JEE Main 2024 6 April Shift 2 Solutions

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1. Let ABC be an equilateral triangle. A new triangle is formed by joining the midpoints of all sides of the triangle ABC, and the same process is repeated infinitely many times. If P is the sum of the perimeters and Q is the sum of areas of all the triangles formed in this process, then:

(1) $P^2 = 36\sqrt{3}Q$

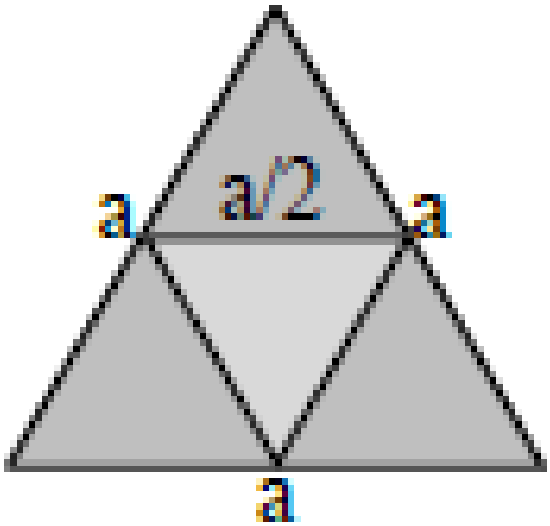
(2) $P^2 = 6\sqrt{3}Q$

(3) $P = 36\sqrt{3}Q^2$

(4) $P^2 = 72\sqrt{3}Q$

Correct Answer: (1).

Solution:



Area of first $\triangle = \frac{\sqrt{3}}{4}a^2$

Area of second $\triangle = \frac{\sqrt{3}}{4} \cdot \frac{a^2}{4} = \frac{\sqrt{3}a^2}{16}$

Area of third $\triangle = \frac{\sqrt{3}a^2}{64}$

Sum of areas $= \frac{\sqrt{3}a^2}{4}\left(1 + \frac{1}{4} + \frac{1}{16} + \ldots\right)$

The sum of this infinite geometric series is:

$Q = \frac{\sqrt{3}}{4} \cdot a^2 \cdot \frac{1}{1-\frac{1}{4}} = \frac{\sqrt{3}}{4} \cdot a^2 \cdot \frac{4}{3} = \frac{\sqrt{3}}{3}a^2$

Perimeter of 1^{st} $\triangle = 3a$

Perimeter of 2^{nd} $\triangle = 3 \cdot \frac{a}{2} = \frac{3a}{2}$

Perimeter of 3^{rd} $\triangle = 3 \cdot \frac{a}{4} = \frac{3a}{4}$

$P = 3a\left(1 + \frac{1}{2} + \frac{1}{4} + \ldots\right)$

The sum of this infinite geometric series is:

$P = 3a \cdot \frac{1}{1-\frac{1}{2}} = 3a \cdot 2 = 6a$

$$a = \frac{P}{6}$$

$$Q = \frac{1}{\sqrt{3}} \cdot \frac{P^2}{36}$$

$$P^2 = 36\sqrt{3}Q$$

Answer: (1) $P = 36\sqrt{3}Q$.

Quick Tip

When dealing with infinite geometric series, remember to use the sum formula $S = \frac{a}{1-r}$, where a is the first term and r is the common ratio.

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2. Let $A = \{1, 2, 3, 4, 5\}$. Let R be a relation on A defined by xRy if and only if $4x \leq 5y$. Let n be the number of elements in R and m be the minimum number of elements from $A \times A$ that are required to be added to R to make it a symmetric relation. Then $m + n$ is equal to:

(1) 24

(2) 23

(3) 25

(4) 26

Correct Answer: (3).

Solution:

Area of first $\triangle = \frac{\sqrt{3}a^2}{4}$

Area of second $\triangle = \frac{\sqrt{3}a^2}{4} \cdot \frac{a^2}{4} = \frac{\sqrt{3}a^2}{16}$

Area of third $\triangle = \frac{\sqrt{3}a^2}{64}$

Sum of area $= \frac{\sqrt{3}a^2}{4}\left(1 + \frac{1}{4} + \frac{1}{16} + \ldots\right)$

$Q = \frac{\sqrt{3}a^2}{4} \cdot \frac{1}{\frac{3}{4}} = \frac{a^2}{\sqrt{3}}$

Perimeter of 1st $\triangle = 3a$

Perimeter of 2nd $\triangle = \frac{3a}{2}$

Perimeter of 3rd $\triangle = \frac{3a}{4}$

$P = 3a\left(1 + \frac{1}{2} + \frac{1}{4} + \ldots\right)$

$P = 3a \cdot 2 = 6a$

$a = \frac{P}{6}$

$Q = \frac{1}{\sqrt{3}} \cdot \frac{P^2}{36}$

$P^2 = 36\sqrt{3}Q$

Quick Tip

To make a relation symmetric, ensure that if $(a, b) \in R$, then (b, a) must also be in R.

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3. If three letters can be posted to any one of the 5 different addresses, then the probability that the three letters are posted to exactly two addresses is:

(1) $\frac{12}{25}$

(2) $\frac{18}{25}$

(3) $\frac{4}{25}$

(4) $\frac{6}{25}$

Correct Answer: (1).

Solution:

Total methods $= 5^3$

Favorable $= {}^3C_2 \cdot (2^3 - 2) = 60$

Probability $= \frac{60}{125} = \frac{12}{25}$

Quick Tip

To calculate probability, divide the number of favorable outcomes by the total number of possible outcomes.

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4. Suppose the solution of the differential equation

$$\frac{dy}{dx} = \frac{(2+\alpha)x - \beta y + 2}{\beta x - 2\alpha y - (\beta y - 4\alpha)}$$

represents a circle passing through origin. Then the radius of this circle is:

(1) $\sqrt{17}$

(2) $\frac{1}{2}$

(3) $\frac{\sqrt{17}}{2}$

(4) 2

Correct Answer: (3).

Sol.

$$\frac{dy}{dx} = \frac{(2+\alpha)x - \beta y + 2}{\beta x - y(2\alpha + \beta) + 4\alpha}$$

$$\beta x dy - (2\alpha + \beta) y dy + 4\alpha dy = (2+\alpha)x dx - \beta y dx + 2dx$$

$$\beta(x dy + y dx) - (2\alpha + \beta) y dy + 4\alpha dy = (2+\alpha)x dx + 2dx$$

$$\beta xy - \frac{(2\alpha + \beta)y^2}{2} + 4\alpha y = \frac{(2+\alpha)x^2}{2}$$

$$\implies \beta = 0 \quad \text{for this to be circle}$$

$$(2+\alpha)\frac{x^2}{2}+\alpha y^2+2x-4\alpha y=0$$

$$\text{coeff. of } x^2=y^2 \implies 2+a=2a$$

$$\implies \alpha=2$$

$$\text{i.e. } 2x^2+2y^2+2x-8y=0$$

$$x^2+y^2+x-4y=0$$

$$\text{rd}=\sqrt{\frac{1}{4}+4}=\frac{\sqrt{17}}{2}$$

Quick Tip

For a differential equation to represent a circle passing through the origin, the coefficients of terms should form a symmetrical relation.

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5. If the locus of the point, whose distances from the point (2, 1) and (1, 3) are in the ratio 5 : 4, is

$$ax^2+by^2+cxy+dx+ey+170=0,$$

then the value of $a^2+2b+3c+4d+e$ **is equal to:**

(1) 5

(2) 27

(3) 37

(4) 437

Correct Answer: (3).

Solution:

Let $P(x, y)$

$$\frac{(x-2)^2+(y-1)^2}{(x-1)^2+(y-3)^2} = \frac{25}{16}$$

Expanding and simplifying:

$$9x^2 + 9y^2 + 14x - 118y + 170 = 0$$

From the equation:

$$a^2 + 2b + 3c + 4d + e = 81 + 18 + 0 + 56 - 118$$

Calculating:

$$= 155 - 118$$

$$= 37$$

Quick Tip

When working with loci involving distances in specific ratios, use the equation of the locus and simplify terms step-by-step to find the required expression.

6.

$$\lim_{n\to\infty} \frac{(1^2-1)(n-1)+(2^2-2)(n-2)+\cdots+((n-1)^2-(n-1))}{(1^3+2^3+\cdots+n^3)-(1^2+2^2+\cdots+n^2)}$$

is equal to:

(1) $\frac{2}{3}$

(2) $\frac{1}{3}$

(3) $\frac{3}{4}$

(4) $\frac{1}{2}$

Correct Answer: (2).

Solution:

$$\lim_{n\to\infty} \sum_{r=1}^{n-1} (r^2 - r)(n - r)$$

$$= \lim_{n\to\infty} \left(\sum_{r=1}^{n} r^3 - \sum_{r=1}^{n} r^2 \right)$$

$$= \lim_{n\to\infty} \sum_{r=1}^{n-1} \left(-r^3 + r^2(n+1) - nr \right)$$

$$\lim_{n\to\infty} \left(\frac{n(n+1)}{2} \right)^2 - \frac{n(n+1)(2n+1)}{6} - \frac{n^2(n-1)}{2}$$

Simplify further:

$$\lim_{n\to\infty} \frac{(n-1)n}{2} + \frac{(n+1)(n-1)n(2n-1) - n^2(n-1)}{6}$$

$$\lim_{n\to\infty} \left[\frac{n(n+1)}{2} + \frac{n(n+1)}{2} + \frac{2n+1}{3} \right]$$

$$\lim_{n\to\infty} \left[\frac{n(n-1)}{2} \left(-n(n-1) + (n+1)(2n-1) \right) \right]$$

$$= \lim_{n\to\infty} \frac{n(n+1)(3n^2+3n-4n-2)}{6}$$

$$= \lim_{n\to\infty} \frac{(n-1)(-3n^2+3+2(2n^2+n-1)-6)}{(n+1)(3n^2-n-2)}$$

$$= \lim_{n\to\infty} \frac{(n-1)(n^2+5n-8)}{(n+1)(3n^2-n-2)} = \frac{1}{3}$$

Quick Tip

For limit calculations involving summation, simplifying each term individually helps in finding an asymptotic behavior for large n.

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7. Let $0 \le r \le n$. If ${}^{n+1}C_{r+1} : {}^{n}C_{r} : {}^{n-1}C_{r-1} = 55 : 35 : 21$, then $2n + 5r$ is equal to:

(1) 60

(2) 62

(3) 50

(4) 55

Correct Answer: (3).

Ans.

$$\frac{{}^{n+1}C_r}{{}^{n}C_r} = \frac{55}{35}$$

$$\frac{\frac{(n+1)!}{(r+1)!(n-r)!}}{\frac{n!}{r!(n-r)!}} = \frac{11}{7}$$

$$\frac{n+1}{r+1} = \frac{11}{7}$$

$$7n = 4 + 11r$$

$$\frac{{}^{n}C_r}{{}^{n-1}C_{r-1}} = \frac{35}{21}$$

$$\frac{\frac{n!}{r!(n-r)!}}{\frac{(n-1)!}{(r-1)!(n-r)!}} = \frac{5}{3}$$

$$\frac{n}{r} = \frac{5}{3}$$

$$3n = 5r$$

Solving for $r = 6$ and $n = 10$:

$$2n + 5r = 50$$

Quick Tip

To solve problems involving ratios of combinations, simplify each combination using factorial notation and set up equations accordingly.

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8. A software company sets up m number of computer systems to finish an assignment in 17 days. If 4 computer systems crashed on the start of the second day, 4 more computer systems crashed on the start of the third day, and so on, then it took 8 more days to finish the assignment. The value of m is equal to:

(1) 125

(2) 150

(3) 180

(4) 160

Correct Answer: (2).

Solution:

$$17m = m + (m - 4) + (m - 4 \times 2) + \ldots + (m - 4 \times 24)$$

$$17m = 25m - 4\,(1 + 2 + \ldots + 24)$$

$$8m = 4 \cdot \frac{24 \cdot 25}{2} = 150$$

Quick Tip

For sequences involving decremental changes, use the formula for the sum of an arithmetic series to simplify calculations.

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9. If z_1, z_2 are two distinct complex numbers such that

$$\left| \frac{z_1 - 2z_2}{\frac{1}{2} - 2z_1\overline{z}_2} \right| = 2,$$

then

(1) either z_1 lies on a circle of radius 1 or z_2 lies on a circle of radius $\frac{1}{2}$

(2) either z_1 lies on a circle of radius $\frac{1}{2}$ or z_2 lies on a circle of radius 1

(3) z_1 lies on a circle of radius $\frac{1}{2}$ and z_2 lies on a circle of radius 1

(4) both z_1 and z_2 lie on the same circle

Correct Answer: (1).

Solution:

$$\frac{z_1 - 2z_2}{\frac{1}{2} - 2z_1\overline{z}_2} \times \frac{\overline{z_1} - 2\overline{z_2}}{\frac{1}{2} - \overline{z_1}z_2} = 4$$

$$|z_1|^2 \left|2z_1\overline{z_2} - 2z_2\overline{z_1} + 4\,|z_2|^2\right|^2$$

$$= 4\left(\frac{1}{4}\left(z_1\overline{z_2} - z_2\overline{z_1}\right)^2 + |z_1|^2\,|z_2|^2\right)$$

$$z_1\overline{z_1} + 2z_2 \cdot 2z_2 - z_1z_2 \cdot 2z_2\overline{z_2} - 1 = 0$$

$$(z_1\overline{z_1} - 1)(1 - 2z_2\overline{z_2}) = 0$$

$$(|z_1|^2 - 1)((2z_2|z_2|^2 - 1) = 0$$

Quick Tip

In problems involving complex numbers and modulus, it can be helpful to multiply by the conjugate to simplify the expression.

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10. If the function $f(x) = \left(\frac{1}{x}\right)^{2x}$; $x > 0$ attains the maximum value at $x = \frac{1}{e}$, then:

(1) $e^{\pi} < \pi^e$

(2) $e^{2\pi} < (2\pi)^e$

(3) $e^{\pi} > \pi^e$

(4) $(2e)^{\pi} > \pi^{2e}$

Correct Answer: (3).

Solution:

Let

$$y = \left(\frac{1}{x}\right)^{2x}$$

Taking natural logarithm on both sides:

$$\ln y = 2x \ln\left(\frac{1}{x}\right)$$

$$\ln y = -2x \ln x$$

Differentiating with respect to x:

$$\frac{1}{y}\frac{dy}{dx} = -2(1+\ln x)$$

$$\frac{dy}{dx} = y \cdot (-2)(1+\ln x)$$

For $x > \frac{1}{e}$, f^n is decreasing.

Thus:

$$e < \pi$$

$$\left(\frac{1}{e}\right)^{2e} > \left(\frac{1}{\pi}\right)^{2\pi}$$

$$e^{\pi} > \pi^{e}$$

Quick Tip

When analyzing the maximum or minimum value of functions, consider taking logarithms to simplify exponentials and apply differentiation to find critical points.

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11. Let $\vec{a} = 6\hat{i}+\hat{j}-\hat{k}$ and $\vec{b} = \hat{i}+\hat{j}$. If $\vec{c}$ is a vector such that $|\vec{c}| \geq 6$, $\vec{a}\cdot\vec{c} = 6|\vec{c}|$, $|\vec{c}-\vec{a}| = 2\sqrt{2}$, and the angle between $\vec{a}\times\vec{b}$ and $\vec{c}$ is 60°, then $|(\vec{a}\times\vec{b})\times\vec{c}|$ is equal to:

(1) $\frac{9}{2}(6-\sqrt{6})$

(2) $\frac{3}{2}\sqrt{3}$

(3) $\frac{3}{2}\sqrt{6}$

(4) $\frac{9}{2}(6+\sqrt{6})$

Correct Answer: (4).

Solution:

$$|(\vec{a}\times\vec{b})\cdot\vec{c}| = |\vec{a}\times\vec{b}||\vec{c}|\cdot\frac{\sqrt{3}}{2}$$

Given:

$$|\vec{c}-\vec{a}| = 2\sqrt{2}$$

Using the formula for magnitude,

$$|\vec{c}|^2 + |\vec{a}|^2 - 2 \cdot \vec{a} \cdot \vec{c} = 8$$

$$|\vec{c}|^2 + 38 - 12|\vec{c}| = 8$$

$$|\vec{c}|^2 - 12|\vec{c}| + 30 = 0$$

Solving this quadratic equation:

$$|\vec{c}| = \frac{12 \pm \sqrt{144 - 120}}{2}$$

$$|\vec{c}| = \frac{12 \pm 2\sqrt{6}}{2}$$

$$|\vec{c}| = 6 + \sqrt{6}$$

Now, calculating $\vec{a} \times \vec{b}$:

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 6 & 1 & -1 \\ 1 & 1 & 0 \end{vmatrix}$$

$$= -\hat{i} + 7\hat{j} + 5\hat{k}$$

$$|\vec{a} \times \vec{b}| = \sqrt{27}$$

Thus,

$$|(\vec{a} \times \vec{b}) \times \vec{c}| = \sqrt{27}(6 + \sqrt{6}) \cdot \frac{\sqrt{3}}{2}$$

$$= \frac{9}{2}(6 + \sqrt{6})$$

Quick Tip

For problems involving cross products and magnitudes, use determinants for vector cross products and simplify with given conditions.

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12. If all the words with or without meaning made using all the letters of the word "NAGPUR" are arranged as in a dictionary, then the word at the 315th position in this arrangement is:

(1) NRAGUP

(2) NRAGPU

(3) NRAPGU

(4) NRAPUG

Correct Answer: (3).

Solution:

Arranging the letters in alphabetical order: NAGPUR

Starting with A: $5! = 120$ positions

Starting with G: $5! = 120$ positions, cumulative: 240

Starting with N and A: $4! = 24$ positions, cumulative: 264

Starting with N and G: $4! = 24$ positions, cumulative: 288

Starting with N and P: $4! = 24$ positions, cumulative: 312

Now, starting with N, R, and A:

NRAGUP $= 1$, cumulative: 313

NRAGPU $= 1$, cumulative: 314

NRAPGU $= 1$, cumulative: 315

Thus, the word at the 315^{th} position is **NRAPGU**.

Quick Tip

When finding the position of a word in a dictionary order arrangement, count the permutations of each starting letter sequentially.

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13. Suppose for a differentiable function h, $h(0) = 0$, $h(1) = 1$, and $h'(0) = h'(1) = 2$. If $g(x) = h(e^x)e^{h(x)}$, then $g'(0)$ is equal to:

(1) 5

(2) 3

(3) 8

(4) 4

Correct Answer: (4).

Solution:

Differentiating with respect to x:

$$g(x) = h(e^x) \cdot e^{h(x)}$$

$$g'(x) = h'(e^x) \cdot e^{h(x)} \cdot h'(x) + e^{h(x)} h'(e^x) \cdot e^x$$

$$g'(0) = h(1)e^{h(0)}h'(0) + e^{h(0)}h'(1)$$

$$= 2 + 2 = 4$$

Quick Tip

When differentiating products of functions, apply the product rule carefully and substitute known values after differentiating.

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14. Let $P(\alpha, \beta, \gamma)$ be the image of the point $Q(3, -3, 1)$ in the line

$$\frac{x-0}{1} = \frac{y-3}{1} = \frac{z-1}{-1}$$

and let R be the point $(2, 5, -1)$. If the area of the triangle PQR is λ and $\lambda^2 = 14K$, then K is equal to:

(1) 36

(2) 72

(3) 18

(4) 81

Correct Answer: (4).

Solution:

The coordinates of Q are $(3, -3, 1)$ and R is at $(2, 5, -1)$.

Step 1: Calculating RQ:

$$RQ = \sqrt{(2-3)^2 + (5+3)^2 + (-1-1)^2} = \sqrt{1 + 64 + 4} = \sqrt{69}$$

Step 2: Representing $\overrightarrow{RQ}$:

$$\overrightarrow{RQ} = -\hat{i} + 8\hat{j} - 2\hat{k}$$

Step 3: Representing $\overrightarrow{RS}$:

$$\overrightarrow{RS} = \hat{i} + \hat{j} - \hat{k}$$

Step 4: Finding cosine of the angle θ between vectors $\overrightarrow{RQ}$ and $\overrightarrow{RS}$:

$$\cos\theta = \frac{\overrightarrow{RQ} \cdot \overrightarrow{RS}}{|\overrightarrow{RQ}||\overrightarrow{RS}|}$$

$$= \frac{(-1 \cdot 1) + (8 \cdot 1) + (-2 \cdot -1)}{\sqrt{69} \cdot \sqrt{3}} = \frac{-1 + 8 + 2}{\sqrt{69} \cdot \sqrt{3}} = \frac{9}{3\sqrt{23}}$$

Step 5: Using sine of the angle:

$$\sin\theta = \sqrt{1 - \cos^2\theta} = \frac{\sqrt{23}}{\sqrt{69}}$$

Step 6: Finding area of triangle PQR:

$$\text{Area} = \frac{1}{2} \times |\overrightarrow{RQ}| \times |\overrightarrow{RS}| \times \sin\theta = \frac{1}{2} \times \sqrt{69} \times \sqrt{3} \times \frac{\sqrt{23}}{\sqrt{69}} = \frac{\sqrt{3} \times \sqrt{23}}{2}$$

Step 7: Given $\lambda^2 = 14K$:

$$\lambda^2 = 81.14 = 14K \implies K = 81$$

Quick Tip

When working with vector geometry, remember to use vector dot products to find cosines of angles and apply cross products to find areas of triangles.

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15. If $P(6, 1)$ be the orthocentre of the triangle whose vertices are $A(5, -2)$, $B(8, 3)$, and $C(h, k)$, then the point C lies on the circle:

(1) $x^2 + y^2 - 65 = 0$

(2) $x^2 + y^2 - 74 = 0$

(3) $x^2 + y^2 - 61 = 0$

(4) $x^2 + y^2 - 52 = 0$

Correct Answer: (1).

Solution:

To find the coordinates of C, we proceed with the slopes of sides and the equations of lines.

1. Slope of AD:

$$\text{Slope of } AD = 3$$

2. Slope of BC:

$$\text{Slope of } BC = -\frac{1}{3}$$

Equation of BC:

$$3y + x - 17 = 0$$

3. Slope of BE:

$$\text{Slope of } BE = 1$$

4. Slope of AC:

$$\text{Slope of } AC = -1$$

Equation of AC:

$$x + y - 3 = 0$$

Solving these equations, we find:

$$\text{Point } C \text{ is } (-4, 7)$$

Since C lies on the circle, we have:

$$x^2 + y^2 - 65 = 0$$

Quick Tip

To find the orthocentre and circumcircle relations, determine slopes of altitudes and solve line equations based on perpendicularity.

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16. Let $f(x) = \frac{1}{7-\sin 5x}$ be a function defined on $\mathbb{R}$. Then the range of the function $f(x)$ is equal to:

(1) $\left[\frac{1}{8}, \frac{1}{5}\right]$

(2) $\left[\frac{1}{7}, \frac{1}{6}\right]$

(3) $\left[\frac{1}{7}, \frac{1}{5}\right]$

(4) $\left[\frac{1}{8}, \frac{1}{6}\right]$

Correct Answer: (4).

Solution:

1. Since $\sin 5x \in [-1, 1]$, we have:

$$-\sin 5x \in [-1, 1]$$

Therefore:

$$7 - \sin 5x \in [6, 8]$$

2. Thus, the function $f(x) = \frac{1}{7-\sin 5x}$ takes values in the interval:

$$\frac{1}{7 - \sin 5x} \in \left[\frac{1}{8}, \frac{1}{6}\right]$$

Therefore, the range of $f(x)$ is $\left[\frac{1}{8}, \frac{1}{6}\right]$.

Quick Tip

To determine the range of a function involving trigonometric expressions, find the range of the trigonometric component first, then apply transformations accordingly.

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17. Let $\vec{a} = 2\hat{i} + \hat{j} - \hat{k}$, $\vec{b} = ((\vec{a} \times (\hat{i} + \hat{j})) \times \hat{i}) \times \hat{i}$. Then the square of the projection of $\vec{a}$ on $\vec{b}$ is:

(1) $\frac{1}{5}$

(2) 2

(3) $\frac{1}{3}$

(4) $\frac{2}{3}$

Correct Answer: (2).

Solution:

Step 1: Calculate $\vec{a} \times (\hat{i} + \hat{j})$:

$$\vec{a} \times (\hat{i} + \hat{j}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -1 \\ 1 & 1 & 0 \end{vmatrix} = -\hat{i} + \hat{k}$$

Step 2: Calculate $(\vec{a} \times (\hat{i} + \hat{j})) \times \hat{i}$:

$$(\vec{a} \times (\hat{i} + \hat{j})) \times \hat{i} = (-\hat{i} + \hat{k}) \times \hat{i} = \hat{k} + \hat{j}$$

Step 3: Calculate $((\vec{a} \times (\hat{i} + \hat{j})) \times \hat{i}) \times \hat{i}$:

$$((\vec{a} \times (\hat{i} + \hat{j})) \times \hat{i}) \times \hat{i} = (\hat{k} + \hat{j}) \times \hat{i} = \hat{j} - \hat{k}$$

Thus, $\vec{b} = \hat{j} - \hat{k}$.

Step 4: Find the projection of $\vec{a}$ on $\vec{b}$:

$$\text{Projection of } \vec{a} \text{ on } \vec{b} = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|}$$

Calculating $\vec{a} \cdot \vec{b}$ and $|\vec{b}|$:

$$\vec{a} \cdot \vec{b} = (2)(0) + (1)(1) + (-1)(-1) = 1 + 1 = 2$$

$$|\vec{b}| = \sqrt{1^2 + (-1)^2} = \sqrt{2}$$

$$\text{Projection of } \vec{a} \text{ on } \vec{b} = \frac{2}{\sqrt{2}} = \sqrt{2}$$

Therefore, the square of the projection is:

$$(\sqrt{2})^2 = 2$$

Quick Tip

For finding projections, calculate the dot product and divide by the magnitude of the vector being projected onto.

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18. If the area of the region

$$\left\{(x, y) : \frac{a}{x^2} \leq y \leq \frac{1}{x},\ 1 \leq x \leq 2,\ 0 < a < 1\right\}$$

is

$$(\log_2 2) - \frac{1}{7}$$

then the value of $7a - 3$ **is equal to:**

(1) 2

(2) 0

(3) -1

(4) 1

Correct Answer: (3).

Solution:

The area of the region is given by:

$$\text{Area} = \int_1^2 \left(\frac{1}{x} - \frac{a}{x^2}\right) dx$$

Evaluating this integral:

$$= \left[\ln x + \frac{a}{x}\right]_1^2$$

$$= \ln 2 + \frac{a}{2} - a = \log_2 2 - \frac{1}{7}$$

Equating and solving for a:

$$-\frac{a}{2} = -\frac{1}{7}$$

$$a = \frac{2}{7}$$

Now, calculating $7a - 3$:

$$7a = 2$$

$$7a - 3 = -1$$

Quick Tip

For calculating areas with given inequalities, set up the definite integral with the given limits and solve step-by-step.

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19. If

$$\textbf{If } \int \frac{1}{a^2 \sin^2 x + b^2 \cos^2 x}\, dx = \frac{1}{12} \tan^{-1}(3 \tan x) + \textbf{ constant,}$$

then the maximum value of $a \sin x + b \cos x$ **is:**

(1) $\sqrt{40}$

(2) $\sqrt{39}$

(3) $\sqrt{42}$

(4) $\sqrt{41}$

Correct Answer: (1).

$$\text{Sol.} \quad \int \frac{\sec^2 x\, dx}{a^2 \tan^2 x + b^2}$$

Let $\tan x = t$, then $\sec^2 x\, dx = dt$.

$$= \int \frac{dt}{a^2 t^2 + b^2}$$

$$= \frac{1}{a^2} \int \frac{dt}{t^2 + \left(\frac{b}{a}\right)^2}$$

$$= \frac{1}{ab} \tan^{-1}\left(\frac{ta}{b}\right) + c$$

$$= \frac{1}{ab} \tan^{-1}\left(\frac{a}{b} \tan x\right) + c$$

On comparing, $\frac{a}{b} = 3$

$$ab = 12$$

$$a = 6,\ b = 2$$

Maximum value of $6 \sin x + 2 \cos x$ is $\sqrt{40}$.

Quick Tip

For expressions of the form $a \sin x + b \cos x$, the maximum value can be found using $\sqrt{a^2 + b^2}$.

---

20. If A is a square matrix of order 3 such that

$\det(A) = 3 and$

$\det(\textbf{adj}(-4\,\textbf{adj}(-3\,\textbf{adj}(3\,\textbf{adj}((2A)^{-1}))))) = 2^m 3^n,$

then **m +** $|2n|$ **is equal to:**

(1) 3

(2) 2

(3) 4

(4) 6

Correct Answer: (3).

Solution:

Given $|A| = 3$, we start with:

$$\left|\text{adj}\left(-4\,\text{adj} - 3\,\text{adj}\left(3\,\text{adj}\left(2A^{-1}\right)\right)\right)\right|$$

Step 1: Simplify the innermost expression:

$$= \left|-4\,\text{adj}\left(-3\,\text{adj}\left(3\,\text{adj}\left(2A^{-1}\right)\right)\right)\right|^2$$

Step 2: Expand the outer term:

$$= 4^5\left|\text{adj}\left(-3\,\text{adj}\left(3\,\text{adj}\left(2A^{-1}\right)\right)\right)\right|^2$$

Step 3: Replace the outermost adj with its expression:

$$= 2^{12}\cdot 3^{12}\cdot\left|3\,\text{adj}\left(2A^{-1}\right)\right|^8$$

Step 4: Simplify the term inside the absolute value:

$$= 2^{12}\cdot 3^{12}\cdot 3^8\cdot\left|\text{adj}\left(2A^{-1}\right)\right|^8$$

Step 5: Use the property of adjugates:

$$= 2^{12}\cdot 3^{20}\cdot\left|2A^{-1}\right|^{16}$$

Step 6: Replace $\left|2A^{-1}\right|^{16}$ with its determinant form:

$$= 2^{12}\cdot 3^{20}\cdot\frac{1}{|2A|^{16}}$$

Step 7: Substitute $|2A|^{16} = 2^{16}\cdot|A|^{16}$:

$$= 2^{12}\cdot 3^{20}\cdot\frac{1}{2^{48}\cdot|A|^{16}}$$

Step 8: Replace $|A| = 3$:

$$= 2^{12}\cdot 3^{20}\cdot\frac{1}{2^{48}\cdot 3^{16}}$$

Step 9: Simplify powers of 2 and 3:

$$= \frac{2^{12}}{2^{48}} \cdot \frac{3^{20}}{3^{16}}$$

Step 10: Further simplify:

$$= \frac{1}{2^{36}} \cdot 3^4$$

Step 11: Combine the terms:

$$= 2^{-36} \cdot 3^4$$

Step 12: Write the values of m and n:

$$m = -36, \quad n = 20$$

Step 13: Final result:

$$m + 2n = 4$$

Quick Tip

When working with determinants involving adjugates, remember that $\det(\text{adj}(A)) = (\det(A))^{n-1}$ for an $n \times n$ matrix.

---

21. Let $\lfloor t \rfloor$ denote the greatest integer less than or equal to t. Let $f : [0, \infty) \to \mathbb{R}$ be a function defined by

$$f(x) = \left\lfloor \frac{x}{2} + 3 \right\rfloor - \lfloor \sqrt{x} \rfloor .$$

Let S be the set of all points in the interval $[0, 8]$ at which f is not continuous. Then $\sum_{a \in S} a$ is equal to _____.

Correct Answer: (17).

Solution:

$\left\lfloor \frac{x}{2} + 3 \right\rfloor$ is discontinuous at $x = 2, 4, 6, 8$

$\sqrt{x}$ is discontinuous at $x = 1, 4$

$F(x)$ is discontinuous at $x = 1, 2, 6, 8$

Summing the values:

$$\sum a = 1 + 2 + 6 + 8 = 17$$

Quick Tip

For functions involving floor and ceiling functions, identify points of discontinuity by locating jumps in each component separately.

---

22. The length of the latus rectum and directrices of a hyperbola with eccentricity e are 9 and

$$x = \pm\frac{4}{\sqrt{3}},$$

respectively. Let the line

$$y - \sqrt{3}x + \sqrt{3} = 0$$

touch this hyperbola at (x_0, y_0). If m is the product of the focal distances of the point (x_0, y_0), then $4e^2 + m$ is equal to _____.

NTA Ans. (61)

Correct Answer: (Bonus)

Solution:

Given

$$\frac{2b^2}{a} = 9 \quad \text{and} \quad \frac{a}{e} = \pm\frac{4}{\sqrt{3}}$$

The equation of the tangent is:

$$y - \sqrt{3}x + \sqrt{3} = 0$$

By the equation of the tangent:

$$\text{Let slope } S = \sqrt{3}$$

$$\text{Constant } = -\sqrt{3}$$

By the condition of tangency:

$$6 = 6a^2 - 9a$$

Simplifying:

$$\Rightarrow a = 2,\ b^2 = 9$$

The equation of the hyperbola is:

$$\frac{x^2}{4} - \frac{y^2}{9} = 1$$

And for the tangent:

$$\text{Point of contact is } (4, 3\sqrt{3}) = (x_0, y_0)$$

Now,

$$e = \sqrt{1 + \frac{9}{4}} = \frac{\sqrt{13}}{2}$$

Again, product of focal distances:

$$m = (x_0 + a)(x_0 - a)$$

$$m + 4e^2 = 20e^2 - a^2$$

Substituting values:

$$= 20 \times \frac{13}{4} - 4 = 61$$

Note: (There is a printing mistake in the equation of the directrix $x = \pm\frac{4}{\sqrt{3}}$).

Corrected equation:

$$x = \pm\frac{4}{\sqrt{13}} \quad \text{for directrix, as eccentricity must be greater than 1.}$$

Conclusion: Therefore, this question must be considered as a bonus.

Note: There is a printing mistake in the equation of the directrix as $x = \pm\frac{4}{\sqrt{3}}$.

Corrected equation: The correct equation for the directrix should be $x = \pm\frac{4}{\sqrt{13}}$, as eccentricity must be greater than one, so this question might be a bonus.

Quick Tip

For hyperbolas, the eccentricity e and directrix play a critical role in defining tangency conditions. Check for consistency in given values, especially with directrices and eccentricity.

---

23. If $S(x) = (1+x)+2(1+x)^2+3(1+x)^3+\ldots+60(1+x)^{60}$**,** $x \neq 0$**, and** $(60)^2 S(60) = a(b)^b + b$**, where** $a, b \in \mathbb{N}$**, then** $(a+b)$ **is equal to** _____**.**

Correct Answer: (3660)

Solution:

Starting with the series:

$$S(x) = (1+x) + 2(1+x)^2 + 3(1+x)^3 + \ldots + 60(1+x)^{60}$$

Multiplying both sides by $(1+x)$, we get:

$$(1+x)S = (1+x) + 2(1+x)^2 + 3(1+x)^3 + \ldots + 60(1+x)^{61}$$

Now, subtracting S from $(1+x)S$, we obtain:

$$-xS = \frac{(1+x)(1+x)^{60} - 1}{x} - 60(1+x)^{61}$$

Now, put $x = 60$:

$$-60S = \frac{61((61)^{60} - 1)}{60} - 60 \cdot (61)^{61}$$

Solving this equation gives:

$$S = 3660$$

Quick Tip

When dealing with series involving powers, multiplying the series by a factor and then subtracting can simplify the expression by creating telescoping terms.

---

24. Let $\lfloor t \rfloor$ denote the largest integer less than or equal to t. If

$$\int_0^3 \left(\lfloor x^2 \rfloor + \left\lfloor \frac{x^2}{2} \right\rfloor \right) dx = a + b\sqrt{2} - \sqrt{3} - \sqrt{5} + c\sqrt{6} - \sqrt{7},$$

where $a, b, c \in \mathbb{Z}$, then $a + b + c$ is equal to _____.

Correct Answer: (23)

Solution:

Breaking the integral into intervals based on values of x:

$$\int_0^3 \left(\lfloor x^2 \rfloor + \left\lfloor \frac{x^2}{2} \right\rfloor \right) dx$$

$$= \int_0^1 0\,dx + \int_1^{\sqrt{2}} 1\,dx + \int_{\sqrt{2}}^{\sqrt{3}} 2\,dx + \int_{\sqrt{3}}^{\sqrt{5}} 3\,dx + \int_{\sqrt{5}}^{\sqrt{6}} 4\,dx + \int_{\sqrt{6}}^{\sqrt{7}} 5\,dx$$

$$+\int_{\sqrt{7}}^{\sqrt{8}} 6\,dx + \int_{\sqrt{8}}^{3} 7\,dx$$

After evaluating each integral, we get:

$$a = 31, \quad b = -6, \quad c = -2$$

Thus,

$$a + b + c = 31 - 6 - 2 = 23$$

Quick Tip

When integrating functions with floor values, break the integral into intervals where the floor function remains constant, then integrate over each interval.

---

25. From a lot of 12 items containing 3 defectives, a sample of 5 items is drawn at random. Let the random variable X denote the number of defective items in the sample. Let items in the sample be drawn one by one without replacement. If the variance of X is $\frac{m}{n}$, where $\gcd(m, n) = 1$, then $n - m$ is equal to _____.

Correct Answer: (71)

Sol. $$a = 1 - \frac{{}^{3}C_5}{{}^{12}C_5}$$

$$b = 3 \cdot \frac{{}^{9}C_4}{{}^{12}C_5}$$

$$c = 3 \cdot \frac{{}^{9}C_3}{{}^{12}C_5}$$

$$d = 1 \cdot \frac{{}^{9}C_2}{{}^{12}C_5}$$

$$u = 0 \cdot a + 1 \cdot b + 2 \cdot c + 3 \cdot d = 1.25$$

$$\sigma^2 = 0 \cdot a + 1 \cdot b + 4 \cdot c + 9 \cdot d - u^2$$

$$\sigma^2 = \frac{105}{176}$$

$$\text{Ans.} \quad 176 - 105 = 71$$

Quick Tip

In problems involving sampling without replacement, use combinatorial probabilities to determine the mean and variance of the random variable.

---

26. In a triangle ABC, $BC = 7$, $AC = 8$, $AB = \alpha \in \mathbb{N}$, and $\cos A = \frac{2}{3}$. If $49\cos(3C)+42 = \frac{m}{n}$, where $\gcd(m, n) = 1$, then $m + n$ is equal to _____.

Correct Answer: (39)

Solution:

Using the cosine rule for $\cos A$:

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

Substitute $b = 8$, $c = 7$, and $\cos A = \frac{2}{3}$:

$$\frac{2}{3} = \frac{8^2 + 7^2 - \alpha^2}{2 \times 8 \times 7}$$

$$\Rightarrow \alpha^2 = 9$$

$$\Rightarrow \alpha = 3$$

Now, calculate $\cos C$ using the cosine rule:

$$\cos C = \frac{7^2 + 8^2 - 9^2}{2 \times 7 \times 8} = \frac{2}{7}$$

Then, for $\cos(3C)$, we use the triple angle formula:

$$49\cos(3C) + 42 = 49(4\cos^3 C - 3\cos C) + 42$$

Substituting $\cos C = \frac{2}{7}$:

$$= 49\left(4\left(\frac{2}{7}\right)^3 - 3 \cdot \frac{2}{7}\right) + 42$$

$$= \frac{32}{7}$$

Thus, $m = 32$ and $n = 7$, so

$$m + n = 32 + 7 = 39$$

Quick Tip

For problems involving multiple angle identities in trigonometry, remember to apply triple angle formulas carefully, especially when finding values of $\cos(3\theta)$.

---

27. If the shortest distance between the lines

$$\frac{x-\lambda}{3} = \frac{y-2}{-1} = \frac{z-1}{1}$$

and

$$\frac{x+2}{-3} = \frac{y+5}{2} = \frac{z-4}{4}$$

is

$$\frac{44}{\sqrt{30}},$$

then the largest possible value of $|\lambda|$ is equal to _____.

Correct Answer: (43)

Solution:

Let

$$\vec{a_1} = \lambda\hat{i} + 2\hat{j} + \hat{k}$$

$$\vec{a_2} = -2\hat{i} - 5\hat{j} + 4\hat{k}$$

$$\vec{p} = 3\hat{i} - \hat{j} + \hat{k}$$

$$\vec{q} = 3\hat{i} + 2\hat{j} + 4\hat{k}$$

Now, the vector $\vec{a_1} - \vec{a_2}$ is given by:

$$(\lambda + 2)\hat{i} + 7\hat{j} - 3\hat{k}$$

Calculate $\vec{p} \times \vec{q}$:

$$\vec{p} \times \vec{q} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & 1 \\ 3 & 2 & 4 \end{vmatrix} = -6\hat{i} - 15\hat{j} + 3\hat{k}$$

The shortest distance d is given by:

$$d = \frac{|(\vec{a_1} - \vec{a_2}) \cdot (\vec{p} \times \vec{q})|}{|\vec{p} \times \vec{q}|}$$

Substitute the values:

$$\frac{44}{\sqrt{30}} = \frac{|-6\lambda - 12 - 105 + 9|}{\sqrt{(-6)^2 + (-15)^2 + 3^2}}$$
$$= \frac{|6\lambda + 126|}{3\sqrt{30}}$$

Equating, we get:

$$132 = |6\lambda + 126|$$

Solving for λ:

$$\lambda = 1 \quad \text{or} \quad \lambda = -43$$

Thus, the largest possible value of $|\lambda|$ is:

$$|\lambda| = 43$$

Quick Tip

For finding the shortest distance between skew lines, use the formula $d = \frac{|(\vec{a_1} - \vec{a_2}) \cdot (\vec{p} \times \vec{q})|}{|\vec{p} \times \vec{q}|}$ and substitute values carefully to avoid sign errors.

---

28. Let α, β be roots of $x^2 + \sqrt{2}x - 8 = 0$. If $U_n = \alpha^n + \beta^n$, then

$$\frac{U_{10} + \sqrt{12}U_9}{2U_8}$$

is equal to _____.

Correct Answer: (4)

Solution:

We have:

$$U_{10} = \alpha^{10} + \beta^{10} \quad \text{and} \quad U_9 = \alpha^9 + \beta^9$$

Then:

$$\frac{U_{10} + \sqrt{12}U_9}{2U_8} = \frac{\alpha^{10} + \beta^{10} + \sqrt{2}\left(\alpha^9 + \beta^9\right)}{2\left(\alpha^8 + \beta^8\right)}$$

Further simplifying, we get:

$$= \frac{\alpha^8 \left(\alpha^2 + \sqrt{2}\alpha\right) + \beta^8 \left(\beta^2 + \sqrt{2}\beta\right)}{2\left(\alpha^8 + \beta^8\right)}$$

Grouping terms, we have:

$$= \frac{8\alpha^8 + 8\beta^8}{2\left(\alpha^8 + \beta^8\right)} = 4$$

Quick Tip

For expressions involving powers of roots, use recurrence relations or identities based on the properties of roots to simplify calculations.

---

29. If the system of equations

$$2x + 7y + \lambda z = 3$$

$$3x + 2y + 5z = 4$$

$$x + \mu y + 32z = -1$$

has infinitely many solutions, then $(\lambda - \mu)$ **is equal to** _____.

Correct Answer: (38)

Solution:

For the system to have infinitely many solutions, the determinant of the coefficient matrix must be zero. We set:

$$D = D_1 = D_2 = D_3 = 0$$

Calculating D_3:

$$D_3 = \begin{vmatrix} 2 & 7 & 3 \\ 3 & 2 & 4 \\ 1 & \mu & -1 \end{vmatrix} = 0$$

Expanding, we get:

$$2\begin{vmatrix} 2 & 4 \\ \mu & -1 \end{vmatrix} - 7\begin{vmatrix} 3 & 4 \\ 1 & -1 \end{vmatrix} + 3\begin{vmatrix} 3 & 2 \\ 1 & \mu \end{vmatrix} = 0$$

Solving for μ, we find:

$$\mu = -39$$

Now, calculating D with λ in place:

$$D = \begin{vmatrix} 2 & 7 & \lambda \\ 3 & 2 & 5 \\ 1 & -39 & 32 \end{vmatrix} = 0$$

Solving this determinant, we get:

$$\lambda = -1$$

Thus, $\lambda - \mu = -1 - (-39) = 38$.

Quick Tip

When solving systems of equations for infinite solutions, set the determinant of the coefficient matrix to zero and solve for parameters.

---

30. If the solution $y(x)$ of the given differential equation

$$(e^y + 1)\cos x\, dx + e^y \sin x\, dy = 0$$

passes through the point $\left(\frac{\pi}{2}, 0\right)$, then the value of $e^{y\left(\frac{\pi}{6}\right)}$ is equal to _____.

Correct Answer: (3)

Solution:

Starting with the differential equation:

$$(e^y + 1)\cos x\, dx + e^y \sin x\, dy = 0$$

Rewrite as:

$$\Rightarrow d\left((e^y + 1)\sin x\right) = 0$$

Integrating, we get:

$$(e^y + 1)\sin x = C$$

Since the solution passes through $\left(\frac{\pi}{2}, 0\right)$, substitute $x = \frac{\pi}{2}$ and $y = 0$:

$$e^0 + 1 = C \Rightarrow C = 2$$

Now, let $x = \frac{\pi}{6}$:

$$(e^y + 1)\sin\frac{\pi}{6} = 2$$

$$\Rightarrow \frac{e^y + 1}{2} = 2$$

$$e^y = 3$$

Thus, $e^{y\left(\frac{\pi}{6}\right)} = 3$.

Quick Tip

For differential equations involving separable functions, integrate each term carefully and apply initial conditions to find constants of integration.

---

31. The longest wavelength associated with Paschen series is: (Given $R_H = 1.097 \times 10^7$ SI unit)

(1) 1.094×10^{-6} m

(2) 2.973×10^{-6} m

(3) 3.646×10^{-6} m

(4) 1.876×10^{-6} m

Correct Answer: (4)

Solution:

For the longest wavelength in the Paschen series:

$$\frac{1}{\lambda} = R\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

For the longest wavelength, $n_1 = 3$ and $n_2 = 4$.

$$\frac{1}{\lambda} = R\left[\frac{1}{(3)^2} - \frac{1}{(4)^2}\right]$$

$$= R\left[\frac{1}{9} - \frac{1}{16}\right]$$

$$= R \cdot \frac{16 - 9}{144} = R \cdot \frac{7}{144}$$

Now, substitute $R = 1.097 \times 10^7$:

$$\frac{1}{\lambda} = \frac{7 \times 1.097 \times 10^7}{144}$$

$$\lambda = \frac{144}{7 \times 1.097 \times 10^7} = 1.876 \times 10^{-6}\,\text{m}$$

Quick Tip

In spectral series calculations, use the Rydberg formula carefully, substituting the correct values for n_1 and n_2 based on the series to find the longest or shortest wavelength.

---

32. A total of 48 J heat is given to one mole of helium kept in a cylinder. The temperature of helium increases by 2°C. The work done by the gas is: (Given, $R = 8.3$ J K^{-1}mol^{-1})

(1) 72.9 J

(2) 24.9 J

(3) 48 J

(4) 23.1 J

Correct Answer: (4)

Solution:

Using the first law of thermodynamics:

$$\Delta Q = \Delta U + W$$

Given:

$$+48 = nC_V \Delta T + W$$

For helium (a monoatomic gas), $C_V = \frac{3R}{2}$:

$$48 = (1)\left(\frac{3R}{2}\right)(2) + W$$

Simplifying:

$$W = 48 - 3 \times R$$

Substitute $R = 8.3$:

$$W = 48 - 3 \times (8.3)$$

$$W = 23.1 \,\text{Joule}$$

Quick Tip

For thermodynamic calculations involving monoatomic gases, use $C_V = \frac{3R}{2}$ and apply the first law of thermodynamics to find the work done.

33. In finding out the refractive index of a glass slab, the following observations were made through a travelling microscope: 50 vernier scale division = 49 MSD; 20 divisions on the main scale in each cm.

For mark on paper:

MSR = 8.45 cm, VC = 26

For mark on paper seen through slab:

MSR = 7.12 cm, VC = 41

For powder particle on the top surface of the glass slab:

MSR = 4.05 cm, VC = 1

(MSR = Main Scale Reading, VC = Vernier Coincidence)

The refractive index of the glass slab is:

(1) 1.42

(2) 1.52

(3) 1.24

(4) 1.35

Correct Answer: (1)

Solution:

1. Calculating the Least Count (LC):

$$1\text{ MSD} = \frac{1\,\text{cm}}{20} = 0.05\,\text{cm}$$

$$1\text{ VSD} = \frac{49}{50}\text{ MSD} = \frac{49}{50} \times 0.05\,\text{cm} = 0.049\,\text{cm}$$

$$\text{LC} = 1\text{ MSD} - 1\text{ VSD} = 0.05\,\text{cm} - 0.049\,\text{cm} = 0.001\,\text{cm}$$

2. For mark on paper, L_1:

$$L_1 = 8.45\,\text{cm} + 26 \times 0.001\,\text{cm} = 8.45\,\text{cm} + 0.026\,\text{cm} = 8.476\,\text{cm}$$

3. For mark on paper seen through the slab, L_2:

$$L_2 = 7.12\,\text{cm} + 41 \times 0.001\,\text{cm} = 7.12\,\text{cm} + 0.041\,\text{cm} = 7.161\,\text{cm}$$

4. For powder particle on the top surface, ZE:

$$ZE = 4.05\,\text{cm} + 1 \times 0.001\,\text{cm} = 4.051\,\text{cm}$$

5. Calculating the thickness of the slab:

$$\text{actual } L_1 = 8.476 - 4.051 = 4.425\,\text{cm}$$

$$\text{actual } L_2 = 7.161 - 4.051 = 3.110\,\text{cm}$$

6. Refractive index μ:

$$\mu = \frac{L_1}{L_2} = \frac{4.425}{3.110} = 1.42$$

Quick Tip

When calculating refractive index using a travelling microscope, accurately determine the least count and adjust measurements accordingly for consistent results.

---

34. In the given electromagnetic wave

$$E_y = 600\sin(\omega t - kx)\,\textbf{Vm}^{-1},$$

the intensity of the associated light beam is (in W/m^2); (Given $\varepsilon_0 = 9 \times 10^{-12}$ C^{2}N^{-1}m^{-2})

(1) 486

(2) 243

(3) 729

(4) 972

Correct Answer: (1)

Solution:

The intensity I of an electromagnetic wave is given by:

$$I = \frac{1}{2}\varepsilon_0 E_0^2 c$$

where $E_0 = 600\,\text{Vm}^{-1}$ and $c = 3 \times 10^8$ m/s.

Substitute the values:

$$I = \frac{1}{2} \times 9 \times 10^{-12} \times (600)^2 \times 3 \times 10^8$$

$$= \frac{9}{2} \times 36 \times 3 = 486\,\text{W/m}^2$$

Quick Tip

The intensity of an electromagnetic wave can be calculated using $I = \frac{1}{2}\varepsilon_0 E_0^2 c$, where E_0 is the peak electric field.

---

35. Assuming the earth to be a sphere of uniform mass density, a body weighed 300 N on the surface of the earth. How much would it weigh at $R/4$ depth under the surface of the earth?

(1) 75 N

(2) 375 N

(3) 300 N

(4) 225 N

Correct Answer: (4)

Solution:

At the surface:

$$mg = 300\,\text{N}$$

$$m = \frac{300}{g_s}$$

At depth $\frac{R}{4}$:

$$g_d = g_s\left(1 - \frac{d}{R}\right)$$

where $d = \frac{R}{4}$.

$$g_d = g_s\left(1 - \frac{R}{4R}\right) = g_s \cdot \frac{3}{4}$$

The weight at depth $\frac{R}{4}$ is:

$$\text{Weight} = m \times g_d = m \times \frac{3g_s}{4}$$

$$= \frac{3}{4} \times 300 = 225\,\text{N}$$

Quick Tip

The gravitational acceleration at a depth d inside the earth is given by $g_d = g_s\left(1 - \frac{d}{R}\right)$, where g_s is the surface gravity.

---

36. The acceptor level of a p-type semiconductor is 6 eV. The maximum wavelength of light which can create a hole would be: (Given $hc = 1240$ eV nm)

(1) 407 nm

(2) 414 nm

(3) 207 nm

(4) 103.5 nm

Correct Answer: (3)

Solution:

The energy E of a photon is given by:

$$E = \frac{hc}{\lambda}$$

Substitute $E = 6$ eV and $hc = 1240$ eV nm:

$$6 = \frac{1240}{\lambda(\text{nm})}$$

Rearrange to find λ:

$$\lambda = \frac{1240}{6} = 207\,\text{nm}$$

Quick Tip

To find the maximum wavelength that can create a hole in a semiconductor, use $\lambda = \frac{hc}{E}$, where E is the energy of the acceptor level.

---

37. A car of 800 kg is taking a turn on a banked road of radius 300 m and angle of banking 30°. If the coefficient of static friction is 0.2, then the maximum speed with which the car can negotiate the turn safely is: ($g = 10$ m/s^2, $\sqrt{3} = 1.73$)

(1) 70.4 m/s

(2) 51.4 m/s

(3) 264 m/s

(4) 102.8 m/s

Correct Answer: (2)

Solution:

Given:

$$m = 800\,\text{kg}, \quad r = 300\,\text{m}, \quad \theta = 30^\circ, \quad \mu = 0.2$$

The maximum speed V_{max} for safe negotiation of the turn is given by:

$$V_{\text{max}} = \sqrt{rg\frac{\tan\theta + \mu}{1 - \mu\tan\theta}}$$

Substitute the values:

$$V_{\text{max}} = \sqrt{300 \times 10 \times \frac{\tan 30^\circ + 0.2}{1 - 0.2 \times \tan 30^\circ}}$$

$$= \sqrt{300 \times 10 \times \frac{0.57 + 0.2}{1 - 0.2 \times 0.57}}$$

$$= \sqrt{3000 \times \frac{0.77}{0.886}}$$

$$V_{\text{max}} = 51.4\,\text{m/s}$$

Quick Tip

For banked turns with friction, use the formula $V_{\text{max}} = \sqrt{rg\frac{\tan\theta+\mu}{1-\mu\tan\theta}}$ to find the maximum speed for safe turning.

---

38. Two identical conducting spheres P and S with charge Q on each, repel each other with a force of 16 N. A third identical uncharged conducting sphere R is successively brought in contact with the two spheres. The new force of repulsion between P and S is:

(1) 4 N

(2) 6 N

(3) 1 N

(4) 12 N

Correct Answer: (2)

Solution:

The initial force of repulsion between P and S is:

$$F_{PS} \propto Q^2$$

$$F_{PS} = 16\,\text{N}$$

1. When the uncharged sphere R is brought in contact with P, the charge on P and R redistributes equally because they are identical spheres. Thus, after contact with P:

$$\text{Charge on each of } P \text{ and } R = \frac{Q}{2}$$

2. Next, R is brought in contact with S, and the charge will again redistribute equally between R and S. After this contact:

$$\text{Charge on each of } S \text{ and } R = \frac{3Q}{4}$$

Now, P has a charge of $\frac{Q}{2}$ and S has a charge of $\frac{3Q}{4}$. The new force of repulsion between P and S is:

$$F_{PS} \propto \frac{Q}{2} \times \frac{3Q}{4} = \frac{3Q^2}{8}$$

Since the initial force F_{PS} was 16 N, we have:

$$F_{PS} = \frac{3}{8} \times 16 = 6\,\text{N}$$

Quick Tip

When identical conductors are brought into contact, charge redistributes equally among them. Use this principle to calculate the resulting forces.

---

39. In a coil, the current changes from -2 A to $+2$ A in 0.2 s and induces an emf of 0.1 V. The self-inductance of the coil is:

(1) 5 mH

(2) 1 mH

(3) 2.5 mH

(4) 4 mH

Correct Answer: (1)

Solution:

The induced emf in a coil is given by:

$$(\text{Emf})_{\text{induced}} = -L\frac{di}{dt}$$

In terms of magnitude:

$$|\text{Emf}_{\text{induced}}| = \left|L\frac{di}{dt}\right|$$

Given:

$$|\text{Emf}_{\text{induced}}| = 0.1\,\text{V}$$

$$\frac{di}{dt} = \frac{2-(-2)}{0.2} = \frac{4}{0.2} = 20\,\text{A/s}$$

Now, solve for L:

$$0.1 = L \times 20$$

$$L = \frac{0.1}{20} = 0.005\,\text{H} = 5\,\text{mH}$$

Quick Tip

To find self-inductance, use $L = \frac{\text{Emf}_{\text{induced}}}{\frac{di}{dt}}$ and ensure units are consistent when calculating.

---

40. For the thin convex lens, the radii of curvature are at 15 cm and 30 cm respectively. The focal length of the lens is 20 cm. The refractive index of the material is:

(1) 1.2

(2) 1.4

(3) 1.5

(4) 1.8

Correct Answer: (3)

Solution:

Using the lens maker's formula:

$$\frac{1}{f} = \left(\frac{\mu_{\text{lens}}}{\mu_{\text{air}}} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Given:

$$f = 20\,\text{cm}, \quad R_1 = 15\,\text{cm}, \quad R_2 = -30\,\text{cm}$$

Substitute into the formula:

$$\frac{1}{20} = (\mu - 1)\left(\frac{1}{15} - \frac{1}{-30}\right)$$

Simplify the expression:

$$\frac{1}{20} = (\mu - 1)\left(\frac{3}{30}\right)$$

$$\Rightarrow \mu - 1 = \frac{1}{2}$$

$$\Rightarrow \mu = 1 + \frac{1}{2} = \frac{3}{2} = 1.5$$

Quick Tip

For thin lenses, the lens maker's formula $\frac{1}{f} = \left(\frac{\mu}{\mu_{\text{medium}}} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$ helps in calculating the refractive index when focal length and radii of curvature are known.

---

41. Energy of 10 non-rigid diatomic molecules at temperature T is:

(1) $\frac{7}{2}RT$

(2) $70\,k_BT$

(3) $35\,RT$

(4) $35\,k_BT$

Correct Answer: (4)

Solution:

For a non-rigid diatomic molecule, the degrees of freedom f are given by:

$$f = 5 + 2(3N - 5)$$

Since $N = 2$ (for diatomic molecules):

$$f = 5 + 2(3 \times 2 - 5) = 7$$

The energy of one molecule is:

$$\text{Energy} = \frac{f}{2}k_BT = \frac{7}{2}k_BT$$

For 10 molecules, the total energy is:

$$10 \times \frac{7}{2}k_BT = 35\,k_BT$$

Quick Tip

For calculating the total energy of multiple molecules, use $E = \frac{f}{2}k_BT$ per molecule, where f is the degrees of freedom, and multiply by the number of molecules.

---

42. A body of weight 200 N is suspended from a tree branch through a chain of mass 10 kg. The branch pulls the chain by a force equal to (if $g = 10\,\text{m/s}^2$):

(1) 150 N

(2) 300 N

(3) 200 N

(4) 100 N

Correct Answer: (2)

Solution:

The total weight acting on the chain is the sum of the weight of the body and the weight of the chain itself.

1. Weight of the body: $200\,\text{N}$ 2. Weight of the chain: $10 \times 10 = 100\,\text{N}$

Thus, the total weight is:

$$\text{Total weight} = 200 + 100 = 300\,\text{N}$$

Since the chain-block system is in equilibrium, the tension T in the chain must balance the total weight:

$$T = 300\,\text{N}$$

Quick Tip

When calculating the tension in a suspended system, consider the weights of all components in the system and add them to find the total force in equilibrium.

---

43. When UV light of wavelength 300 nm is incident on the metal surface having work function 2.13 eV, electron emission takes place. The stopping potential is: (Given $hc = 1240$ eV nm)

(1) 4 V

(2) 4.1 V

(3) 2 V

(4) 1.5 V

Correct Answer: (3)

Solution:

The energy of the incident photon is given by:

$$\frac{hc}{\lambda} = eV_s + \phi$$

Substitute the given values:

$$\frac{1240}{300}\,\text{eV} - 2.13\,\text{eV} = eV_s$$

$$4.13\,\text{eV} - 2.13\,\text{eV} = eV_s$$

$$V_s = 2\,\text{V}$$

Quick Tip

In photoelectric effect problems, use the equation $\frac{hc}{\lambda} = eV_s + \phi$ to find the stopping potential, where ϕ is the work function of the material.

---

44. The number of electrons flowing per second in the filament of a 110 W bulb operating at 220 V is: (Given $e = 1.6 \times 10^{-19}$ C)

(1) 31.25×10^{17}

(2) 6.25×10^{18}

(3) 6.25×10^{17}

(4) 1.25×10^{19}

Correct Answer: (1)

Solution:

Given:

$$\text{Power (P)} = V \cdot I$$

$$110 = 220 \times I$$

$$I = 0.5\,\text{A}$$

Now, we know:

$$I = \frac{n \cdot e}{t}$$

Substitute the values:

$$0.5 = \frac{n \times (1.6 \times 10^{-19})}{t}$$

Rearrange to solve for n:

$$\frac{n}{t} = \frac{0.5}{1.6 \times 10^{-19}}$$
$$\frac{n}{t} = 31.25 \times 10^{17}$$

Quick Tip

To calculate the number of electrons flowing per second, use $I = \frac{n \cdot e}{t}$ and solve for n when current and charge of an electron are given.

---

45. When the kinetic energy of a body becomes 36 times its original value, the percentage increase in the momentum of the body will be:

(1) 500%

(2) 600%

(3) 6%

(4) 60%

Correct Answer: (1)

Solution:

The kinetic energy K is given by:

$$K = \frac{P^2}{2m}$$

Therefore, momentum P can be expressed as:

$$P = \sqrt{2mK}$$

If the final kinetic energy K_f is 36 times the initial kinetic energy K_i, we have:

$$K_f = 36\, K_i$$

Thus, the final momentum P_f will be:

$$P_f = \sqrt{2m \cdot 36K_i} = 6\, P_i$$

The percentage increase in momentum is:

$$\text{Percentage increase} = \frac{P_f - P_i}{P_i} \times 100\%$$

$$= \frac{6P_i - P_i}{P_i} \times 100\%$$

$$= \frac{5P_i}{P_i} \times 100\% = 500\%$$

Quick Tip

When kinetic energy changes by a factor, the momentum changes by the square root of that factor. Use this relationship to find the percentage increase in momentum.

---

46. Pressure inside a soap bubble is greater than the pressure outside by an amount:

(Given: R = Radius of bubble, S = Surface tension of bubble)

(1) $\frac{4S}{R}$

(2) $\frac{4R}{S}$

(3) $\frac{S}{R}$

(4) $\frac{2S}{R}$

Correct Answer: (1)

Solution:

For a soap bubble, there are two liquid-air surfaces, so the excess pressure ΔP inside the bubble is given by:

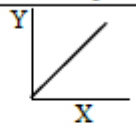
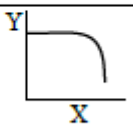
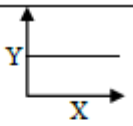
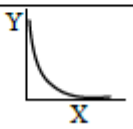
$$\Delta P = 2\left(\frac{2S}{R}\right) = \frac{4S}{R}$$

Quick Tip

In a soap bubble, the excess pressure is calculated considering both inner and outer surfaces. Use $\Delta P = \frac{4S}{R}$ for soap bubbles.

---

47. Match List-I with List-II

List-I (Y vs X)		List-II (Shape of Graph)	
(A)	Y = magnetic susceptibility X = magnetising field	(I)	
(B)	Y = magnetic field X = distance from centre of a current carrying wire for x < a (where a=radius of wire)	(II)	
(C)	Y = magnetic field X = distance from centre of a current carrying wire for x > a (where a = radius of wire)	(III)	
(D)	Y= magnetic field inside solenoid X = distance from center	(IV)	

Choose the correct answer from the options given below:

(1) (A)-(III), (B)-(I), (C)-(IV), (D)-(II)

(2) (A)-(I), (B)-(III), (C)-(II), (D)-(IV)

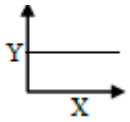
(3) (A)-(IV), (B)-(I), (C)-(II), (D)-(III)

(4) (A)-(III), (B)-(IV), (C)-(I), (D)-(II)

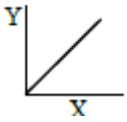
Correct Answer: (4)

Solution:

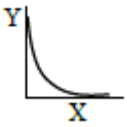
- (A) The graph between magnetic susceptibility and magnetising field is as shown in (III).



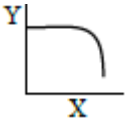
- (B) For $x < a$, the magnetic field due to a current-carrying wire is given by $B = \frac{\mu_0 I x}{2\pi a^2}$, matching graph (IV).



- (C) For $x > a$, the magnetic field due to a current-carrying wire is given by $B = \frac{\mu_0 I}{2\pi x}$, matching graph (I).



- (D) The magnetic field inside a solenoid varies with distance as shown in (II).



Quick Tip

To match graphs with physical quantities, analyze the relationship or formula that describes each scenario and identify whether it shows linearity, proportionality, or inverse behavior.

---

48. In a vernier caliper, when both jaws touch each other, zero of the vernier scale shifts towards the left, and its 4^{th} division coincides exactly with a certain division on the main scale. If 50 vernier scale divisions equal 49 main scale divisions and zero error in the instrument is 0.04 mm, then how many main scale divisions are there in 1 cm?

(1) 40

(2) 5

(3) 20

(4) 10

NTA Ans. (3)

Solution:

Since the 4^{th} division coincides with the 3^{rd} division on the main scale, we have:

$$0.004\,\text{cm} = 4\text{VSD} - 3\text{MSD}$$

Given that 49 MSD = 50 VSD.

Now, calculate the length of 1 MSD:

$$1\text{MSD} = \frac{1}{N}\,\text{cm}$$

Using the zero error formula:

$$0.004 = 4\left(\frac{49}{50}\text{MSD}\right) - 3\text{MSD}$$

$$0.004 = \frac{196}{50}\text{MSD} - 3\text{MSD}$$

Simplifying further:

$$0.004 = \left(\frac{196 - 150}{50}\right)\text{MSD}$$

$$0.004 = \frac{46}{50} \times \frac{1}{N}$$

Solve for N:

$$N = \frac{46 \times 1000}{4 \times 50} = 230$$

Therefore, there are 20 main scale divisions in 1 cm.

Quick Tip

When calculating divisions in a vernier caliper, use the relation between vernier and main scale divisions and adjust for zero error.

---

49. Given below are two statements:

Statement (I): Dimensions of specific heat is $[L^2T^{-2}K^{-1}]$

Statement (II): Dimensions of gas constant is $[ML^2T^{-2}K^{-1}]$

(1) Statement (I) is incorrect but statement (II) is correct

(2) Both statement (I) and statement (II) are incorrect

(3) Statement (I) is correct but statement (II) is incorrect

(4) Both statement (I) and statement (II) are correct

Correct Answer: (3)

Solution:

$$\Delta Q = mS\Delta T$$

$$s = \frac{\Delta Q}{m\Delta T}$$

$$[s] = \left[\frac{ML^2T^{-2}}{MK}\right]$$

$$[s] = [L^2T^{-2}K^{-1}]$$

Statement-(I) is correct.

From $PV = nRT$, we have:

$$R = \frac{PV}{nT}$$

Substitute dimensions:

$$[R] = \left[\frac{ML^{-1}T^{-2}L^3}{[\text{mol}][K]}\right]$$

Simplify:

$$[R] = [ML^2T^{-2}\ \text{mol}^{-1}K^{-1}]$$

Statement-(II) is incorrect.

Quick Tip

To determine the dimensional formula, analyze the fundamental equation related to the quantity and simplify based on units for mass, length, time, and temperature.

---

50. A body projected vertically upwards with a certain speed from the top of a tower reaches the ground in t_1. If it is projected vertically downwards from the same point with the same speed, it reaches the ground in t_2. Time required to reach the ground, if it is dropped from the top of the tower, is:

(1) $\sqrt{t_1 t_2}$

(2) $\sqrt{t_1 - t_2}$

(3) $\sqrt{\frac{t_1}{t_2}}$

(4) $\sqrt{t_1 + t_2}$

Correct Answer: (1)

Solution:

Given:

$$t_1 = \frac{u + \sqrt{u^2 + 2gh}}{g}$$

$$t_2 = \frac{-u + \sqrt{u^2 + 2gh}}{g}$$

For the time t required if the body is dropped (i.e., initial velocity $u = 0$):

$$t = \sqrt{\frac{2gh}{g^2}} = \frac{\sqrt{2gh}}{g}$$

Now, using the equations for t_1 and t_2:

$$t_1 t_2 = \frac{(u^2 + 2gh) - u^2}{g^2} = \frac{2gh}{g^2} = t^2$$

Thus:

$$t = \sqrt{t_1 t_2}$$

Quick Tip

When a body is projected upwards or downwards from a height, use kinematic equations to derive time relations for different initial velocities.

51. In the Franck-Hertz experiment, the first dip in the current-voltage graph for hydrogen is observed at $10.2\,\text{V}$. The wavelength of light emitted by the hydrogen atom when excited to the first excitation level is ____ nm.
(Given $hc = 1245\,\text{eV nm}$, $e = 1.6 \times 10^{-19}\,\text{C}$)

Correct Answer: (122)

Solution:

The energy corresponding to the first dip is given as:

$$10.2\,\text{eV} = \frac{hc}{\lambda}$$

Rearrange to solve for λ:

$$\lambda = \frac{1245\,\text{eV} - \text{nm}}{10.2\,\text{eV}}$$

$$\lambda = 122.06\,\text{nm}$$

Quick Tip

Use the formula $E = \frac{hc}{\lambda}$ to relate the energy of an electron transition to the wavelength of the emitted photon. Ensure that energy and hc are in consistent units.

---

52. For a given series LCR circuit, it is found that maximum current is drawn when the value of variable capacitance is $2.5\,\text{nF}$. If resistance of $200\,\Omega$ and $100\,\text{mH}$ inductor is being used in the given circuit, the frequency of the AC source is ____ $\times 10^3$ Hz. (Given $\pi^2 = 10$)

Correct Answer: (10)

Solution:

For maximum current, the circuit must be in resonance.

The resonant frequency f_0 is given by:

$$f_0 = \frac{1}{2\pi\sqrt{L \cdot C}}$$

Substitute $L = 100 \times 10^{-3}$ H and $C = 2.5 \times 10^{-9}$ F:

$$f_0 = \frac{1}{2\pi\sqrt{100 \times 10^{-3} \times 2.5 \times 10^{-9}}}$$

Simplify under the square root:

$$f_0 = \frac{1}{2\pi\sqrt{25 \times 10^{-11}}}$$

$$f_0 = \frac{1}{2\pi \times 5 \times 10^{-6}}$$

Using $\pi^2 = 10$:

$$f_0 = \frac{10^5\sqrt{10}}{10}$$

$$f_0 = 10 \times 10^3 \text{ Hz}$$

Quick Tip

In a series LCR circuit, maximum current is drawn at the resonant frequency, which is calculated using $f_0 = \frac{1}{2\pi\sqrt{LC}}$.

53.A particle moves in a straight line so that its displacement x at any time t is given by:

$$x^2 = 1 + t^2.$$

Its acceleration at any time t is x^{-n}, where $n =$ ____.

Correct Answer: (3)

Solution:

$$x^2 = 1 + t^2$$

Differentiating with respect to t:

$$2x\frac{dx}{dt} = 2t$$

$$xv = t \quad (\text{where } v = \frac{dx}{dt})$$

Differentiating again:

$$x\frac{dv}{dt} + v\frac{dx}{dt} = 1$$

$$x \cdot a + v^2 = 1 \quad (\text{where } a = \frac{dv}{dt})$$

Simplify:

$$a = \frac{1 - v^2}{x} = \frac{1 - t^2/x^2}{x}$$

$$a = \frac{1}{x^3} = x^{-3}$$

Quick Tip

When differentiating equations involving products, apply the product rule and solve systematically to find expressions for velocity and acceleration.

---

54. Three balls of masses 2 kg, 4 kg, and 6 kg respectively are arranged at the center of the edges of an equilateral triangle of side 2 m. The moment of inertia of the system about an axis through the centroid and perpendicular to the plane of the triangle will be ____ kg m^2.

Correct Answer: (4)

Solution:

For an equilateral triangle, the distance r from the center of each edge to the centroid C is:

$$r = \frac{1}{\sqrt{3}}$$

The moment of inertia I about point C and perpendicular to the plane is given by:

$$I = r^2 \left[2 + 4 + 6\right]$$

Substitute $r = \frac{1}{\sqrt{3}}$:

$$I = \left(\frac{1}{\sqrt{3}}\right)^2 \times 12$$

$$I = \frac{1}{3} \times 12 = 4\,\text{kg m}^2$$

Quick Tip

For calculating the moment of inertia of masses arranged symmetrically around a centroid, use $I = \sum mr^2$, where r is the distance from each mass to the axis.

---

55. A coil having 100 turns, area of 5×10^{-3} m^2, carrying a current of 1 mA is placed in a uniform magnetic field of 0.20 T such that the plane of the coil is perpendicular to the magnetic field. The work done in turning the coil through 90° is ____ μJ.

Correct Answer: (100)

Solution:

The work done $W = \Delta U = U_f - U_i$:

$$W = -(\mu B)_f - (-\mu B)_i$$

Initially, the magnetic moment μ is perpendicular to the magnetic field, so:

$$W = 0 + (\mu B)$$

Substitute the values:

$$\mu = (100 \times 5 \times 10^{-3} \times 1 \times 10^{-3})\,\text{A} \cdot \text{m}^2$$

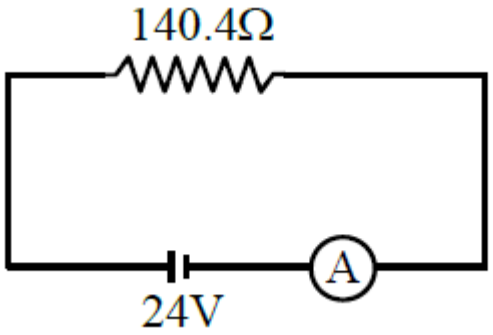
$$W = (1 \times 10^{-4}) \times 0.2\,\text{J} = 1 \times 10^{-5}\,\text{J} = 100\,\mu\text{J}$$

Quick Tip

To find the work done in rotating a magnetic dipole in a uniform magnetic field, use $W = \Delta U = \mu B(1 - \cos\theta)$ where θ is the angle of rotation.

---

56. In the given figure, an ammeter A consists of a $240\,\Omega$ coil connected in parallel to a $10\,\Omega$ shunt. The reading of the ammeter is ____ mA.



Correct Answer: (160)

Solution:

The circuit has a total resistance composed of a $140.4\,\Omega$ resistor in series with the parallel combination of $240\,\Omega$ and $10\,\Omega$ resistors.

The equivalent resistance R_{eq} of the parallel combination is:

$$R_{\text{eq}} = \frac{240 \times 10}{240 + 10} = \frac{2400}{250} = 9.6\,\Omega$$

Thus, the total resistance in the circuit becomes:

$$R_{\text{total}} = 140.4 + 9.6 = 150\,\Omega$$

Now, the current I in the circuit is:

$$I = \frac{V}{R_{\text{total}}} = \frac{24}{150} = 0.16\,\text{A} = 160\,\text{mA}$$

Therefore, the current in the ammeter is $160\,\text{mA}$.

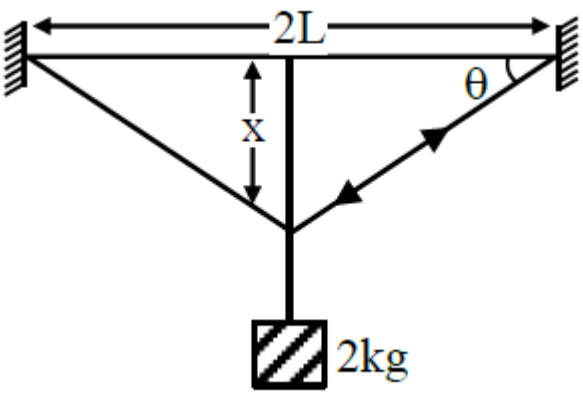
Quick Tip

For a parallel and series resistor combination, calculate the equivalent resistance for parallel components first, then add series resistances.

---

57. A wire of cross-sectional area A, modulus of elasticity $2 \times 10^{11}\,\text{Nm}^{-2}$, and length 2 m is stretched between two vertical rigid supports. When a mass of 2 kg is suspended at the middle, it sags lower from its original position making an angle $\theta = \frac{1}{100}$ radian on the points of support. The value of A is ____ $\times 10^{-4}\,\text{m}^2$.

(Given: $g = 10\,\text{m/s}^2$**)**



Correct Answer: (1)

Solution:

In the vertical direction:

$$2T\sin\theta = 20$$

Using the small angle approximation $\sin\theta \approx \theta$:

$$\theta = \frac{1}{100} \Rightarrow T = \frac{10}{\theta} = 1000\,\text{N}$$

The change in length ΔL is given by:

$$\Delta L = 2\sqrt{x^2 + L^2} - 2L$$

$$\Delta L \approx 2L\left(\frac{x^2}{2L^2}\right) = \frac{x^2}{L}$$

Modulus of elasticity E is defined as:

$$E = \frac{\text{stress}}{\text{strain}}$$

Substitute $E = 2 \times 10^{11}\,\text{Nm}^{-2}$:

$$2 \times 10^{11} = \frac{10^3}{A \times \frac{x^2}{L}} \times 2L$$

Solve for A:

$$A = 1 \times 10^{-4}\,\text{m}^2$$

Quick Tip

For small displacements, use the small-angle approximation $\sin\theta \approx \theta$ and relate stress and strain using modulus of elasticity.

---

58. Two coherent monochromatic light beams of intensities I and $4I$ are superimposed. The difference between maximum and minimum possible intensities in the resulting beam is x. The value of x is ____.

Correct Answer: (8)

Solution:

The maximum intensity I_{max} in the superimposed beam is given by:

$$I_{\text{max}} = \left(\sqrt{I} + \sqrt{4I}\right)^2 = \left(\sqrt{I} + 2\sqrt{I}\right)^2 = (3\sqrt{I})^2 = 9I$$

The minimum intensity I_{min} is given by:

$$I_{\text{min}} = \left(\sqrt{4I} - \sqrt{I}\right)^2 = (2\sqrt{I} - \sqrt{I})^2 = (\sqrt{I})^2 = I$$

Therefore, the difference $I_{\text{max}} - I_{\text{min}}$ is:

$$x = I_{\text{max}} - I_{\text{min}} = 9I - I = 8I$$

Quick Tip

For two coherent light beams with intensities I_1 and I_2, the maximum and minimum intensities when they interfere are given by $I_{\text{max}} = (\sqrt{I_1}+\sqrt{I_2})^2$ and $I_{\text{min}} = (\sqrt{I_1}-\sqrt{I_2})^2$.

---

59. Two open organ pipes of length 60 cm and 90 cm resonate at 6$^{\text{th}}$ and 5$^{\text{th}}$ harmonics respectively. The difference of frequencies for the given modes is ____ Hz.

(Velocity of sound in air $v = 333$ m/s)

Correct Answer: (740)

Solution:

The frequency f of an open organ pipe is given by:

$$f = \frac{nv}{2L}$$

The difference in frequency Δf for the two pipes is:

$$\Delta f = \frac{6v}{2 \times 0.6} - \frac{5v}{2 \times 0.9}$$

Substitute $v = 333\,\text{m/s}$:

$$\Delta f = \frac{6 \times 333}{2 \times 0.6} - \frac{5 \times 333}{2 \times 0.9}$$

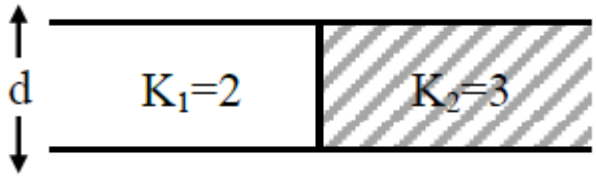
$$\Delta f = 740\,\text{Hz}$$

Quick Tip

For open organ pipes, the frequency for the n-th harmonic is given by $f = \frac{nv}{2L}$. Calculate differences by plugging in the values for each harmonic and length.

---

60. A capacitor of 10 μF capacitance whose plates are separated by 10 mm through air and each plate has an area $4\,\text{cm}^2$ is now filled with two dielectric media of $K_1 = 2, K_2 = 3$ respectively as shown in the figure. If new force between the plates is 8 N, the supply voltage is ____ V.



Correct Answer: (80)

Solution:

The total capacitance C_{eq} of the capacitor is the sum of the two capacitances C_1 and C_2:

$$C_{\text{eq}} = C_1 + C_2$$

Capacitance C_1 with dielectric constant $K_1 = 2$ is:

$$C_1 = \frac{\varepsilon_0 A}{d} \cdot K_1 = 2 \cdot \frac{4 \times 10^{-4}}{10 \times 10^{-3}}\,\mu F = 10\,\mu F$$

Capacitance C_2 with dielectric constant $K_2 = 3$ is:

$$C_2 = \frac{\varepsilon_0 A}{d} \cdot K_2 = 3 \cdot \frac{4 \times 10^{-4}}{10 \times 10^{-3}} \mu F = 15 \, \mu F$$

Thus, the total capacitance is:

$$C_{\text{eq}} = 10 \, \mu F + 15 \, \mu F = 25 \, \mu F$$

Now, the charge on the capacitors is given by:

$$C_1 = 10 \, \text{V}, \quad C_2 = 1.5 \, \text{V}$$

The force between the plates is:

$$F = \frac{Q^2}{2A\varepsilon_0}$$

Substitute the values:

$$\frac{100V^2 \times 10^{-12}}{2 \times 2 \times 10^{-4} \times \varepsilon_0} + \frac{225V^2 \times 10^{-12}}{2 \times 2 \times 10^{-4} \times \varepsilon_0} = 8$$

Thus, the supply voltage is:

$$325 \, V^2 = 8 \times 4 \times 10^{-4} \times 8.85$$

$$V^2 = \frac{32 \times 8.85 \times 10^{-4}}{325}$$

$$\therefore \, V = \sqrt{\frac{283.2 \times 10^{-4}}{325}}$$

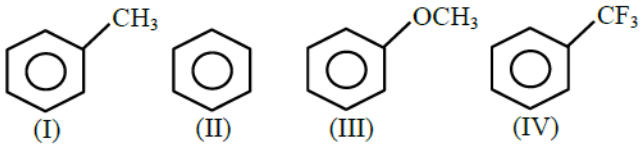
$$V = 0.93 \times 10^{-2}$$

Quick Tip

For capacitors with multiple dielectric materials, compute the capacitance for each section and then use the total capacitance in the force equation.

---

61. The correct arrangement for decreasing order of electrophilic substitution for the above compounds is ____.



(1) $(IV) > (I) > (II) > (III)$

(2) $(III) > (I) > (II) > (IV)$

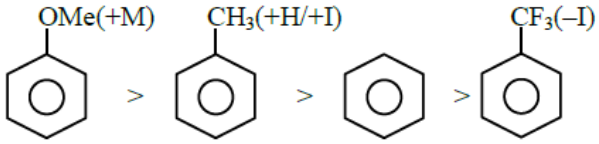
(3) $(II) > (IV) > (III) > (I)$

(4) $(III) > (IV) > (II) > (I)$

Correct Answer: (2)

Solution:

The compounds are arranged in order of their electron-donating and electron-withdrawing effects:



Thus, the decreasing order for electrophilic substitution is:

Compound (III) > Compound (I) > Compound (II) > Compound (IV)

Quick Tip

The electron-donating groups increase the rate of electrophilic substitution, whereas electron-withdrawing groups decrease it.

---

62. Molality (m) of 3 M aqueous solution of NaCl is:

(Given: Density of solution = 1.25 g mL^{-1}, Molar mass in g mol^{-1}: Na = 23, Cl = 35.5)

(1) 2.90 m

(2) 2.79 m

(3) 1.90 m

(4) 3.85 m

Correct Answer: (2)

Solution:

For a 3 M solution, 3 moles of NaCl are present in 1 liter of solution.

The formula for molality m is:

$$\text{molality} = \frac{\text{moles of solute} \times 1000}{\text{mass of solvent in grams}}$$

Calculate the mass of the solution:

$$\text{Mass of solution} = \text{Density} \times \text{Volume} = 1.25 \times 1000 = 1250\,\text{g}$$

Now, calculate the mass of solute (NaCl):

$$\text{Mass of solute} = \text{moles} \times \text{molar mass} = 3 \times 58.5 = 175.5\,\text{g}$$

Therefore, the mass of the solvent (water) is:

$$\text{Mass of solvent} = 1250 - 175.5 = 1074.5\,\text{g}$$

Substitute the values to find molality:

$$\text{molality} = \frac{3 \times 1000}{1074.5} = 2.79\,\text{m}$$

Quick Tip

Molality is calculated as moles of solute per kilogram of solvent, which makes it independent of temperature.

---

63. The incorrect statements regarding enzymes are:

(A) Enzymes are biocatalysts.

(B) Enzymes are non-specific and can catalyse different kinds of reactions.

(C) Most Enzymes are globular proteins.

(D) Enzyme - oxidase catalyses the hydrolysis of maltose into glucose.

Choose the correct answer from the options given below:

(1) (B) and (C)

(2) (B), (C) and (D)

(3) (B) and (D)

(4) (A), (D) and (C)

Correct Answer: (3)

Solution:

Statement (A) is correct because enzymes function as biocatalysts, accelerating biochemical reactions.
Statement (B) is incorrect because enzymes are specific in their action and typically catalyze only particular types of reactions.
Statement (C) is correct as most enzymes are indeed globular proteins.
Statement (D) is incorrect because oxidase enzymes do not catalyze the hydrolysis of maltose into glucose; instead, they are involved in oxidation-reduction reactions.

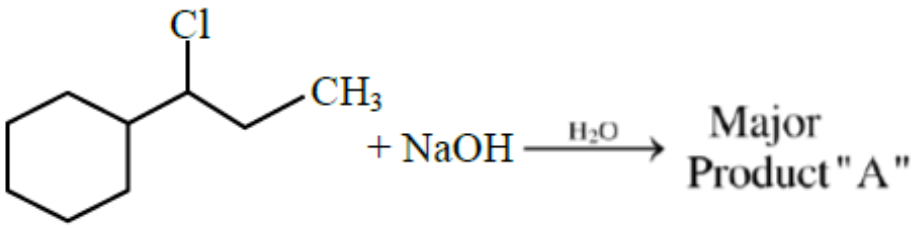
Thus, the incorrect statements are (B) and (D).

Quick Tip

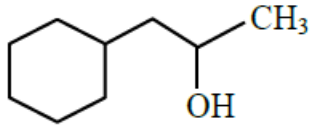
Enzymes are specific catalysts, often catalyzing only a particular type of reaction due to their specific active sites.

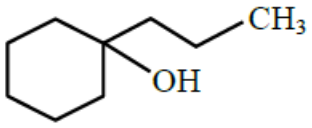
---

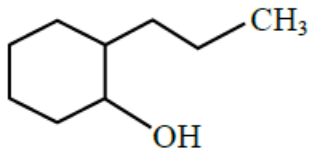
64. Consider the above chemical reaction. Product ”A” is:

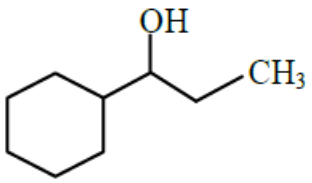


Consider the above chemical reaction. Product "A" is:

(1) 

(2) 

(3) 

(4) 

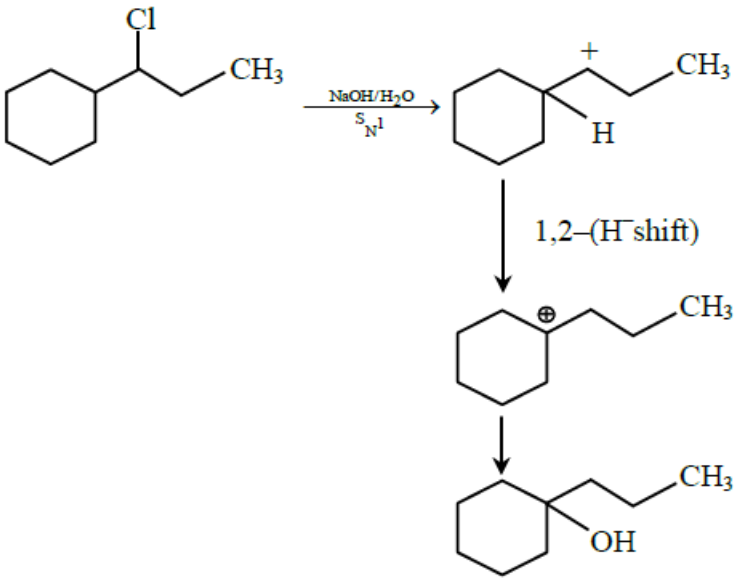
Correct Answer: (2)

Solution:

The reaction proceeds via an S_N1 mechanism where the chlorine group leaves, forming a carbocation.

After the formation of the carbocation, a 1,2-hydride shift occurs to stabilize the carbocation, resulting in a more stable tertiary carbocation.

Subsequent attack by hydroxide ion (OH^-) leads to the formation of the major product:



(Major Product)

The major product is a tertiary alcohol.

Quick Tip

In S_N1 reactions, carbocation rearrangements, such as hydride or alkyl shifts, often occur to form more stable carbocations.

---

65. During the detection of acidic radical present in a salt, a student gets a pale yellow precipitate soluble with difficulty in NH_4OH solution when sodium carbonate extract was first acidified with dilute HNO_3 and then $AgNO_3$ solution was added. This indicates the presence of:

(1) Br^-

(2) CO_3^{2-}

(3) I^-

(4) Cl^-

Correct Answer: (1)

Solution:

The reactions with Ag^+ ions produce different colored precipitates depending on the halide present:

$$Ag^+ + I^- \rightarrow AgI \quad \text{(Yellow ppt.)}$$

$$Ag^+ + Cl^- \rightarrow AgCl \quad \text{(White ppt.)}$$

$$Ag^+ + Br^- \rightarrow AgBr \quad \text{(Pale yellow ppt.)}$$

Since a pale yellow precipitate was obtained, it indicates the presence of Br^- ions.

Quick Tip

Halide ions form colored precipitates with $AgNO_3$: Yellow for AgI, white for AgCl, and pale yellow for AgBr.

---

66. How can an electrochemical cell be converted into an electrolytic cell?

(1) Applying an external opposite potential greater than E^0_{cell}

(2) Reversing the flow of ions in the salt bridge.

(3) Applying an external opposite potential lower than E^0_{cell}.

(4) Exchanging the electrodes at anode and cathode.

Correct Answer: (1)

Solution:

To convert an electrochemical cell into an electrolytic cell, an external potential needs to be applied in the opposite direction. This applied potential should be greater than the standard cell potential E^0_{cell}. When this condition is met, the cell reaction reverses, and the electrochemical cell functions as an electrolytic cell.

Quick Tip

To reverse an electrochemical cell reaction, apply an external potential greater than the cell's standard potential in the opposite direction.

---

67. Arrange the following elements in the increasing order of number of unpaired electrons in it.

(A) Sc

(B) Cr

(C) V

(D) Ti

(E) Mn

Choose the correct answer from the options given below:

(1) $(C) < (E) < (B) < (A) < (D)$

(2) $(B) < (C) < (D) < (E) < (A)$

(3) $(A) < (D) < (C) < (B) < (E)$

(4) $(A) < (D) < (C) < (E) < (B)$

Correct Answer: (4)

Solution:

The electronic configurations and the number of unpaired electrons for each element are as follows:

Sc:	$[\text{Ar}]\, 4s^2\, 3d^1$	(1 unpaired electron)
Cr:	$[\text{Ar}]\, 4s^1\, 3d^5$	(6 unpaired electrons)
V:	$[\text{Ar}]\, 4s^2\, 3d^3$	(3 unpaired electrons)
Ti:	$[\text{Ar}]\, 4s^2\, 3d^2$	(2 unpaired electrons)
Mn:	$[\text{Ar}]\, 4s^2\, 3d^5$	(5 unpaired electrons)

Arranging them in increasing order of unpaired electrons, we get:

$$\text{Sc (A)} < \text{Ti (D)} < \text{V (C)} < \text{Mn (E)} < \text{Cr (B)}$$

Quick Tip

The number of unpaired electrons in transition elements depends on their electronic configuration, particularly the arrangement in d-orbitals.

---

68. Match List-I with List-II.

List-I	**List-II**
Alkali Metal	Emission Wavelength in nm
(A) Li	(I) 589.2
(B) Na	(II) 455.5
(C) Rb	(III) 670.8
(D) Cs	(IV) 780.0

Choose the correct answer from the options given below:

(1) (A)-(I), (B)-(IV), (C)-(III), (D)-(II)

(2) (A)-(III), (B)-(I), (C)-(IV), (D)-(II)

(3) (A)-(II), (B)-(I), (C)-(III), (D)-(IV)

(4) (A)-(III), (B)-(IV), (C)-(I), (D)-(II)

Correct Answer: (2)

Solution:

This is a fact-based question regarding the emission wavelengths of alkali metals. The correct matches based on known data are:

(A) Li –(III) 670.8 nm

(B) Na –(I) 589.2 nm

(C) Rb –(IV) 780.0 nm

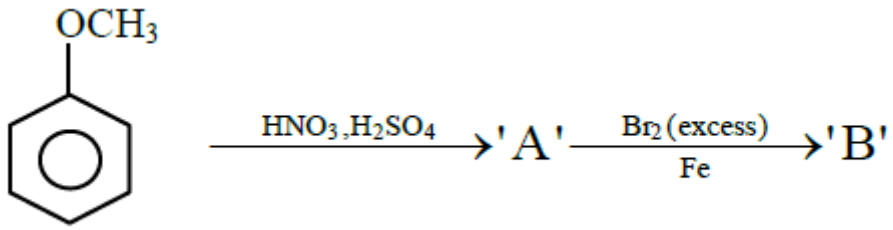
(D) Cs –(II) 455.5 nm

Quick Tip

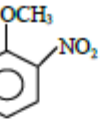
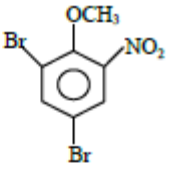
Alkali metals exhibit characteristic emission wavelengths when subjected to flame tests, allowing for their identification.

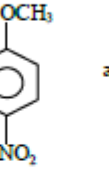
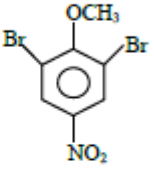
---

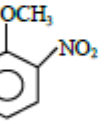
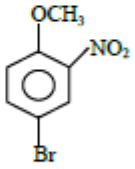
69. The major products formed:

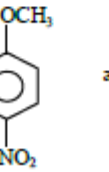
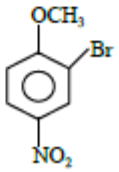


A and B respectively are:

(1)  and 

(2)  and 

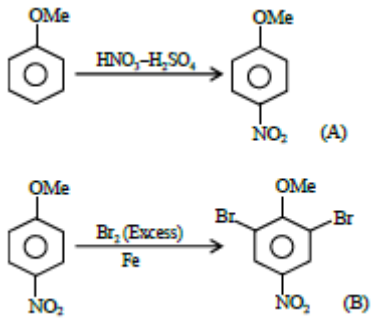
(3)  and 

(4)  and 

Correct Answer: (2)

Solution:

The reaction proceeds as follows: 1. The anisole (methoxybenzene) undergoes nitration with HNO_3/H_2SO_4 to form the product (A), which is 4-nitroanisole. 2. Subsequently, bromination with Br_2 (in the presence of excess iron) results in the brominated product (B), 4-bromo-2-nitroanisole.



Thus, the major products are (A) and (B) as represented in option (2).

Quick Tip

Anisole (OMe-substituted benzene) directs electrophilic substitution to the para and ortho positions, with nitration and bromination resulting in selective substitution patterns.

---

70. The incorrect statement regarding the geometrical isomers of 2-butene is:

(1) cis-2-butene and trans-2-butene are not interconvertible at room temperature.

(2) cis-2-butene has less dipole moment than trans-2-butene.

(3) trans-2-butene is more stable than cis-2-butene.

(4) cis-2-butene and trans-2-butene are stereoisomers.

Correct Answer: (2)

Solution:

The structures of cis-2-butene and trans-2-butene are as follows:

$$\text{cis-2-butene:} \quad CH_3 - CH = CH - CH_3 \quad \text{(Polar)}$$

$$\text{trans-2-butene:} \quad H - CH = CH - CH_3 \quad \text{(Non-Polar)}$$

Cis-but-2-ene has a higher dipole moment than trans-but-2-ene due to the arrangement of the substituent groups. The cis configuration results in a net dipole moment, while the trans configuration cancels out the dipole due to symmetry.

Quick Tip

In geometric isomers, the cis form generally has a higher dipole moment than the trans form because the substituent groups are on the same side in the cis configuration, leading to a net dipole.

---

71. Given below are two statements:

Statement I: PF_5 and BrF_5 both exhibit sp^3d hybridisation.

Statement II: Both SF_6 and $[Co(NH_3)_6]^{3+}$ exhibit sp^3d^2 hybridisation.

In the light of the above statements, choose the correct answer from the options given below:

(1) Statement I is true but Statement II is false

(2) Both Statement I and Statement II are true

(3) Both Statement I and Statement II are false

(4) Statement I is false but Statement II is true

Correct Answer: (3)

Solution:

The hybridisation of the given molecules is as follows:

$$\begin{aligned} PF_5 &: sp^3d \\ BrF_5 &: sp^3d^2 \\ SF_6 &: sp^3d^2 \\ [Co(NH_3)_6]^{3+} &: d^2sp^3 \end{aligned}$$

Thus, both Statement I and Statement II are false.

Quick Tip

The hybridisation of a molecule depends on the number of regions of electron density around the central atom. For example, SF_6 with six bonding pairs exhibits sp^3d^2 hybridisation, while PF_5 with five bonding pairs exhibits sp^3d hybridisation.

---

72. The number of ions from the following that are expected to behave as oxidising agents is:

Sn^{4+}, Sn^{2+}, Pb^{2+}, Tl^{3+}, Pb^{4+}, Tl^{+}

(1) 3

(2) 4

(3) 1

(4) 2

Correct Answer: (4)

Solution:

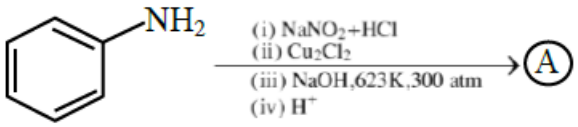
Due to the inert pair effect, Tl^{3+} and Pb^{4+} can behave as oxidising agents.

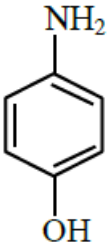
Quick Tip

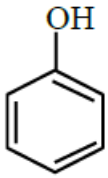
The inert pair effect makes higher oxidation states less stable for heavier elements in the group, thus ions in higher oxidation states like Tl^{3+} and Pb^{4+} act as oxidising agents by accepting electrons to attain a more stable lower oxidation state.

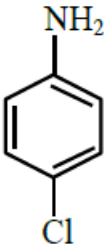
---

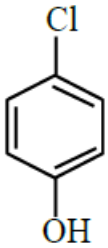
73. Identify the product A in the following reaction.



(1) 

(2) 

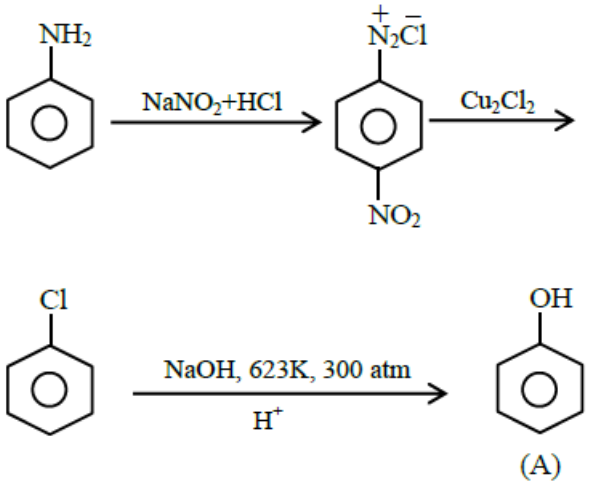
(3) 

(4) 

Correct Answer: (2)

Solution:

The reaction sequence is as follows:



Quick Tip

This reaction involves diazotization, followed by the Sandmeyer reaction, and then further conversion to the final product through hydrolysis.

---

74. The correct statements among the following, for a "chromatography" purification method is:

(1) Organic compounds run faster than solvent in the thin layer chromatographic plate.

(2) Non-polar compounds are retained at top and polar compounds come down in column chromatography.

(3) R_f of a polar compound is smaller than that of a non-polar compound.

(4) R_f is an integral value.

Correct Answer: (3)

Solution:

Non-polar compounds have a higher value of R_f than polar compounds in chromatography.

Quick Tip

The R_f (retention factor) value indicates how far a compound travels in chromatography. Non-polar compounds tend to have higher R_f values than polar compounds, as they are less attracted to the stationary phase.

---

75. Evaluate the following statements related to group 14 elements for their correctness.

(A) Covalent radius decreases down the group from C to Pb in a regular manner.

(B) Electronegativity decreases from C to Pb down the group gradually.

(C) Maximum covalence of C is 4 whereas other elements can expand their covalence due to presence of d orbitals.

(D) Heavier elements do not form π-π bonds.

(E) Carbon can exhibit negative oxidation states.

Choose the correct answer from the options given below:

(1) (C), (D) and (E) Only

(2) (A) and (B) Only

(3) (A), (B) and (C) Only

(4) (C) and (D) Only

Correct Answer: (1)

Solution:

(A) Down the group, radius increases.

(B) EN does not decrease gradually from C to Pb.

(C) Correct.

(D) Correct.

(E) Range of oxidation state of carbon: -4 to +4

Quick Tip

In group 14, the covalent radius generally increases down the group. Carbon's ability to form π-π bonds is unique due to its small size and high electronegativity, which the heavier elements lack.

---

76. Match List-I with the List-II

List-I (Reaction)	**List-II (Type of redox reaction)**
(A) $N_2(g) + O_2(g) \rightarrow 2NO(g)$	(I) Decomposition
(B) $2Pb(NO_3)_{2(s)} \rightarrow 2PbO_{(s)} + 4NO_{2(g)} + O_{2(g)}$	(II) Displacement
(C) $2Na(s) + 2H_2O \rightarrow 2NaOH_{(aq)} + H_2(g)$	(III) Disproportionation
(D) $2NO_2(g) + 2^{-}OH_{(aq)} \rightarrow NO^{-}_{2(aq)} + NO^{-}_{3(aq)} + H_2O_{(l)}$	(IV) Combination

Choose the correct answer from the options given below:

(1) (A)-(I), (B)-(II), (C)-(III), (D)-(IV)

(2) (A)-(III), (B)-(II), (C)-(IV), (D)-(I)

(3) (A)-(II), (B)-(IV), (C)-(III), (D)-(I)

(4) (A)-(IV), (B)-(I), (C)-(II), (D)-(III)

Correct Answer: (4)

Solution:

- **(A)** $N_2(g) + O_2(g) \rightarrow 2NO(g)$: This reaction is a combination reaction because two reactants combine to form a single product. Thus, $A \rightarrow (IV)$.

- **(B)** $2Pb(NO_3)_{2(s)} \rightarrow 2PbO_{(s)} + 4NO_{2(g)} + O_{2(g)}$**:** This is a decomposition reaction because a single compound breaks down into multiple products. Thus, $B \rightarrow (I)$.

- **(C)** $2Na(s) + 2H_2O \rightarrow 2NaOH_{(aq)} + H_2(g)$ This is a displacement reaction as sodium displaces hydrogen from water. Thus, $C \rightarrow (II)$.

- **(D)** $2NO_2(g) + 2^{-}OH_{(aq)} \rightarrow NO^{-}_{2(aq)} + NO^{-}_{3(aq)} + H_2O_{(l)}$**:** This reaction involves disproportionation because the same species (NO_2) is oxidized and reduced simultaneously. Thus, $D \rightarrow (III)$.

Final Matching:

$$A \to (IV),\ B \to (I),\ C \to (II),\ D \to (III)$$

Quick Tip

In redox reactions, combination reactions involve combining elements to form compounds, decomposition breaks down compounds, displacement involves element substitution, and disproportionation occurs when an element in a compound is simultaneously oxidized and reduced.

---

77. Consider the given reaction, identify the major product P.

$$CH_3-COOH \xrightarrow[\text{(iv) } H_2O/\bar{O}H,\Delta]{\text{(i) } LiAlH_4 \quad \text{(ii) PCC} \quad \text{(iii) } HCN/\bar{O}H} \text{"P"}$$

(1) $CH_3-CH_2-CH_2-OH$

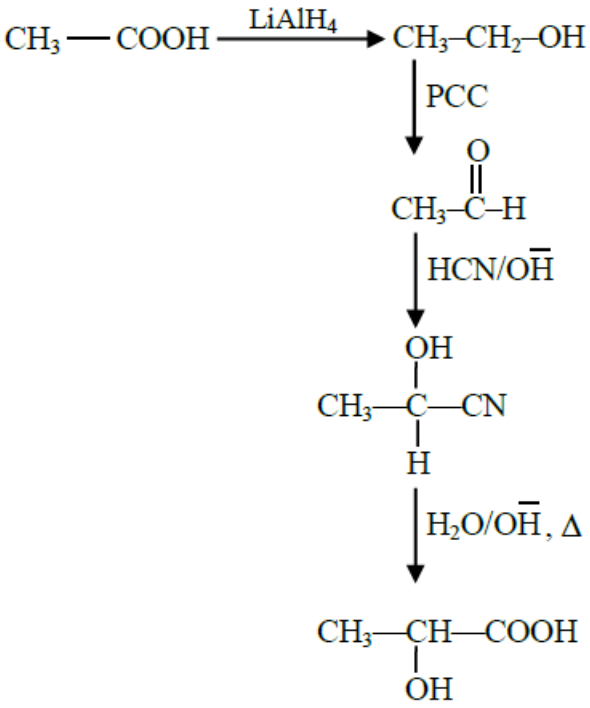
(2) $CH_3-CH_2-\overset{\overset{O}{\|}}{C}-NH_2$

(3) $CH_3-\overset{\overset{O}{\|}}{C}-CH_2CH_3$

(4) $CH_3-\overset{\overset{OH}{|}}{CH}-COOH$

Correct Answer: (4)

Solution:



Quick Tip

In organic synthesis, $LiAlH_4$ is commonly used for the reduction of carboxylic acids to alcohols, while PCC is a mild oxidizing agent that converts alcohols to aldehydes.

---

78. The correct IUPAC name of $[PtBr_2(PMe_3)_2]$ is:

1. bis(trimethylphosphine)dibromoplatinum(II)
2. bis[bromo(trimethylphosphine)]platinum(II)
3. dibromobis(trimethylphosphine)platinum(II)
4. dibromodi(trimethylphosphine)platinum(II)

Correct Answer: (3)

Solution:

Dibromo bis(trimethylphosphine) platinum (II)

Quick Tip

In coordination chemistry, ligands are named alphabetically, and prefixes like ”bis” are used when ligands are polydentate or complex.

---

79. Match List-I with List-II

List-I	**List-II**
Tetrahedral Complex	Electronic configuration
(A) $TiCl_4$	(I) e^2, t_2^1
(B) $[FeO_4]^{2-}$	(II) e^4, t_2^2
(C) $[FeCl_4]^{-}$	(III) e^0, t_2^0
(D) $[CoCl_4]^{2-}$	(IV) e^2, t_2^2

Choose the correct answer from the option given below:

(1) (A)-(I), (B)-(III), (C)-(IV), (D)-(II)

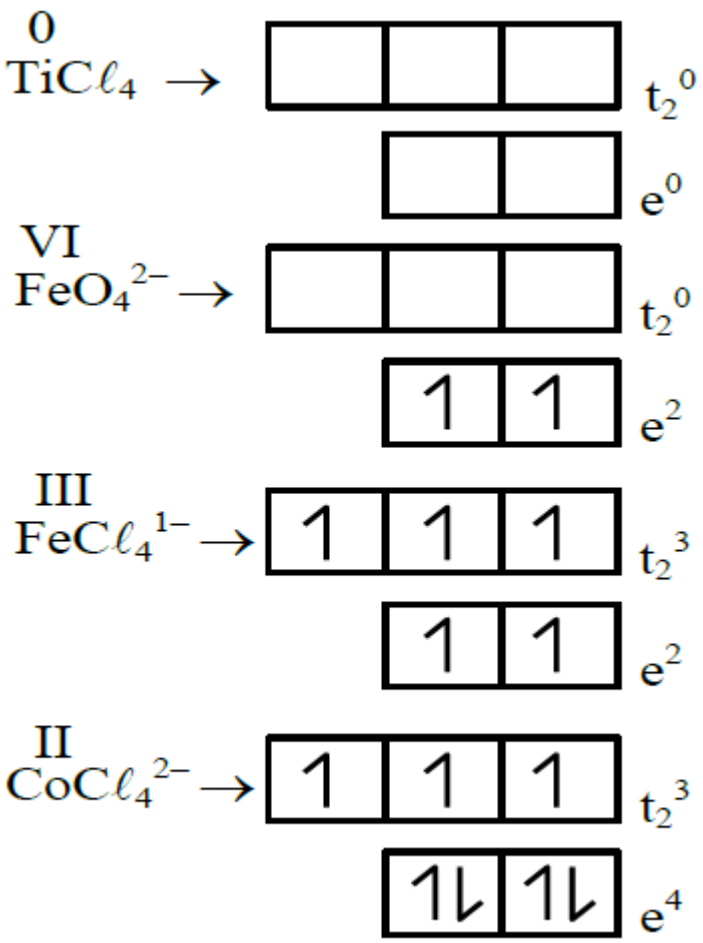
(2) (A)-(IV), (B)-(III), (C)-(I), (D)-(II)

(3) (A)-(III), (B)-(IV), (C)-(II), (D)-(I)

(4) (A)-(III), (B)-(I), (C)-(IV), (D)-(II)

Correct Answer: (4)

Solution:



Quick Tip

In crystal field theory, the electronic configuration for tetrahedral complexes is based on the splitting of d-orbitals into e and t_2 sets.

---

80. The ratio $\frac{K_P}{K_C}$ for the reaction:

$$\mathbf{CO}_{(g)} + \frac{1}{2}\mathbf{O}_2(g) \rightarrow \mathbf{CO}_2(g)$$

is:

1. $(RT)^{1/2}$
2. RT
3. 1
4. $\frac{1}{\sqrt{RT}}$

Correct Answer: (4)

Solution:

$$CO_{(g)} + \frac{1}{2}O_2(g) \rightarrow CO_2(g)$$

$$\Delta n_g = 1 - \left(1 + \frac{1}{2}\right) = -\frac{1}{2}$$

$$\frac{K_P}{K_C} = (RT)^{\Delta n_g} = \frac{1}{\sqrt{RT}}$$

Quick Tip

For gas-phase reactions, the relationship between K_P and K_C is given by $\frac{K_P}{K_C} = (RT)^{\Delta n_g}$, where Δn_g is the change in the number of moles of gas.

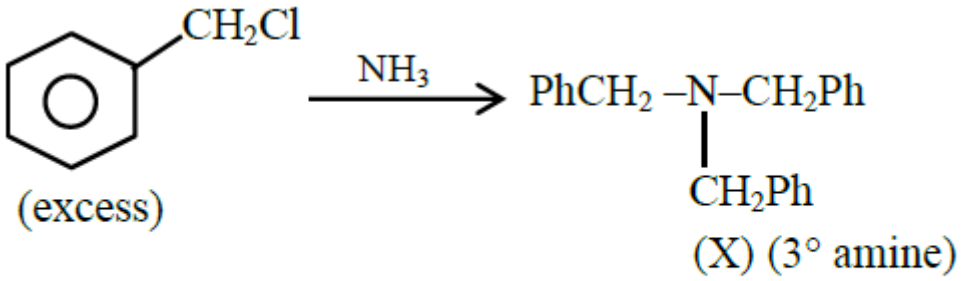
---

81. An amine (X) is prepared by ammonolysis of benzyl chloride. On adding p-toluenesulphonyl chloride to it, the solution remains clear. Molar mass of the amine (X) formed is _____ g mol^{-1}.

(Given molar mass in g mol^{-1} C: 12, H: 1, O: 16, N: 14)

Correct Answer: (287)

Solution:



Molar Mass of (X) is 287 g mol^{-1}

Quick Tip

Ammonolysis of benzyl chloride results in the formation of a tertiary amine with a molar mass of 287 g/mol.

82. Consider the following reactions

$$NiS + HNO_3 + HCl \rightarrow A + NO + S + H_2O$$

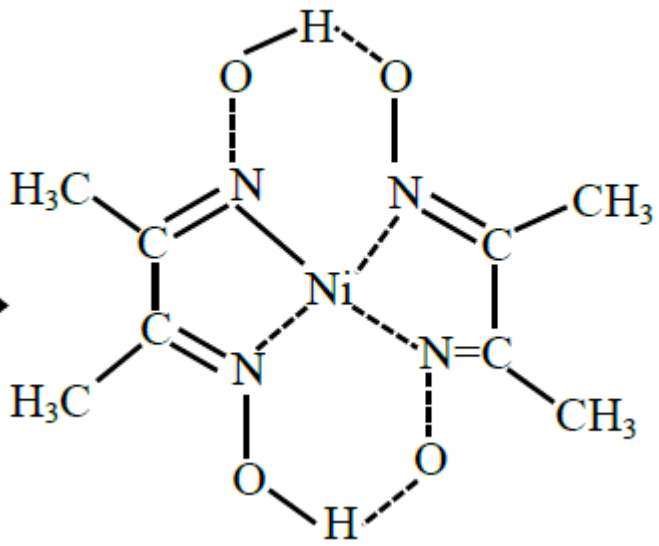
$$A + NH_4OH + \begin{matrix} H_3C - C = N - OH \\ | \\ H_3C - C = N - OH \end{matrix} \longrightarrow B + NH_4Cl + H_2O$$

The number of protons that do not involve in hydrogen bonding in the product B is_____.

Correct Answer: (12)

Solution:

B →



$$3NiS + 2HNO_3 + 6HCl \longrightarrow 3NiCl_2 + 2NO + 3S + 4H_2O$$

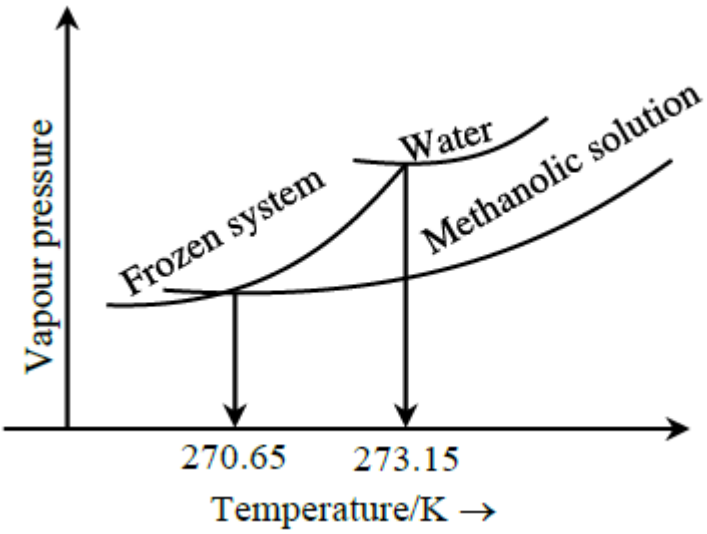
$$NiCl_2 + 2NH_4OH + \begin{matrix} H_3C - C = N - OH \\ | \\ H_3C - C = N - OH \end{matrix} \rightarrow NH_4Cl + H_2O + (B)$$

Quick Tip

Product B is a complex molecule with a structure where 12 protons are not involved in hydrogen bonding.

---

83. When $x \times 10^{-2}$ mL methanol (molar mass = 32 g; density = 0.792 g/cm³) is added to 100 mL water (density = 1 g/cm³), the following diagram is obtained.



x=_____ = (nearest integer)

[Given: Molal freezing point depression constant of water at 273.15 K is 1.86 K kg mol⁻¹.]

Correct Answer: 543

Solution.

- Calculate the freezing point depression (ΔT_f):

$$\Delta T_f = 273.15 - 270.65 = 2.5\,\text{K}$$

- Using the formula for freezing point depression:

$$\Delta T_f = K_f \cdot m = 2.5 = 1.86 \times \frac{n}{0.1}$$

- Solve for moles of methanol (n):

$$n = 0.1344\,\text{moles}$$

- Calculate the mass of methanol (w):

$$w = 0.1344 \times 32 = 4.3\,\text{g}$$

- Calculate the volume of methanol:

$$\text{Volume} = \frac{4.3}{0.792} = 5.43\,\text{mL} = 543 \times 10^{-2}\,\text{mL}$$

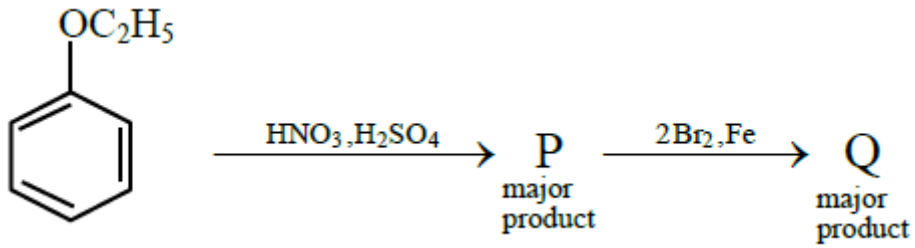
Answer: $x = 543$

Quick Tip

The freezing point depression (ΔT_f) is calculated by subtracting the observed freezing point from the standard freezing point of water. The molality is then determined using the given molal freezing point depression constant.

---

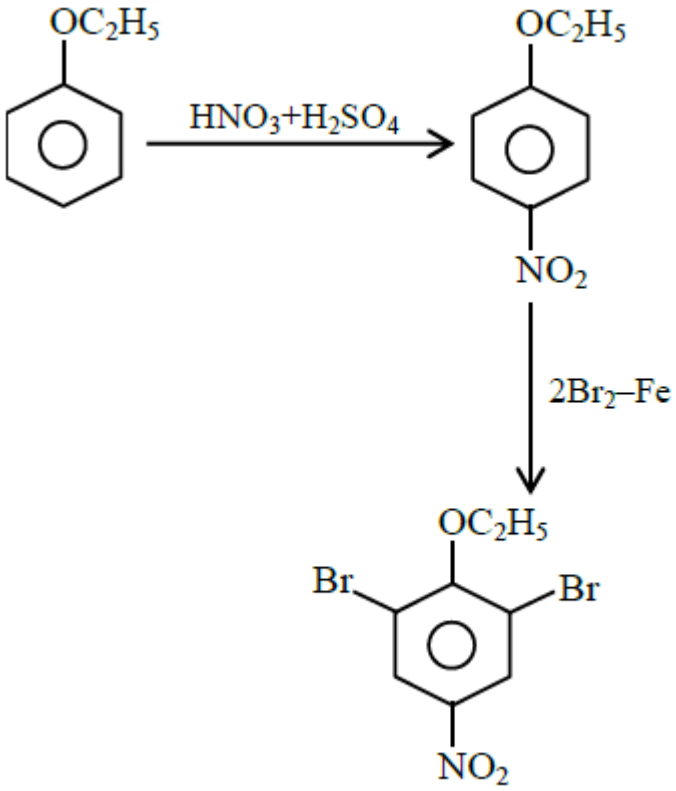
84. The compound with OC_2H_5 group undergoes the following reaction sequence:



The ratio of the number of oxygen atoms to bromine atoms in the product Q is $\_\_\_\_ \times 10^{-1}$.

Correct Answer: 15

Solution:



The initial compound with OC_2H_5 undergoes nitration to introduce a NO_2 group, yielding intermediate P.

$$OC_2H_5 \xrightarrow{HNO_3, H_2SO_4} OC_2H_5NO_2$$

Bromination of P in the presence of Fe introduces two bromine atoms on the aromatic ring, resulting in product Q.

$$OC_2H_5NO_2 \xrightarrow{2\,Br_2, Fe} \text{Br-}OC_2H_5NO_2Br$$

Count the number of oxygen and bromine atoms in product Q:

Oxygen atoms: 3 (one from OC_2H_5 and two from NO_2)

Bromine atoms: 2

The ratio of oxygen to bromine atoms in Q is:

$$\frac{3}{2} \times 10^{-1} = 15 \times 10^{-1}$$

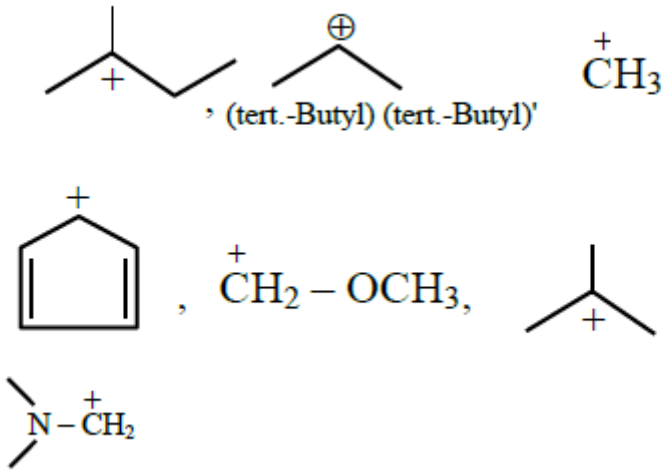
Answer: 15

Quick Tip

In multi-step organic reactions, carefully track each functional group's transformations to determine the final atom count in the product.

---

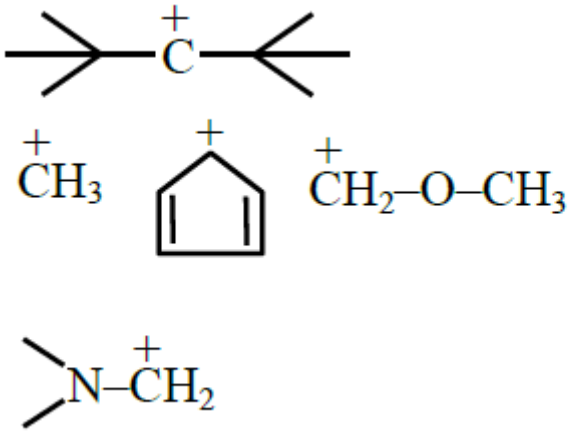
85. Number of carbocation from the following that are not stabilized by hyperconjugation is..... .



Correct Answer: (5)

Solution:

In the given structures, only certain carbocations can be stabilized by hyperconjugation. The carbocations which are not stabilized by hyperconjugation are:



Therefore, the number of carbocations that are not stabilized by hyperconjugation is 5.

Quick Tip

Hyperconjugation is generally possible in carbocations adjacent to alkyl groups, where hydrogen atoms from these groups can participate in delocalization.

---

86. For the reaction at 298 K, $2A+B \rightarrow C$. $\Delta H = 400\,\text{kJ mol}^{-1}$ and $\Delta S = 0.2\,\text{kJ mol}^{-1}\,\text{K}^{-1}$. The reaction will become spontaneous above _____ K.

Correct Answer: (2000)

Solution:

For spontaneity, we use the Gibbs free energy equation where $\Delta G = 0$ at the threshold temperature for spontaneity:

$$T = \frac{\Delta H}{\Delta S} = \frac{400}{0.2} = 2000\,\text{K}$$

Quick Tip

For a reaction to become spontaneous, the Gibbs free energy ΔG must be zero or negative. Calculating the temperature at which $\Delta G = 0$ can help find the threshold for spontaneity.

---

87. Total number of species from the following with central atom utilising sp^2 hybrid orbitals for bonding is ...

NH_3, SO_2, SiO_2, $BeCl_2$, C_2H_2, C_2H_4, BCl_3, HCHO, C_6H_6, BF_3, $C_2H_4Cl_2$

Correct Answer: (6)

Solution:

Central atom utilising sp^2 hybrid orbitals:

SO_2, C_2H_4, BCl_3, HCHO, C_6H_6, BF_3

Quick Tip

sp^2 hybridization occurs when the central atom forms three regions of electron density, usually with one double bond or lone pair contributing to a trigonal planar structure.

---

88. Consider the two different first order reactions given below:

A + B $\rightarrow$ C (Reaction 1)

P $\rightarrow$ Q (Reaction 2)

The ratio of the half-life of Reaction 1 : Reaction 2 is 5 : 2. If t_1 and t_2 represent the time taken to complete $\frac{2}{3}$ and $\frac{4}{5}$ of Reaction 1 and Reaction 2, respectively, then the value of the ratio $t_1 : t_2$ is $\_\_\_ \times 10^{-1}$ (nearest integer).

[Given: $\log_{10}(3) = 0.477$ **and** $\log_{10}(5) = 0.699$**]**

Correct Answer: (17)

Solution:

For first order reactions:

$$K_1 t_1 = \ln\left(\frac{1}{1-\frac{2}{3}}\right) = \ln 3$$

$$K_2 t_2 = \ln\left(\frac{1}{1-\frac{4}{5}}\right) = \ln 5$$

$$\therefore K_1 t_1 = 5 \quad \text{and} \quad K_2 t_2 = 2$$

$$\frac{K_1}{K_2} = \frac{\ln 3}{\ln 5}$$

$$\frac{t_1}{t_2} = \frac{0.477}{0.699} \cdot 5 = 1.7 = 17 \times 10^{-1}$$

Quick Tip

For first order reactions, the time taken for a specific fraction of reaction completion can be determined using the natural logarithm values. Always remember that the half-life ratio can give insights about the rate constants.

---

89. For hydrogen atom, energy of an electron in first excited state is - 3.4 eV, K.E. of the same electron of hydrogen atom is x eV. Value of x is $\_\_\_\_ \times 10^{-1}$ eV. (Nearest integer)

Correct Answer: (34)

Solution:

The total energy (E) of an electron in a hydrogen atom is given by:

$$E = -\frac{13.6}{n^2}\,\text{eV}$$

where n is the principal quantum number.

For the first excited state ($n = 2$):

$$E = -\frac{13.6}{2^2} = -\frac{13.6}{4} = -3.4\,\text{eV}.$$

The total energy (E) of an electron in an atom is related to its potential energy (U) and kinetic energy ($K.E.$) by:

$$E = K.E. + U,$$

where:

$$U = 2E.$$

Thus:

$$K.E. = -E.$$

Substituting $E = -3.4\,\text{eV}$:

$$K.E. = -(-3.4) = 3.4\,\text{eV}.$$

The kinetic energy is expressed as $x \times 10^{-1}\,\text{eV}$. Therefore:

$$x = 34.$$

Final Answer: $x = 34$.

Quick Tip

For hydrogen-like atoms, the kinetic energy in any orbit is half the absolute value of the total energy of the electron in that orbit.

---

90. Among VO_2^+, MnO_4^- and $Cr_2O_7^{2-}$, the spin-only magnetic moment value of the species with least oxidising ability is _____ BM (Nearest integer).

[Given atomic number V = 23, Mn = 25, Cr = 24]

Correct Answer: (0)

Solution:

For 3d transition series:

$$\text{Oxidising power: } V^{+5} < Cr^{+6} < Mn^{+7}$$

For V^{5+} : $[Ar]\, 4s^0\, 3d^0$

Number of unpaired electrons = 0

$$\mu = 0$$

Quick Tip

The magnetic moment for a species with no unpaired electrons is zero. Oxidation states and electron configurations help in predicting magnetic properties.
