\hat{k})$, $\vec{B} = (-\hat{i} + 4\hat{j} + 3\hat{k})$ and $\vec{C} = (-8\hat{i} - \hat{j} + 3\hat{k})$, if $\vec{A} \cdot (\vec{B} \times \vec{C}) = 0$, then the value of x is _________.
(1) 3
(2) 5
(3) 6
(4) 4

Ans. (4)
Solution:
First, compute $\vec{B} \times \vec{C}$:

$$\vec{B} \times \vec{C} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 4 & 3 \\ -8 & -1 & 3 \end{vmatrix}$$

$$\vec{B} \times \vec{C} = \hat{i}\begin{vmatrix} 4 & 3 \\ -1 & 3 \end{vmatrix} - \hat{j}\begin{vmatrix} -1 & 3 \\ -8 & 3 \end{vmatrix} + \hat{k}\begin{vmatrix} -1 & 4 \\ -8 & -1 \end{vmatrix}$$

$$\vec{B} \times \vec{C} = \hat{i}(12 + 3) - \hat{j}(-3 + 24) + \hat{k}(-1 - (-32))$$

$$\vec{B} \times \vec{C} = 15\hat{i} - 21\hat{j} + 33\hat{k}$$

Now, compute $\vec{A} \cdot (\vec{B} \times \vec{C})$:

$$\vec{A} \cdot (\vec{B} \times \vec{C}) = (-x\hat{i} - 6\hat{j} - 2\hat{k}) \cdot (15\hat{i} - 21\hat{j} + 33\hat{k})$$

$$\vec{A} \cdot (\vec{B} \times \vec{C}) = (-x)(15) + (-6)(-21) + (-2)(33)$$

$$\vec{A} \cdot (\vec{B} \times \vec{C}) = -15x + 126 - 66$$

$$\vec{A} \cdot (\vec{B} \times \vec{C}) = -15x + 60$$

Since $\vec{A} \cdot (\vec{B} \times \vec{C}) = 0$, we get:

$$-15x + 60 = 0$$

$$15x = 60$$

$$x = 4$$

Quick Tip

The scalar triple product $\vec{A} \cdot (\vec{B} \times \vec{C})$ is zero if $\vec{A}$, $\vec{B}$, and $\vec{C}$ are coplanar.

Q61. Functional group present in sulphonic acid is:
(1) SO_3H
(2) SO_3H
(3) $-S-OH$
(4) $-SO_2$
Ans. (2)
Solution:
The functional group present in sulphonic acids is as follows:

$$\text{Structure:} \quad \begin{array}{c} \text{O} \\ \| \\ \text{S} - \text{OH} \\ \| \\ \text{O} \end{array}$$

Group present in sulphonic acids.

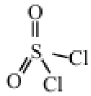
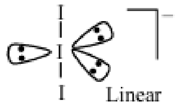
Quick Tip

Sulphonic acids are characterized by the presence of the SO_3H functional group.

Q62. Match List I with List II:

List I (Molecule / Species)	List II (Property / Shape)
A. SO_2Cl_2	I. Paramagnetic
B. NO	II. Diamagnetic
C. NO_2^-	III. Tetrahedral
D. I_3^-	IV. Linear

Choose the correct answer from the options given below:
(1) A-IV, B-I, C-III, D-II
(2) A-III, B-I, C-II, D-IV
(3) A-III, B-III, C-I, D-IV
(4) A-III, B-IV, C-II, D-I
Ans. (2) Solution:

(A)	SO_2Cl_2	sp^3	 Tetrahedral
(B)	NO		Paramagnetic
(C)	NO_2^-		Diamagnetic
(D)	I_3^-	sp^3d	 Linear

Explanation:

(A) SO_2Cl_2 : Hybridization is sp^3, Tetrahedral shape.

(B) NO : Unpaired electron, hence Paramagnetic.

(C) NO_2^- : Paired electrons, hence Diamagnetic.

(D) I_3^- : Linear shape with sp^3d hybridization.

Quick Tip

Molecules with unpaired electrons exhibit paramagnetic behavior, while those with paired electrons are diamagnetic.

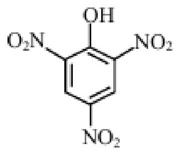
Q63. Given below are two statements:

Statement I: Picric acid is 2, 4, 6-trinitrotoluene.
Statement II: Phenol-2, 4-disulphuric acid is treated with conc. HNO_3 to get picric acid.

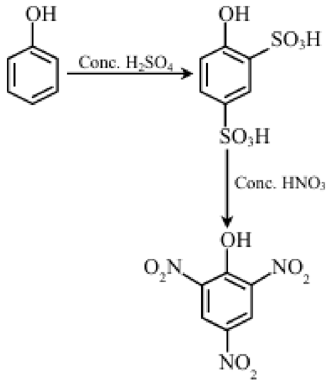
In the light of the above statement, choose the most appropriate answer from the options given below:
(1) Statement I is incorrect but Statement II is correct.
(2) Both Statement I and Statement II are incorrect.
(3) Statement I is correct but Statement II is incorrect.
(4) Both Statement I and Statement II are correct.

Ans. (1)

Solution:
Statement I: Picric acid is **2, 4, 6-trinitrophenol**, not 2, 4, 6-trinitrotoluene.
Statement II: Phenol-2, 4-disulphuric acid is treated with concentrated HNO_3 to produce picric acid.



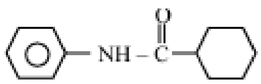
picric acid
(2, 4, 6 – trinitrophenol)



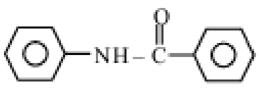
Picric acid

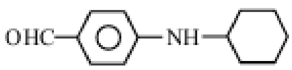
> **Quick Tip**
>
> Remember that 2, 4, 6-trinitrophenol is commonly called picric acid. It is not the same as trinitrotoluene (TNT).

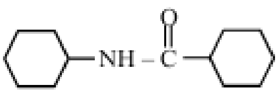
Q64. Which of the following is a metamer of the given compound (X)?

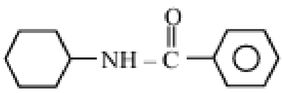


(X)

(1) 

(2) 

(3) 

(4) 

Ans. (4)

Solution:

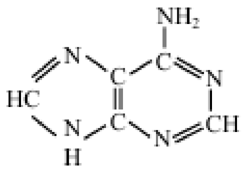
A **metamer** is an isomer having the same molecular formula, the same functional group, but different alkyl/aryl groups on either side of the functional group.
Hence, option (4) is the correct metamer.

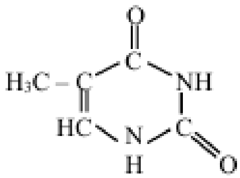
Quick Tip

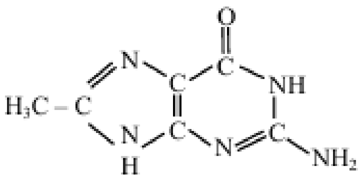
Remember, metamers have the same functional group and molecular formula but differ in the alkyl or aryl groups attached to the functional group.

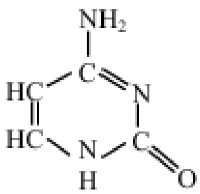
---

65. DNA molecule contains 4 bases whose structure are shown below. One of the structures is not correct. Identify the incorrect base structure.

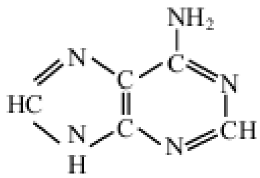
(1) 

(2) 

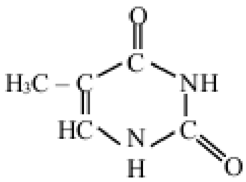
(3) 

(4) 

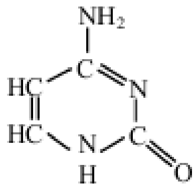
Correct Answer: (3)
Solution:
Are bases of DNA molecule. As DNA contain four bases, which are adenine, guanine, cytosine and thymine.



⇒ Adenine



⇒ Thymine



⇒ Cytosine

Quick Tip

DNA bases include Adenine (A), Thymine (T), Guanine (G), and Cytosine (C). If a structure is mismatched, cross-check against these four.

---

66. Match List I with List II :

List I (Hybridization)	List II (Orientation in Space)
A. sp^3	III. Tetrahedral
B. dsp^2	IV. Square planar
C. sp^3d	I. Trigonal bipyramidal
D. sp^3d^2	II. Octahedral

Options:

(1) A-III, B-I, C-IV, D-II

(2) A-III, B-I, C-II, D-IV

(3) A-IV, B-III, C-I, D-II

(4) A-III, B-IV, C-I, D-II

Correct Answer: (4)
Solution:

$$sp^3 \rightarrow \text{Tetrahedral}$$

$$dsp^2 \rightarrow \text{Square planar}$$

$$sp^3d \rightarrow \text{Trigonal bipyramidal}$$

$$sp^3d^2 \rightarrow \text{Octahedral}$$

Quick Tip

To identify hybridization and shapes:

- sp^3: Tetrahedral (e.g., methane)
- dsp^2: Square planar (e.g., $[PtCl_4]^{2-}$)
- sp^3d: Trigonal bipyramidal (e.g., PCl_5)
- sp^3d^2: Octahedral (e.g., SF_6)

---

67. Given below are two statements:
Statement I: Gallium is used in the manufacturing of thermometers.
Statement II: A thermometer containing gallium is useful for measuring the freezing point (256 K) of brine solution.
In the light of the above statements, choose the correct answer from the options given below:

(1) Both Statement I and Statement II are false.
(2) Statement I is false but Statement II is true.
(3) Both Statement I and Statement II are true.
(4) Statement I is true but Statement II is false.

Correct Answer: (4)
Solution:

- **Statement I:** True. Gallium is used in manufacturing thermometers due to its wide liquid range.
- **Statement II:** False. Gallium is used to measure high temperatures, not low temperatures like 256 K (freezing point of brine).

Quick Tip

Gallium thermometers are ideal for measuring high temperatures due to gallium's melting point (303 K) and boiling point (2477 K), making it unsuitable for low-temperature measurements.

---

68. Which of the following statements are correct?

- A. Glycerol is purified by vacuum distillation because it decomposes at its normal boiling point.
- B. Aniline can be purified by steam distillation as aniline is miscible in water.
- C. Ethanol can be separated from ethanol-water mixture by azeotropic distillation because it forms azeotrope.

- D. An organic compound is pure if mixed melting point remains the same.

Choose the most appropriate answer from the options given below:

(1) A, B, C only
(2) A, C, D only
(3) B, C, D only
(4) A, B, D only

Correct Answer: (2)
Solution:

- **A:** True. Glycerol decomposes at its normal boiling point, so it is purified by vacuum distillation.
- **B:** False. Aniline is **immiscible** in water but can still be purified by steam distillation.
- **C:** True. Ethanol forms an azeotrope with water, making azeotropic distillation effective for separation.
- **D:** True. A pure organic compound exhibits the same melting point even when mixed with another sample of the same compound.

Quick Tip

- Vacuum distillation is used for heat-sensitive compounds.
- Immiscibility or azeotropic properties influence purification methods.
- Mixed melting point analysis is a quick test for purity.

---

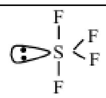
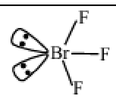
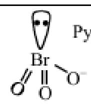
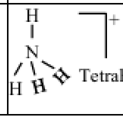
69. Match List I with List II:

List I (Compound/Species)	**List II (Shape/Geometry)**
A. SF_4	I. Tetrahedral
B. BrF_3	II. Pyramidal
C. BrO_3^-	III. See Saw
D. NH_4^+	IV. Bent T-shape

Choose the correct answer from the options given below:

(1) A-II, B-III, C-I, D-IV
(2) A-III, B-IV, C-II, D-I
(3) A-II, B-IV, C-III, D-I
(4) A-III, B-II, C-IV, D-I

Correct Answer: (2)
Solution:

(A)	SF_4	sp^3d hybridisation	
(B)	BrF_3	sp^3d hybridisation	 Bent T-Shape
(C)	BrO_3^-	sp^3 hybridisation	 Pyramidal
(D)	NH_4^+	sp^3 hybridisation	 Tetrahedral

- **(A) SF_4:** The sulfur atom in SF_4 undergoes sp^3d hybridization, resulting in a **see-saw geometry**.
- **(B) BrF_3:** Bromine in BrF_3 exhibits sp^3d hybridization with two lone pairs, resulting in a **bent T-shape geometry**.
- **(C) BrO_3^-:** The bromine atom in BrO_3^- is sp^3 hybridized, resulting in a **pyramidal geometry**.
- **(D) NH_4^+:** The nitrogen atom in NH_4^+ undergoes sp^3 hybridization, forming a **tetrahedral geometry**.

Quick Tip

Lone pairs influence molecular geometry significantly, altering idealized shapes. SF_4 and BrF_3 are classic examples of molecules with lone pairs causing deviation from perfect geometries.

---

70. In Reimer-Tiemann reaction, phenol is converted into salicylaldehyde through an intermediate. The structure of intermediate is:

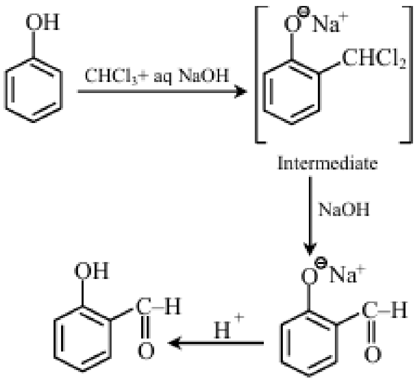
Correct Answer: (4)

Solution:

In the Reimer-Tiemann reaction, phenol reacts with chloroform ($CHCl_3$) and aqueous sodium hydroxide (NaOH) to form salicylaldehyde via the following mechanism:

1. Phenol undergoes ortho-substitution due to the electrophilic attack of dichlorocarbene (:CCl_2), generated in situ from $CHCl_3$ and NaOH.
2. The intermediate formed is the sodium salt of dichlorophenol.
3. On further hydrolysis and protonation, salicylaldehyde is obtained.

The reaction proceeds as:



Quick Tip

- Dichlorocarbene (:CCl_2) is the key intermediate in the Reimer-Tiemann reaction.
- Electrophilic substitution at the ortho-position of phenol occurs due to the activating effect of the hydroxyl (-OH) group.

Q.71. Which of the following material is not a semiconductor?

(1) Germanium (2) Graphite (3) Silicon (4) Copper oxide

Ans. (2)
Solution:

Graphite is a good conductor of electricity due to the free electrons in its structure, and it is not a semiconductor.

Quick Tip

Graphite has a unique structure of delocalized electrons, making it an excellent conductor instead of a semiconductor.

Q.72. Consider the following complexes:

(A) $[CoCl(NH_3)_5]^{2+}$, (B) $[Co(CN)_6]^{3-}$
(C) $[Co(NH_3)_5(H_2O)]^{3+}$, (D) $[Cu(H_2O)_4]^{2+}$

The correct order of A, B, C, and D in terms of wavenumber of light absorbed is:

(1) $C < D < A < B$ (2) $D < A < C < B$ (3) $A < C <$

$B < D$ (4) $B < C < A < D$

Ans. (2)
Solution:

As the ligand field strength increases, the energy of light absorbed by the complex also increases. Since wavenumber $\bar{\nu} \propto$ energy of absorbed light, the order of wavenumber depends on the ligand strength.
- For $[Co(CN)_6]^{3-}$ (B), CN^- is a strong field ligand (highest $\bar{\nu}$). - For $[Co(NH_3)_5(H_2O)]^{3+}$ (C), NH_3 and H_2O have moderate ligand field strength. - For $[Cu(H_2O)_4]^{2+}$ (D), H_2O is a weaker field ligand. - For $[CoCl(NH_3)_5]^{2+}$ (A), Cl^- is the weakest field ligand (lowest $\bar{\nu}$).
Thus, the order is:

$$D < A < C < B.$$

Quick Tip

Ligand strength directly affects the crystal field splitting energy. Stronger field ligands absorb light of higher energy, corresponding to a higher wavenumber.

Q.73. Match List I with List II:

List I (Precipitating reagent and conditions)	List II (Cation)
A. $NH_4Cl + NH_4OH$	I. Mn^{2+}
B. $NH_4OH + Na_2CO_3$	II. Pb^{2+}
C. $NH_4OH + NH_4Cl + H_2S$ gas	III. Al^{3+}
D. Dilute HCl	IV. Sr^{2+}

Choose the correct answer from the options given below:

(1) A-IV, B-III, C-II, D-I
(2) A-IV, B-II, C-I, D-II
(3) A-III, B-IV, C-II, D-I
(4) A-III, B-IV, C-II, D-I

Ans. (3)
Solution:

The matching is based on the chemical properties and specific precipitation conditions for cations:
- A. $NH_4Cl + NH_4OH$ precipitates $Al(OH)_3$, hence matches with **III. Al^{3+}**. - B. $NH_4OH + Na_2CO_3$ precipitates $SrCO_3$, hence matches with **IV. Sr^{2+}**. - C. $NH_4OH + NH_4Cl + H_2S$ gas precipitates PbS, hence matches with **II. Pb^{2+}**. - D. Dilute HCl precipitates $MnCl_2$, hence matches with **I. Mn^{2+}**.

Thus, the correct order is:

$$A - III, B - IV, C - II, D - I.$$

Quick Tip

In qualitative analysis, precipitation reactions are used to identify cations. For example, sulfide ions (H_2S gas) are commonly used to precipitate transition metals, while ammonium carbonate or hydroxide precipitates alkaline earth metals.

Q.74. The electron affinity values are negative for:

A. $Be \rightarrow Be^-$
B. $N \rightarrow N^-$
C. $O \rightarrow O^-$
D. $Na \rightarrow Na^-$
E. $Al \rightarrow Al^-$

Choose the most appropriate answer from the options given below:

(1) D and E only
(2) A, B, D and E only
(3) A and D only
(4) A, B and C only

Allen Ans. (4)
NTA Ans. (1)
Solution:

The electron affinity values for each case are determined as follows: - **(A)** $Be + e^- \rightarrow Be^-$: Electron affinity is negative (**E.A. = −ive**).
(B) $N + e^- \rightarrow N^-$: Electron affinity is negative (**E.A. = −ive**).
(C) $O + e^- \rightarrow O^-$: Electron affinity is positive for $O \rightarrow O^-$ (**E.A. = +ive**).
(D) $Na + e^- \rightarrow Na^-$: Electron affinity is positive (**E.A. = +ive**).
(E) $Al + e^- \rightarrow Al^-$: Electron affinity is positive (**E.A. = +ive**).
Thus, the correct answer is: **A, B and C only**.

Quick Tip

Electron affinity (E.A.) is the energy change when an electron is added to a neutral atom. Elements with half-filled or filled orbitals tend to have positive electron affinity values.

Q.75. The number of elements from the following that do not belong to lanthanoids is:

Eu, Cm, Er, Tb, Yb and Lu

(1) 3
(2) 4
(3) 1
(4) 5

Ans. (3)
Solution:

Among the given elements: - **Eu (Europium)**, **Er (Erbium)**, **Tb (Terbium)**, **Yb (Ytterbium)**, and **Lu (Lutetium)** all belong to the lanthanoid series. - **Cm (Curium)** belongs to the actinide series.

Hence, the number of elements that do not belong to lanthanoids is **1 (Cm)**.

Quick Tip

Lanthanoids include elements with atomic numbers from 57 (La) to 71 (Lu). Elements beyond Lu belong to the actinide series.

Q.76. The density of 'x' M solution ('x' molar) of NaOH is $1.12\,\mathrm{g\,mL^{-1}}$, while in molality, the concentration of the solution is $3\,\mathrm{m}$ (3 molal). Then x is

(1) 3.5
(2) 3.0
(3) 3.8
(4) 2.8

Ans. (2)
Solution:

We know the relationship between molality and molarity:

$$\text{Molality} = \frac{1000 \times M}{1000 \times d - M \times (\text{Molar mass of solute})}$$

Substituting the values:

$$3 = \frac{1000 \times x}{1000 \times 1.12 - x \times 40}$$

Rearranging:

$$3 \times (1000 \times 1.12 - x \times 40) = 1000 \times x$$

Simplify:

$$3 \times 1120 - 120x = 1000x$$

$$3360 = 1120x$$

$$x = 3$$

Thus, the molarity of the solution is **3.0 M**.

Quick Tip

To convert between molarity and molality, always consider the density of the solution and the molar mass of the solute. Use the formula Molality $= \frac{1000\times\text{Molarity}}{1000\times d-\text{Molarity}\times\text{Molar mass}}$.

Q.77. Which among the following aldehydes is most reactive towards nucleophilic addition reactions?

(1) HCHO
(2) C_2H_5CHO
(3) CH_3CHO
(4) C_3H_7CHO

Ans. (1)
Solution:

Formaldehyde (HCHO) is the most reactive towards nucleophilic addition reactions because:

- It has no alkyl groups attached to the carbonyl carbon, leading to the lowest steric hindrance at the carbonyl carbon.
- The partial positive charge on the carbonyl carbon is highest due to the absence of electron-donating groups (alkyl groups), making it highly electrophilic.

Thus, HCHO is the most reactive aldehyde for nucleophilic addition reactions.

Quick Tip

Reactivity of aldehydes and ketones in nucleophilic addition reactions decreases with an increase in steric hindrance and electron-donating alkyl groups around the carbonyl carbon.

Q.78. At $-20°$C and 1 atm pressure, a cylinder is filled with an equal number of H_2, I_2, and HI molecules for the reaction:

$$H_2(g) + I_2(g) \rightleftharpoons 2HI(g), \quad K_p \text{ for the process is } x \times 10^{-1}.\ x = ?$$

Options:

(1) 2
(2) 1
(3) 10
(4) 0.01

Ans. (3)
Solution:

The reaction given is:

$$H_2(g) + I_2(g) \rightleftharpoons 2HI(g)$$

At equilibrium, $\Delta n_g = 0$ (no change in the number of moles of gas). Therefore:

$$K_p = \frac{(n_{HI})^2}{n_{H_2} \cdot n_{I_2}} \cdot \left(\frac{P_T}{P_T}\right)^{\Delta n_g}$$

Since $\Delta n_g = 0$, the pressure term simplifies:

$$K_p = \frac{(n_{HI})^2}{n_{H_2} \cdot n_{I_2}}$$

Given that the number of moles of H_2, I_2, and HI are all initially equal:

$$n_{H_2} = n_{I_2} = 1, \quad n_{HI} = 1$$

Substituting into the formula:

$$K_p = \frac{(1)^2}{1 \cdot 1} = 1 \times 10^1$$

Thus:

$$x = 10$$

Quick Tip

For reactions where $\Delta n_g = 0$, the equilibrium constant K_p depends only on the ratio of mole fractions of reactants and products.

Q.79 Match **List I** with **List II**:

List I (Compound)	**List II (Uses)**
A. Iodoform	I. Fire extinguisher
B. Carbon tetrachloride	II. Insecticide
C. CFC	III. Antiseptic
D. DDT	IV. Refrigerants

Choose the **correct** answer from the options given below:

1. A-I, B-II, C-III, D-IV
2. A-III, B-II, C-IV, D-I

3. A-III, B-I, C-IV, D-II
4. A-II, B-IV, C-I, D-III

Ans. (3)
Solution:

$$\begin{aligned}\textbf{Iodoform} &\rightarrow \text{Antiseptic}\\ \textbf{CCl}_4 &\rightarrow \text{Fire extinguisher}\\ \textbf{CFC} &\rightarrow \text{Refrigerants}\\ \textbf{DDT} &\rightarrow \text{Insecticide}\end{aligned}$$

Thus, the correct match is:

$$\boxed{\text{A-III, B-I, C-IV, D-II}}$$

Quick Tip

CFCs (Chlorofluorocarbons) are widely used as refrigerants due to their stability and low boiling points. Always link compounds to their historical or primary industrial applications for accurate matches.

Q.80 A conductivity cell with two electrodes (dark side) are half filled with infinitely dilute aqueous solution of a weak electrolyte. If volume is doubled by adding more water at constant temperature, the molar conductivity of the cell will:

1. increase sharply
2. remain same or can not be measured accurately
3. decrease sharply
4. depend upon type of electrolyte

Ans. (2)
Solution:
The solution is already infinitely dilute, meaning the molar conductivity is at its maximum value. Adding more water will not dilute the solution further; therefore, there will be no change in molar conductivity. It will remain the same.

$$\text{Molar conductivity } (\Lambda_m) \propto \frac{\kappa}{c}$$

where κ is the conductivity and c is the concentration. For infinitely dilute solutions, c is already approaching zero, and Λ_m becomes constant.

Quick Tip

For infinitely dilute solutions, molar conductivity reaches its limiting value (Λ_m^∞), and no further dilution will change its value.

81. Consider the dissociation of the weak acid HX as given below:

$$HX(aq)H^{+}(aq) + X^{-}(aq),\ K_a = 1.2 \times 10^{-5}$$

[K_a: dissociation constant]
The osmotic pressure of 0.03 M aqueous solution of HX at 300 K is _____ $\times 10^{-2}$ bar (nearest integer).

$$\text{Given: } R = 0.083\,\text{L bar mol}^{-1}\text{K}^{-1}$$

Correct Answer: 76

Solution:

The dissociation of HX is represented as:

$$HXH^{+} + X^{-}$$

Initial concentration of HX: 0.03 M
At equilibrium:

$$[HX] = 0.03 - x, \quad [H^{+}] = x, \quad [X^{-}] = x$$

Using the dissociation constant K_a:

$$K_a = \frac{x^2}{0.03 - x}$$

For $K_a = 1.2 \times 10^{-5}$ and $0.03 - x \approx 0.03$ (since K_a is very small):

$$1.2 \times 10^{-5} = \frac{x^2}{0.03}$$

$$x^2 = 1.2 \times 10^{-5} \times 0.03 = 3.6 \times 10^{-7}$$
$$x = \sqrt{3.6 \times 10^{-7}} = 6 \times 10^{-4}$$

Total solute concentration:

$$C_{\text{total}} = [HX] + [H^{+}] + [X^{-}] = 0.03 - x + x + x = 0.03 + x$$

$$C_{\text{total}} = 0.03 + 6 \times 10^{-4} = 0.0306\,\text{M}$$

Osmotic pressure Π is calculated using:

$$\Pi = C_{\text{total}} RT$$

$$\Pi = (0.0306) \times (0.083) \times (300)$$
$$\Pi = 76.19\,\text{bar}$$

Nearest integer:

$$\Pi = 76 \times 10^{-2}\,\text{bar}$$

Quick Tip

For weak acids with very small K_a, assume that the change in concentration (x) is negligible compared to the initial concentration. This simplifies calculations significantly.

---

82. The difference in the 'spin-only' magnetic moment values of $KMnO_4$ and the manganese product formed during titration of $KMnO_4$ against oxalic acid in acidic medium is _____ BM (nearest integer).

Correct Answer: 6

Solution:

Spin-only magnetic moment of Mn in $KMnO_4$:

$$\mu = 0$$

Spin-only magnetic moment of the manganese product formed during titration of $KMnO_4$ against oxalic acid in acidic medium:

$$\mu = 6\,\text{BM}$$

Difference in magnetic moments:

$$6 - 0 = 6\,\text{BM}$$

Quick Tip

The oxidation state of Mn in $KMnO_4$ is +7 (no unpaired electrons, $\mu = 0$), and the manganese product is Mn^{2+} (5 unpaired electrons, $\mu = 6$ BM).

---

83. Time required for 99.9% completion of a first-order reaction is _____ times the time required for completion of 90% reaction (nearest integer).

Correct Answer: 3

Solution:

The rate constant K for a first-order reaction is given by:

$$K = \frac{1}{t} \ln\left(\frac{100}{100 - \text{completion percentage}}\right)$$

For 99.9% completion:

$$t_{99.9\%} = \frac{\ln(1000)}{K}$$

For 90% completion:

$$t_{90\%} = \frac{\ln(10)}{K}$$

Ratio of times:

$$\frac{t_{99.9\%}}{t_{90\%}} = \frac{\ln(1000)}{\ln(10)} = \frac{3\ln(10)}{\ln(10)} = 3$$

Quick Tip

For a first-order reaction, the time required to reach a certain percentage completion is proportional to the natural log of the remaining percentage. Use the ratio of ln values for quick comparison.

---

84. Number of molecules from the following which can exhibit hydrogen bonding is _____ (nearest integer).

$$CH_3OH,\ H_2O,\ C_6H_6,\ C_6H_5NO_2,\ HF,\ NH_3$$

Correct Answer: 5

Solution:

Molecules that exhibit hydrogen bonding:

$$CH_3OH,\ H_2O,\ HF,\ NH_3,\ C_6H_5NO_2$$

Benzene (C_6H_6) does not form hydrogen bonds. Total number of molecules exhibiting hydrogen bonding:

$$5$$

Quick Tip

Hydrogen bonding occurs when H is directly bonded to highly electronegative atoms like O, N, or F. Look for -OH, -NH, or H-F groups in molecules.

---

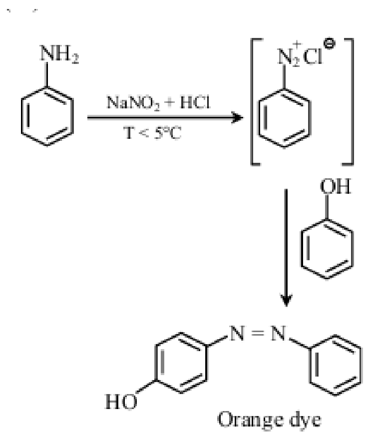
85. 9.3 g of pure aniline upon diazotisation followed by coupling with phenol gives an orange dye. The mass of orange dye produced (assume 100% yield/conversion) is ________ g. (nearest integer).

Correct Answer: 20
Solution:
The reaction is as follows:

$$C_6H_5NH_2 + NaNO_2 + HCl \xrightarrow{T<5^\circ C} C_6H_5\text{-}N_2^+Cl^- \xrightarrow{\text{Phenol}} \text{Orange Dye}$$

1 mole of aniline ($C_6H_5NH_2$) produces 1 mole of orange dye.
Molar mass of aniline = 93 g mol^{-1}.
Molar mass of orange dye = 199 g mol^{-1}.
Moles of aniline used:

$$\text{moles of aniline} = \frac{\text{mass of aniline}}{\text{molar mass of aniline}} = \frac{9.3}{93} = 0.1\,\text{mol}$$



Mass of orange dye produced:

$$\text{mass of orange dye} = \text{moles of aniline} \times \text{molar mass of orange dye} = 0.1 \times 199 = 19.9\,\text{g} \approx 20\,\text{g}$$

Quick Tip

For such stoichiometric calculations: - Use the reaction equation to determine the mole ratio. - Calculate moles using mass/molar mass. - Multiply moles by the molar mass of the product to find the mass.

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86. The major product of the following reaction is P.

$$CH_3C \equiv C\text{-}CH_3 \xrightarrow{\text{(i) Na/liq. } NH_3} \xrightarrow{\text{(ii) dil. } KMnO_4,\, 273\,K} \text{'P'}$$

Number of oxygen atoms present in product 'P' is _______ (nearest integer).

Correct Answer: 2
Solution:
The reaction proceeds as follows:
Step 1: The triple bond in $CH_3C{\equiv}C\text{-}CH_3$ is reduced to a trans-alkene using Na/liq. NH_3:

$$CH_3C \equiv C\text{-}CH_3 \xrightarrow{\text{Na/liq. } NH_3} CH_3CH = CHCH_3$$

Step 2: The trans-alkene undergoes oxidation with dilute $KMnO_4$ at 273 K. This leads to the formation of a vicinal diol:

$$CH_3CH = CHCH_3 \xrightarrow{\text{dil. } KMnO_4} CH_3CH(OH)CH(OH)CH_3$$

The final product 'P' contains two hydroxyl (-OH) groups, contributing two oxygen atoms.
Number of oxygen atoms in product 'P':

$$2 \text{ (from hydroxyl groups)}$$

Quick Tip

In such reactions: - Sodium in liquid ammonia selectively reduces alkynes to trans-alkenes. - Dilute $KMnO_4$ oxidizes alkenes to form vicinal diols (glycols).

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87. Frequency of the de-Broglie wave of an electron in Bohr's first orbit of the hydrogen atom is _____ $\times 10^{13}$ Hz (nearest integer).

Given: R_H (Rydberg constant) $= 2.18\times10^{-18}$ J, h (Planck's constant) $= 6.6\times10^{-34}$ J $\cdot$ s.

Allen Answer: 661
NTA Answer: 658

Correct Answer: 661
Solution:
The de-Broglie wavelength λ is given by:

$$\lambda = \frac{h}{mv}$$

For an electron in motion:

$$\text{Kinetic Energy (K.E.)} = \frac{1}{2}mv^2 \implies v^2 = \frac{2 \cdot \text{K.E.}}{m}.$$

Step 1: Substituting values:

$$\text{K.E.} = R_H = 2.18 \times 10^{-18}\,\text{J}.$$

$$v = \sqrt{\frac{2 \cdot R_H}{m}} = \sqrt{\frac{2 \cdot 2.18 \times 10^{-18}}{9.1 \times 10^{-31}}}.$$

Step 2: Using frequency relation:

$$\nu = \frac{v}{\lambda}, \quad \lambda = \frac{h}{mv}.$$

Step 3: Substituting h and solving for ν:

$$\nu = \frac{\text{K.E.}}{h} = \frac{2.18 \times 10^{-18}}{6.6 \times 10^{-34}}.$$

$$\nu = 660.6 \times 10^{13}\,\text{Hz}.$$

Step 4: Nearest integer:

$$\nu \approx 661 \times 10^{13}\,\text{Hz}.$$

Quick Tip

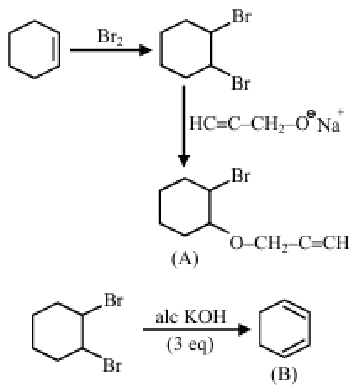
For frequency of de-Broglie waves: - Use $\nu = \frac{\text{K.E.}}{h}$ directly when energy values are given. - Ensure consistent units for energy and Planck's constant.

88. The major products from the following reaction sequence are product A and product B.

$$B \xrightarrow{\text{(i) } Br_2} \xrightarrow{\text{(ii) alc. KOH (3 eq.)}} A$$
$$B \xrightarrow{\text{(i) } Br_2} \xrightarrow{\text{(ii) } Na^+/O^- \text{ (1.0 eq.)}} B$$

The total sum of π electrons in product A and product B are ______ (nearest integer).

Correct Answer: 8 Solution:
The reaction proceeds as follows:



$$\text{Cyclohexane} \xrightarrow{Br_2} C_6H_{10}Br_2$$

1. Product A: - Bromination followed by elimination with alcoholic KOH gives a conjugated diene:

$$C_6H_{10}Br_2 \xrightarrow{\text{alc. KOH (3 eq.)}} C_6H_6 \text{ (benzene)}.$$

- Benzene contains $6\,\pi$ electrons.
2. Product B: - Bromination followed by reaction with sodium hydroxide gives an enolate ion:

$$C_6H_{10}Br_2 \xrightarrow{Na^+/O^-} \text{O-CH}_2\text{CH=CH}_2.$$

- This product contains $2\,\pi$ electrons.

$$\text{Total } \pi \text{ electrons: } 6 + 2 = 8$$

Quick Tip

For calculating π electrons: - Benzene contributes 6π electrons due to its aromatic ring. - Each double bond in alkenes contributes 2π electrons.

89. Among CrO, Cr_2O_3, and CrO_3, the sum of spin-only magnetic moment values of basic and amphoteric oxides is _____ $\times 10^{-2}$ BM (nearest integer).

(Given atomic number of Cr is 24)

Correct Answer: 877

Solution:

1. **CrO** (Basic oxide): - In CrO, chromium exists as Cr^{2+}. - Spin-only magnetic moment (μ):

$$\mu = \sqrt{n(n+2)}\,\text{BM}, \quad n = \text{number of unpaired electrons} = 4.$$

$$\mu = \sqrt{4(4+2)} = 4.90\,\text{BM}.$$

2. **Cr_2O_3** (Amphoteric oxide): - In Cr_2O_3, chromium exists as Cr^{3+}. - Spin-only magnetic moment (μ):

$$\mu = \sqrt{n(n+2)}\,\text{BM}, \quad n = 3.$$

$$\mu = \sqrt{3(3+2)} = 3.87\,\text{BM}.$$

3. **CrO_3** (Acidic oxide): - In CrO_3, chromium exists as Cr^{6+}. - No unpaired electrons ($n = 0$), so $\mu = 0$.

Sum of spin-only magnetic moments of basic and amphoteric oxides:

$$\mu_{\text{total}} = 4.90 + 3.87 = 8.77\,\text{BM}.$$

Expressing in terms of 10^{-2} BM:

$$\mu_{\text{only}} = 8.77 \times 10^{-2}\,\text{BM}.$$

Answer: 877

Quick Tip

For calculating spin-only magnetic moments: - Use $\mu = \sqrt{n(n+2)}$, where n is the number of unpaired electrons. - Remember to determine the oxidation state of the metal ion to calculate n.

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90. An ideal gas, $C_V = \frac{5}{2}R$, is expanded adiabatically against a constant pressure of 1 atm until it doubles in volume. If the initial temperature and pressure is 298 K and 5 atm, respectively, then the final temperature is ______ K (nearest integer).

(C_V is the molar heat capacity at constant volume.)

Correct Answer: 274

Solution:

For an adiabatic process:

$$\Delta U = q + w, \quad q = 0 \implies \Delta U = w$$

Step 1: Using the first law of thermodynamics:

$$nC_V \Delta T = -P_{\text{ext}}(V_2 - V_1)$$

Since $V_2 = 2V_1$, substitute and simplify:

$$nR\frac{T_2}{P_2} = 2nR\frac{T_1}{P_1}.$$

Step 2: Relation between P_2 and T_2:

$$P_2 = \frac{5T_2}{2 \times 298}.$$

Step 3: Using C_V:

$$\frac{5}{2}nR(T_2 - T_1) = -nRT_1\left(\frac{P_2}{P_1} - 1\right).$$

Step 4: Substitute and solve:

$$T_2 = \frac{5T_2}{2 \times 298}.$$

From the equation:

$$T_2 \approx 274.16\,\text{K}.$$

Nearest integer:

$$T_2 \approx 274\,\text{K}.$$

Quick Tip

In adiabatic processes with constant external pressure: - Use $\Delta U = w$ and C_V for calculations. - Relate final and initial states using P, V, and T relations.
