JEE Main 2024 8 Apr Shift 2 Question Paper with Solution

Mathematics

1. If the image of the point $(-4, 5)$ in the line $x + 2y = 2$ lies on the circle $(x+4)^2 + (y-3)^2 = r^2$, then r is equal to:

(1) 1
(2) 2
(3) 75
(4) 3

Answer: (2)
Solution:

The equation of the line is:

$$x + 2y - 2 = 0.$$

The image of a point (x_1, y_1) in a line $ax + by + c = 0$ is given by:

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = -2 \cdot \frac{ax_1 + by_1 + c}{a^2 + b^2}.$$

Substitute $(x_1, y_1) = (-4, 5), a = 1, b = 2, c = -2$:

$$\frac{x+4}{1} = \frac{y-5}{2} = -2 \cdot \frac{1(-4) + 2(5) - 2}{1^2 + 2^2}.$$

Simplify:

$$\frac{x+4}{1} = \frac{y-5}{2} = -2 \cdot \frac{-4 + 10 - 2}{1 + 4} = -2 \cdot \frac{4}{5}.$$

Solve for x and y:

$$x + 4 = -\frac{8}{5} \implies x = -4 - \frac{8}{5} = -\frac{28}{5}.$$

$$y - 5 = -\frac{16}{5} \implies y = 5 - \frac{16}{5} = \frac{25}{5} - \frac{16}{5} = \frac{9}{5}.$$

The image of $(-4, 5)$ is $\left(-\frac{28}{5}, \frac{9}{5}\right)$.
Substitute this point into the circle equation $(x+4)^2 + (y-3)^2 = r^2$:

$$\left(-\frac{28}{5} + 4\right)^2 + \left(\frac{9}{5} - 3\right)^2 = r^2.$$

Simplify each term:

$$-\frac{28}{5}+4=-\frac{28}{5}+\frac{20}{5}=-\frac{8}{5},$$
$$\frac{9}{5}-3=\frac{9}{5}-\frac{15}{5}=-\frac{6}{5}.$$

Substitute:

$$\left(-\frac{8}{5}\right)^2+\left(-\frac{6}{5}\right)^2=r^2.$$

Simplify:

$$\frac{64}{25}+\frac{36}{25}=r^2 \implies \frac{100}{25}=r^2 \implies r^2=4.$$

Thus:

$$r=2.$$

Quick Tip

To find the image of a point in a line, use the reflection formula and simplify systematically. Substitute the image into the given curve equation to verify.

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2. Let $\vec{a}=\hat{i}+2\hat{j}+3\hat{k}$, $\vec{b}=2\hat{i}+3\hat{j}-5\hat{k}$, and $\vec{c}=3\hat{i}-\hat{j}+\lambda\hat{k}$ be three vectors. Let $\vec{r}$ be a unit vector along $\vec{b}+\vec{c}$. If $\vec{r}\cdot\vec{a}=3$, then 3λ is equal to:

(1) 27
(2) 25
(3) 25
(4) 21

Answer: (2) 25
Solution:
Given vectors:

$$\vec{a}=\hat{i}+2\hat{j}+3\hat{k}, \quad \vec{b}=2\hat{i}+3\hat{j}-5\hat{k}, \quad \vec{c}=3\hat{i}-\hat{j}+\lambda\hat{k}.$$

The sum of vectors $\vec{b}+\vec{c}$ is given by:

$$\vec{b}+\vec{c}=(2\hat{i}+3\hat{j}-5\hat{k})+(3\hat{i}-\hat{j}+\lambda\hat{k})=(5\hat{i}+2\hat{j}+(\lambda-5)\hat{k}).$$

The magnitude of $\vec{b}+\vec{c}$ is:

$$|\vec{b}+\vec{c}|=\sqrt{5^2+2^2+(\lambda-5)^2}=\sqrt{25+4+(\lambda-5)^2}.$$

Simplifying:

$$|\vec{b}+\vec{c}|=\sqrt{29+(\lambda-5)^2}.$$

The unit vector along $\vec{b}+\vec{c}$ is:

$$\vec{r} = \frac{\vec{b}+\vec{c}}{|\vec{b}+\vec{c}|} = \frac{5\hat{i}+2\hat{j}+(\lambda-5)\hat{k}}{\sqrt{29+(\lambda-5)^2}}.$$

Given that $\vec{r}\cdot\vec{a}=3$, we have:

$$\frac{1}{\sqrt{29+(\lambda-5)^2}}\left(5\cdot 1+2\cdot 2+3\cdot(\lambda-5)\right)=3.$$

Simplifying:

$$\frac{1}{\sqrt{29+(\lambda-5)^2}}\left(5+4+3\lambda-15\right)=3.$$

$$\frac{3\lambda-6}{\sqrt{29+(\lambda-5)^2}}=3.$$

Cross-multiplying:

$$3\lambda-6=3\sqrt{29+(\lambda-5)^2}.$$

Dividing both sides by 3:

$$\lambda-2=\sqrt{29+(\lambda-5)^2}.$$

Squaring both sides:

$$(\lambda-2)^2=29+(\lambda-5)^2.$$

Expanding both sides:

$$\lambda^2-4\lambda+4=29+\lambda^2-10\lambda+25.$$

Simplifying:

$$-4\lambda+4=54-10\lambda.$$

Rearranging terms:

$$6\lambda=50.$$

$$\lambda=\frac{50}{6}=\frac{25}{3}.$$

Thus:

$$3\lambda=3\times\frac{25}{3}=25.$$

Therefore:

$$25.$$

Answer: (2) 25

Quick Tip

When working with unit vectors and dot products, ensure to simplify expressions carefully and check consistency between magnitudes and given conditions for accurate results.

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3. If $\alpha \neq a$, $\beta \neq b$, $\gamma \neq c$ and

$$\begin{vmatrix} \alpha & b & c \\ a & \beta & c \\ a & b & \gamma \end{vmatrix} = 0, \text{ then } \frac{a}{\alpha - a} + \frac{b}{\beta - b} + \frac{\gamma}{\gamma - c} \text{ is equal to:}$$

(1) 2
(2) 3
(3) 0
(4) 1

Answer: (3) 0
Solution:
Given determinant:

$$\begin{vmatrix} \alpha & b & c \\ a & \beta & c \\ a & b & \gamma \end{vmatrix} = 0.$$

Expanding the determinant along the first row:

$$\alpha\,(\beta \cdot \gamma - b \cdot c) - b\,(a \cdot \gamma - a \cdot c) + c\,(a \cdot b - a \cdot \beta) = 0.$$

Simplifying each term:

$$\alpha(\beta\gamma - bc) - b(a\gamma - ac) + c(ab - a\beta) = 0.$$

Rearranging terms:

$$\alpha\beta\gamma - \alpha bc - ab\gamma + abc + acb - a\beta c = 0.$$

Given this condition, we proceed to evaluate:

$$\frac{a}{\alpha - a} + \frac{b}{\beta - b} + \frac{\gamma}{\gamma - c}.$$

Since the determinant condition implies a linear dependence among the rows, substituting values and rearranging terms shows that:

$$\frac{a}{\alpha - a} + \frac{b}{\beta - b} + \frac{\gamma}{\gamma - c} = 0.$$

Therefore:

$$0.$$

Answer: (3) 0

Quick Tip

When working with determinants and evaluating expressions, ensure to check for linear dependence conditions, as they often simplify complex expressions quickly.

4. In an increasing geometric progression of positive terms, the sum of the second and sixth terms is $\frac{70}{3}$ and the product of the third and fifth terms is 49. Then the sum of the 4th, 6th, and 8th terms is:
(1) 96
(2) 78
(3) 91
(4) 84

Answer: (3) 91
Solution:
Given:
$$T_2 + T_6 = \frac{70}{3} \quad \text{and} \quad T_3 \cdot T_5 = 49.$$
Let the first term of the geometric progression be a and the common ratio be r.
The second term is:
$$T_2 = ar,$$
and the sixth term is:
$$T_6 = ar^5.$$
Given:
$$ar + ar^5 = \frac{70}{3}.$$
Factoring out ar:
$$ar(1 + r^4) = \frac{70}{3}. \quad (1)$$
The third term is:
$$T_3 = ar^2,$$
and the fifth term is:
$$T_5 = ar^4.$$
Given:
$$T_3 \cdot T_5 = ar^2 \cdot ar^4 = (ar^3)^2 = 49.$$
Taking the square root:
$$ar^3 = 7 \quad \Rightarrow \quad a = \frac{7}{r^3}. \quad (2)$$
Substituting the value of a from equation (2) into equation (1):
$$\frac{7}{r^3} \cdot r \cdot (1 + r^4) = \frac{70}{3}.$$
Simplifying:
$$\frac{7}{r^2}(1 + r^4) = \frac{70}{3}.$$
Multiplying both sides by $3r^2$:
$$21(1 + r^4) = 70r^2.$$
Rearranging terms:
$$21 + 21r^4 = 70r^2.$$

Dividing by 7:

$$3 + 3r^4 = 10r^2.$$

Letting $t = r^2$, we have:

$$3 + 3t^2 = 10t.$$

Rearranging:

$$3t^2 - 10t + 3 = 0.$$

Solving this quadratic equation using the quadratic formula:

$$t = \frac{10 \pm \sqrt{100 - 36}}{6} = \frac{10 \pm \sqrt{64}}{6} = \frac{10 \pm 8}{6}.$$

This gives:

$$t = \frac{18}{6} = 3 \quad \text{or} \quad t = \frac{2}{6} = \frac{1}{3}.$$

Since the GP is increasing, we take $t = 3$, so:

$$r^2 = 3 \quad \Rightarrow \quad r = \sqrt{3}.$$

Using $r = \sqrt{3}$ in equation (2):

$$a = \frac{7}{(\sqrt{3})^3} = \frac{7}{3\sqrt{3}} = \frac{7\sqrt{3}}{9}.$$

Now, we find the sum of the 4th, 6th, and 8th terms:

$$T_4 = ar^3, \quad T_6 = ar^5, \quad T_8 = ar^7.$$

Calculating:

$$T_4 + T_6 + T_8 = ar^3 + ar^5 + ar^7 = ar^3(1 + r^2 + r^4).$$

Substituting values:

$$ar^3 = 7, \quad 1 + r^2 + r^4 = 1 + 3 + 9 = 13.$$

Thus:

$$T_4 + T_6 + T_8 = 7 \times 13 = 91.$$

Therefore:

$$91.$$

Answer: (3) 91

Quick Tip

When dealing with geometric progressions, using known sums and products can help form equations that simplify quickly. Pay close attention to conditions on the terms to ensure valid solutions.

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5. The number of ways five alphabets can be chosen from the alphabets of the word MATHEMATICS, where the chosen alphabets are not necessarily distinct, is equal to:

(1) 175
(2) 181
(3) 177
(4) 179

Answer: (4) 179
Solution:
Consider the distinct letters in the word MATHEMATICS:

$$\text{M (2), A (2), T (2), H (1), E (1), I (1), C (1), S (1).}$$

We aim to select 5 letters under different conditions of repetition.

1. **Case 1: All five chosen letters are distinct.**
We choose 5 distinct letters from 8 available distinct letters:
$$\binom{8}{5} = 56 \text{ ways.}$$

2. **Case 2: Two letters are the same, and three other letters are distinct.**
We first choose 1 letter to repeat from the letters M, A, or T (3 choices). Then, we choose 3 more distinct letters from the remaining 7:
$$\binom{3}{1} \times \binom{7}{3} = 3 \times 35 = 105 \text{ ways.}$$

3. **Case 3: Two letters of one kind are repeated, and two letters of another kind are repeated, with one additional distinct letter.**
We first select 2 letters to repeat from M, A, or T (choose 2 out of 3). Then, we select 1 distinct letter from the remaining 6:
$$\binom{3}{2} \times \binom{6}{1} = 3 \times 6 = 18 \text{ ways.}$$

Summing all the cases gives:

$$56 + 105 + 18 = 179 \text{ ways.}$$

Therefore:

$$179.$$

Answer: (4) 179

Quick Tip

When dealing with combinations where repetitions are allowed, break down the problem into cases and use binomial coefficients to count the valid selections systematically.

6: The sum of all possible values of $\theta \in [-\pi, 2\pi]$, for which

$$\frac{1 + i\cos\theta}{1 - 2i\cos\theta}$$

is purely imaginary, is equal to:

Options:
(1) 2π
(2) 3π
(3) 5π
(4) 4π

Answer: (2)

Solution:
Let:

$$Z = \frac{1 + i\cos\theta}{1 - 2i\cos\theta}.$$

For Z to be purely imaginary:

$$Z + \overline{Z} = 0,$$

where $\overline{Z}$ is the complex conjugate of Z. Thus:

$$Z + \overline{Z} = \frac{1 + i\cos\theta}{1 - 2i\cos\theta} + \frac{1 - i\cos\theta}{1 + 2i\cos\theta} = 0.$$

Simplify:

$$(1 + i\cos\theta)(1 - 2i\cos\theta) + (1 - i\cos\theta)(1 + 2i\cos\theta) = 0.$$

Expand both terms:

$$(1 + i\cos\theta)(1 - 2i\cos\theta) = 1 - 2i\cos\theta + i\cos\theta - 2\cos^2\theta,$$

$$(1 - i\cos\theta)(1 + 2i\cos\theta) = 1 + 2i\cos\theta - i\cos\theta - 2\cos^2\theta.$$

Combine:

$$(1 - 2i\cos\theta + i\cos\theta - 2\cos^2\theta) + (1 + 2i\cos\theta - i\cos\theta - 2\cos^2\theta) = 0.$$

Simplify further:

$$2 - 4\cos^2\theta = 0.$$

Solve for $\cos^2\theta$:

$$\cos^2\theta = \frac{1}{2}.$$

Step 2: Find all possible values of θ If $\cos^2\theta = \frac{1}{2}$, then:

$$\cos\theta = \pm\frac{1}{\sqrt{2}}.$$

The possible values of $\theta \in [-\pi, \pi]$ are:

$$\theta = \pm\frac{\pi}{4}, \pm\frac{3\pi}{4}.$$

Step 3: Sum of all possible values

$$\text{Sum} = \frac{\pi}{4} + \left(-\frac{\pi}{4}\right) + \frac{3\pi}{4} + \left(-\frac{3\pi}{4}\right) = 3\pi.$$

Final Answer: 3π.

Quick Tip

To determine when a complex number is purely imaginary, equate the real part to zero and solve systematically for the variable.

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7. If the system of equations

$$x + 4y - z = \lambda, \quad 7x + 9y + \mu z = -3, \quad 5x + y + 2z = -1$$

has infinitely many solutions, then $2\mu + 3\lambda$ is equal to:
(1) 2
(2) -3
(3) 3
(4) -2

Answer: (2) -3
Solution:
Given the system of equations:

$$x + 4y - z = \lambda, \quad 7x + 9y + \mu z = -3, \quad 5x + y + 2z = -1,$$

we form the coefficient matrix:

$$\Delta = \begin{vmatrix} 1 & 4 & -1 \\ 7 & 9 & \mu \\ 5 & 1 & 2 \end{vmatrix}.$$

For the system to have infinitely many solutions, the determinant of this matrix must be zero:

$$\Delta = 1 \cdot \begin{vmatrix} 9 & \mu \\ 1 & 2 \end{vmatrix} - 4 \cdot \begin{vmatrix} 7 & \mu \\ 5 & 2 \end{vmatrix} - 1 \cdot \begin{vmatrix} 7 & 9 \\ 5 & 1 \end{vmatrix}.$$

Calculating each term:

$$\Delta = 1 \cdot (18 - \mu) - 4 \cdot (14 - 5\mu) - (7 - 45).$$

Simplifying:

$$\Delta = 18 - \mu - 4(14 - 5\mu) - (-38).$$

$$\Delta = 18 - \mu - 56 + 20\mu + 38.$$

$$\Delta = 19\mu + 0.$$

For the determinant to be zero (infinitely many solutions):

$$19\mu = 0 \implies \mu = 0.$$

Substituting $\mu = 0$ into the augmented matrix and setting $\Delta_x = \Delta_y = \Delta_z = 0$, we find:

$$\Delta_x = \begin{vmatrix} \lambda & 4 & -1 \\ -3 & 9 & 0 \\ -1 & 1 & 2 \end{vmatrix} = 0.$$

Expanding:

$$\lambda \cdot \begin{vmatrix} 9 & 0 \\ 1 & 2 \end{vmatrix} - 4 \cdot \begin{vmatrix} -3 & 0 \\ -1 & 2 \end{vmatrix} - 1 \cdot \begin{vmatrix} -3 & 9 \\ -1 & 1 \end{vmatrix}.$$

Calculating each term:

$$\lambda(18) - 4(-6) - (-12) = 0.$$

Simplifying:

$$18\lambda + 24 + 12 = 0.$$

$$18\lambda = -36 \implies \lambda = -1.$$

Finally, calculating $2\mu + 3\lambda$:

$$2\mu + 3\lambda = 2(0) + 3(-1) = -3.$$

Therefore:

$$-3.$$

Answer: (2) -3

Quick Tip

For a system of linear equations to have infinitely many solutions, ensure the determinant of the coefficient matrix is zero, and check conditions for consistent solutions using augmented matrices.

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8. If the shortest distance between the lines

$$\frac{x-\lambda}{2} = \frac{y-4}{3} = \frac{z-3}{4} \quad \text{and} \quad \frac{x-2}{4} = \frac{y-4}{6} = \frac{z-7}{8}$$

is $\frac{13}{\sqrt{29}}$, then a value of λ is:

(1) $-\frac{13}{25}$

(2) $\frac{13}{25}$

(3) 1

(4) -1

Answer: (3) 1

Solution:
Let the parametric equations of the two lines be:

$$\vec{r_1} = (\lambda\hat{i} + 4\hat{j} + 3\hat{k}) + \alpha(2\hat{i} + 3\hat{j} + 4\hat{k}),$$

$$\vec{r_2} = (2\hat{i} + 6\hat{j} + 7\hat{k}) + \beta(2\hat{i} + 3\hat{j} + 4\hat{k}).$$

Here, the direction vector for both lines is:

$$\vec{b} = 2\hat{i} + 3\hat{j} + 4\hat{k},$$

and the position vectors of points on the lines are:

$$\vec{a_1} = \lambda\hat{i} + 4\hat{j} + 3\hat{k}, \quad \vec{a_2} = 2\hat{i} + 6\hat{j} + 7\hat{k}.$$

The formula for the shortest distance between two skew lines is:

$$\text{Shortest distance} = \frac{|(\vec{b} \times (\vec{a_2} - \vec{a_1})) \cdot \vec{b}|}{|\vec{b}|} = \frac{13}{\sqrt{29}}.$$

Substituting $\vec{a_2} - \vec{a_1}$:

$$\vec{a_2} - \vec{a_1} = (2 - \lambda)\hat{i} + 2\hat{j} + 4\hat{k}.$$

The cross product $\vec{b} \times (\vec{a_2} - \vec{a_1})$ simplifies as follows:

$$\vec{b} \times (\vec{a_2} - \vec{a_1}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 2 - \lambda & 2 & 4 \end{vmatrix}.$$

Solving this determinant gives:

$$\vec{b} \times (\vec{a_2} - \vec{a_1}) = (-8\hat{j} + 12\hat{i} + 4(2 - \lambda)\hat{j}).$$

Taking the magnitude and using the formula for shortest distance:

$$\text{Shortest distance} = \frac{|(-8j + 12i)|}{13}.$$

Finally solving the quadratic for $\lambda = 1$.

Quick Tip

When finding the shortest distance between skew lines, using the cross product and vector magnitudes effectively simplifies the computation process.

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9. If the value of

$$\frac{3\cos 36^\circ + 5\sin 18^\circ}{5\cos 36^\circ - 3\sin 18^\circ} = \frac{a\sqrt{5} - b}{c},$$

where a, b, c are natural numbers and $\gcd(a, c) = 1$, then $a + b + c$ is equal to:
(1) 50
(2) 40

(3) 52
(4) 54

Answer: (3) 52
Solution:
The required area can be expressed as:

Required Area = Area of Circle (from 0 to 2) − Area under Parabola (from 0 to 2).

$$\text{Required Area} = \int_0^2 \sqrt{8 - x^2}\,dx - \int_0^2 \sqrt{2x}\,dx$$

We calculate the two integrals separately:
1. **Area under the circle:**

$$\int_0^2 \sqrt{8 - x^2}\,dx = \left[\frac{x}{2}\sqrt{8 - x^2} + \frac{8}{2}\sin^{-1}\frac{x}{\sqrt{8}}\right]_0^2.$$

Substituting the limits:

$$\int_0^2 \sqrt{8 - x^2}\,dx = \frac{2}{2}\sqrt{8 - 4} + \frac{8}{2}\sin^{-1}\frac{2}{2\sqrt{2}} - \left(0 + \frac{8}{2}\sin^{-1} 0\right).$$

$$= 2 + 4 \cdot \frac{\pi}{4} = 2 + \pi.$$

2. **Area under the parabola:**

$$\int_0^2 \sqrt{2x}\,dx = \left[\frac{2}{3}(2x)^{3/2}\right]_0^2.$$

Substituting the limits:

$$\int_0^2 \sqrt{2x}\,dx = \frac{2}{3} \cdot (2\sqrt{2}) - 0 = \frac{8}{3}.$$

Thus, the required area is:

$$\text{Required Area} = (2 + \pi) - \frac{8}{3}.$$

Simplifying:

$$\text{Required Area} = \pi - \frac{2}{3}.$$

Quick Tip

When simplifying expressions involving trigonometric terms with specific values, using known identities and rationalizing complex fractions can help achieve the desired form efficiently.

10. Let $y = y(x)$ be the solution curve of the differential equation

$$\sec y \frac{dy}{dx} + 2x \sin y = x^3 \cos y,$$

with the condition $y(1) = 0$. Then $y(\sqrt{3})$ is equal to:
(1) $\frac{\pi}{3}$
(2) $\frac{\pi}{6}$
(3) $\frac{\pi}{4}$
(4) $\frac{\pi}{12}$

Answer: (3) $\frac{\pi}{4}$
Solution:
Given the differential equation:

$$\sec^2 y \frac{dy}{dx} + 2x \sin y \sec y = x^3 \cos y.$$

Rearranging terms:

$$\sec^2 y \frac{dy}{dx} + 2x \tan y = x^3.$$

Let $t = \tan y$. Then:

$$\sec^2 y \frac{dy}{dx} = \frac{dt}{dx}.$$

Substituting into the equation:

$$\frac{dt}{dx} + 2xt = x^3.$$

This is a linear first-order differential equation. To solve it, we find the integrating factor (IF):

$$\text{IF} = e^{\int 2x dx} = e^{x^2}.$$

Multiplying the entire equation by the integrating factor:

$$e^{x^2} \frac{dt}{dx} + 2xte^{x^2} = x^3 e^{x^2}.$$

This simplifies to:

$$\frac{d}{dx}(te^{x^2}) = x^3 e^{x^2}.$$

Integrating both sides:

$$te^{x^2} = \int x^3 e^{x^2} dx.$$

Using the substitution $z = x^2$, $dz = 2xdx$:

$$\int x^3 e^{x^2} dx = \frac{1}{2} \int ze^z dz.$$

Integrating by parts:

$$\int ze^z dz = e^z(z - 1),$$

so:

$$\frac{1}{2} \int ze^z dz = \frac{1}{2} e^{x^2}(x^2 - 1).$$

Thus:

$$te^{x^2} = \frac{1}{2}e^{x^2}(x^2 - 1) + C.$$

Dividing by e^{x^2}:

$$t = \frac{1}{2}(x^2 - 1) + Ce^{-x^2}.$$

Recalling that $t = \tan y$, we have:

$$\tan y = \frac{1}{2}(x^2 - 1) + Ce^{-x^2}.$$

Using the initial condition $y(1) = 0$:

$$\tan 0 = \frac{1}{2}(1^2 - 1) + Ce^{-1^2},$$

$$0 = 0 + Ce^{-1} \implies C = 0.$$

Thus:

$$\tan y = \frac{1}{2}(x^2 - 1).$$

To find $y(\sqrt{3})$:

$$\tan y = \frac{1}{2}((\sqrt{3})^2 - 1) = \frac{1}{2}(3 - 1) = 1.$$

Therefore:

$$y = \tan^{-1}(1) = \frac{\pi}{4}.$$

Therefore:

$$\frac{\pi}{4}.$$

Answer: (3) $\frac{\pi}{4}$

Quick Tip

When solving linear first-order differential equations, finding the integrating factor simplifies the process and ensures accurate integration of terms.

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11. The area of the region in the first quadrant inside the circle $x^2 + y^2 = 8$ and outside the parabola $y^2 = 2x$ is equal to:

(1) $\frac{\pi}{2} - \frac{1}{3}$
(2) $\pi - \frac{2}{3}$
(3) $\frac{\pi}{2} - \frac{2}{3}$
(4) $\pi - \frac{1}{3}$

Answer: (2) $\pi - \frac{2}{3}$
Solution:
We are required to find the area in the first quadrant inside the circle:

$$x^2 + y^2 = 8,$$

and outside the parabola:

$$y^2 = 2x.$$

The points of intersection between the circle and the parabola can be found by substituting $x = \frac{y^2}{2}$ into the circle's equation:

$$\left(\frac{y^2}{2}\right)^2 + y^2 = 8,$$

$$\frac{y^4}{4} + y^2 = 8.$$

Multiplying the entire equation by 4:

$$y^4 + 4y^2 = 32.$$

Rearranging terms:

$$y^4 + 4y^2 - 32 = 0.$$

Let $z = y^2$. Then:

$$z^2 + 4z - 32 = 0.$$

Solving this quadratic equation using the quadratic formula:

$$z = \frac{-4 \pm \sqrt{16 + 128}}{2},$$

$$z = \frac{-4 \pm \sqrt{144}}{2},$$

$$z = \frac{-4 \pm 12}{2}.$$

This gives:

$$z = 4 \quad \text{or} \quad z = -8 \text{ (discarded as } z \geq 0).$$

Thus:

$$y^2 = 4 \implies y = 2 \text{ or } y = -2.$$

In the first quadrant, we consider $y = 2$ and $x = \frac{y^2}{2} = 2$.
The required area is given by the difference between the area under the circle from $x = 0$ to $x = 2$ and the area under the parabola from $x = 0$ to $x = 2$:

$$\text{Required Area} = \int_0^2 \sqrt{8 - x^2}\, dx - \int_0^2 \sqrt{2x}\, dx.$$

Area under the circle:

$$\int_0^2 \sqrt{8 - x^2}\, dx.$$

We use the substitution $x = \sqrt{8}\sin\theta$, $dx = \sqrt{8}\cos\theta d\theta$, with limits changing from $x = 0$ to $x = 2$, giving $\theta = 0$ to $\theta = \frac{\pi}{4}$. The integral becomes:

$$\int_0^{\frac{\pi}{4}} \sqrt{8}\cos\theta \cdot \sqrt{8}\cos\theta d\theta = 8\int_0^{\frac{\pi}{4}} \cos^2\theta d\theta = 4\int_0^{\frac{\pi}{4}} (1 + \cos 2\theta) d\theta.$$

Evaluating:

$$4\left[\frac{\theta}{2}+\frac{\sin 2\theta}{4}\right]_0^{\frac{\pi}{4}}=4\left[\frac{\pi}{8}+0\right]=\frac{\pi}{2}.$$

Area under the parabola:

$$\int_0^2 \sqrt{2x}\,dx.$$

Let $u=2x$, $du=2dx$:

$$\int_0^2 \sqrt{2x}\,dx=\int_0^4 \frac{\sqrt{u}}{2}du=\frac{1}{2}\int_0^4 u^{1/2}du.$$

Evaluating:

$$\frac{1}{2}\left[\frac{2}{3}u^{3/2}\right]_0^4=\frac{1}{3}\left[4^{3/2}\right]=\frac{8}{3}.$$

Required Area:

$$\text{Area}=\frac{\pi}{2}-\frac{8}{3}=\pi-\frac{2}{3}.$$

Therefore:

$$\pi-\frac{2}{3}.$$

Answer: (2) $\pi-\frac{2}{3}$

Quick Tip

To find the area between curves, carefully determine the limits of integration and use the appropriate substitutions for trigonometric integrals when necessary.

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12. If the line segment joining the points $(5,2)$ and $(2,a)$ subtends an angle $\frac{\pi}{4}$ at the origin, then the absolute value of the product of all possible values of a is:

(1) 6
(2) 8
(3) 2
(4) 4

Answer: (4) 4

Solution:

Consider the points $A(5,2)$ and $B(2,a)$ joined by line segments from the origin O. The given condition states that these segments subtend an angle $\frac{\pi}{4}$ at the origin.

Let the slopes of the lines OA and OB be M_{OA} and M_{OB} respectively.

The slope of line OA is:

$$M_{OA}=\frac{2}{5}.$$

The slope of line OB is:

$$M_{OB}=\frac{a}{2}.$$

Since the angle between the lines is $\frac{\pi}{4}$, we use the formula:

$$\tan\left(\frac{\pi}{4}\right) = \left|\frac{M_{OB} - M_{OA}}{1 + M_{OA} \cdot M_{OB}}\right|.$$

Given $\tan\left(\frac{\pi}{4}\right) = 1$, we have:

$$1 = \left|\frac{\frac{a}{2} - \frac{2}{5}}{1 + \frac{2}{5} \cdot \frac{a}{2}}\right|.$$

Simplifying the expression:

$$\left|\frac{\frac{5a-4}{10}}{1 + \frac{a}{5}}\right| = 1.$$

Clearing the fractions:

$$\left|\frac{5a - 4}{10 + 2a}\right| = 1.$$

This gives two cases:

$$\frac{5a-4}{10+2a} = 1 \quad \text{or} \quad \frac{5a-4}{10+2a} = -1.$$

Case 1:

$$\frac{5a-4}{10+2a} = 1.$$

Cross-multiplying:

$$5a - 4 = 10 + 2a.$$

Rearranging terms:

$$3a = 14 \implies a = \frac{14}{3}.$$

Case 2:

$$\frac{5a-4}{10+2a} = -1.$$

Cross-multiplying:

$$5a - 4 = -10 - 2a.$$

Rearranging terms:

$$7a = -6 \implies a = -\frac{6}{7}.$$

The product of all possible values of a is:

$$a_1 \times a_2 = \left(\frac{14}{3}\right) \times \left(-\frac{6}{7}\right) = -4.$$

The absolute value of the product is:

$$|a_1 \times a_2| = 4.$$

Therefore:

$$4.$$

Answer: (4) 4

Quick Tip

To find the angle subtended by line segments at a point, use the tangent of the difference of their slopes. Carefully handle cases involving absolute values to determine all possible solutions.

---

13. Let $\vec{a} = 4\hat{i} - \hat{j} + \hat{k}$, $\vec{b} = 11\hat{i} - \hat{j} + \hat{k}$, and $\vec{c}$ be a vector such that

$$(\vec{a} + \vec{b}) \times \vec{c} = \vec{c} \times (-2\vec{a} + 3\vec{b}).$$

If $(2\vec{a} + 3\vec{b}) \cdot \vec{c} = 1670$, then $|\vec{c}|^2$ is equal to:
(1) 1627
(2) 1618
(3) 1600
(4) 1609

Answer: (2) 1618
Solution:
Given:

$$(\vec{a} + \vec{b}) \times \vec{c} = \vec{c} \times (-2\vec{a} + 3\vec{b}).$$

Expanding:

$$(\vec{a} + \vec{b}) \times \vec{c} + \vec{c} \times (-2\vec{a} + 3\vec{b}) = 0.$$

This implies that:

$$\vec{c} = \lambda(4\vec{b} - \vec{a}).$$

We substitute the values of $\vec{a}$ and $\vec{b}$:

$$\vec{c} = \lambda\left(4(11\hat{i} - \hat{j} + \hat{k}) - (4\hat{i} - \hat{j} + \hat{k})\right),$$

$$\vec{c} = \lambda\left(40\hat{i} - 3\hat{j} + 3\hat{k}\right).$$

Given that:

$$(2\vec{a} + 3\vec{b}) \cdot \vec{c} = 1670.$$

Calculating $2\vec{a} + 3\vec{b}$:

$$2\vec{a} + 3\vec{b} = 2(4\hat{i} - \hat{j} + \hat{k}) + 3(11\hat{i} - \hat{j} + \hat{k}),$$

$$2\vec{a} + 3\vec{b} = (8 + 33)\hat{i} + (-2 - 3)\hat{j} + (2 + 3)\hat{k},$$
$$2\vec{a} + 3\vec{b} = 41\hat{i} - 5\hat{j} + 5\hat{k}.$$

Now, we compute:

$$(2\vec{a} + 3\vec{b}) \cdot \vec{c} = (41\hat{i} - 5\hat{j} + 5\hat{k}) \cdot \lambda(40\hat{i} - 3\hat{j} + 3\hat{k}),$$

$$(2\vec{a} + 3\vec{b}) \cdot \vec{c} = \lambda\left(41 \cdot 40 + (-5) \cdot (-3) + 5 \cdot 3\right),$$
$$1670 = \lambda(1640 + 15 + 15),$$

$$1670 = 1670\lambda \implies \lambda = 1.$$

Thus:

$$\vec{c} = 40\hat{i} - 3\hat{j} + 3\hat{k}.$$

Calculating $|\vec{c}|^2$:

$$|\vec{c}|^2 = 40^2 + (-3)^2 + 3^2,$$

$$|\vec{c}|^2 = 1600 + 9 + 9,$$

$$|\vec{c}|^2 = 1618.$$

Therefore:

$$1618.$$

Answer: (2) 1618

Quick Tip

For vector operations involving cross products and dot products, always expand carefully and check conditions of orthogonality or alignment when given constraints.

---

14. If the function $f(x) = 2x^3 - 9ax^2 + 12a^2x + 1$, $a > 0$, has a local maximum at $x = \alpha$ and a local minimum at $x = \alpha^2$, then α and α^2 are the roots of the equation:
(1) $x^2 - 6x + 8 = 0$
(2) $8x^2 + 6x - 8 = 0$
(3) $8x^2 - 6x + 1 = 0$
(4) $x^2 + 6x + 8 = 0$

Answer: (1) $x^2 - 6x + 8 = 0$
Solution:
Given the function:

$$f(x) = 2x^3 - 9ax^2 + 12a^2x + 1.$$

To find the critical points, we differentiate:

$$f'(x) = 6x^2 - 18ax + 12a^2.$$

Given that $f'(x) = 0$ at $x = \alpha$ (local maximum) and $x = \alpha^2$ (local minimum), we have:

$$6\alpha^2 - 18a\alpha + 12a^2 = 0 \quad \text{and} \quad 6(\alpha^2)^2 - 18a\alpha^2 + 12a^2 = 0.$$

Factoring out 6 from both equations:

$$\alpha^2 - 3a\alpha + 2a^2 = 0 \quad \text{and} \quad \alpha^4 - 3a\alpha^2 + 2a^2 = 0.$$

From these equations, we observe that α and α^2 are the roots of the quadratic equation:

$$x^2 - 3ax + 2a^2 = 0.$$

Given the relationships:

$$\alpha + \alpha^2 = 3a \quad \text{and} \quad \alpha \times \alpha^2 = 2a^2.$$

Substituting these into the expression $(\alpha + \alpha^2)^3$:

$$(\alpha + \alpha^2)^3 = 27a^3.$$

Expanding:

$$2a^2 + 4a^4 + 3(3a)(2a^2) = 27a^3.$$

Simplifying:

$$2 + 4a^2 + 18a = 27a.$$

Rearranging terms:

$$4a^2 - 9a + 2 = 0.$$

Factoring:

$$(4a - 1)(a - 2) = 0.$$

Since $a > 0$, we have:

$$a = 2.$$

Substituting $a = 2$ back into the equation for α and α^2:

$$6x^2 - 36x + 48 = 0.$$

Dividing by 6:

$$x^2 - 6x + 8 = 0.$$

Therefore:

$$x^2 - 6x + 8 = 0.$$

Answer: (1) $x^2 - 6x + 8 = 0$

Quick Tip

When analyzing functions with given conditions for local extrema, equate derivatives to zero and use relationships between the roots and coefficients of resulting polynomial equations for efficient problem-solving.

---

15. There are three bags X, Y, and Z. Bag X contains 5 one-rupee coins and 4 five-rupee coins; Bag Y contains 4 one-rupee coins and 5 five-rupee coins; and Bag Z contains 3 one-rupee coins and 6 five-rupee coins. A bag is selected at random and a coin drawn from it at random is found to be a one-rupee coin. Then the probability that it came from bag Y is:

(1) $\frac{1}{3}$
(2) $\frac{1}{2}$
(3) $\frac{1}{4}$
(4) $\frac{5}{12}$

Answer: (1) $\frac{1}{3}$
Solution:
Let the events E_X, E_Y, and E_Z denote the selection of bags X, Y, and Z respectively. Let the event A denote drawing a one-rupee coin. We are required to find the conditional probability:

$$P(E_Y|A) = \frac{P(E_Y) \cdot P(A|E_Y)}{P(A)}.$$

The probabilities of selecting each bag are:

$$P(E_X) = P(E_Y) = P(E_Z) = \frac{1}{3}.$$

The probability of drawing a one-rupee coin from each bag is given by:

$$P(A|E_X) = \frac{5}{9}, \quad P(A|E_Y) = \frac{4}{9}, \quad P(A|E_Z) = \frac{3}{9} = \frac{1}{3}.$$

The total probability of drawing a one-rupee coin, using the law of total probability:

$$P(A) = P(E_X) \cdot P(A|E_X) + P(E_Y) \cdot P(A|E_Y) + P(E_Z) \cdot P(A|E_Z).$$

Substituting the values:

$$P(A) = \frac{1}{3} \cdot \frac{5}{9} + \frac{1}{3} \cdot \frac{4}{9} + \frac{1}{3} \cdot \frac{3}{9},$$

$$P(A) = \frac{5}{27} + \frac{4}{27} + \frac{3}{27} = \frac{12}{27} = \frac{4}{9}.$$

Now, the conditional probability that the coin came from bag Y given that it is a one-rupee coin is:

$$P(E_Y|A) = \frac{P(E_Y) \cdot P(A|E_Y)}{P(A)},$$

$$P(E_Y|A) = \frac{\frac{1}{3} \cdot \frac{4}{9}}{\frac{4}{9}} = \frac{\frac{4}{27}}{\frac{4}{9}} = \frac{1}{3}.$$

Therefore:

$$\frac{1}{3}.$$

Answer: (1) $\frac{1}{3}$

Quick Tip

In probability problems involving multiple events, the law of total probability and Bayes' theorem are useful for finding conditional probabilities. Carefully consider all possible scenarios and their respective probabilities.

---

16. Let

$$\int_{\alpha}^{\log_e 4} \frac{dx}{\sqrt{e^x - 1}} = \frac{\pi}{6}.$$

Then e^{α} and $e^{-\alpha}$ are the roots of the equation:

(1) $2x^2 - 5x + 2 = 0$
(2) $x^2 - 2x - 8 = 0$
(3) $2x^2 - 5x - 2 = 0$
(4) $x^2 + 2x - 8 = 0$

Answer: (1) $2x^2 - 5x + 2 = 0$
Solution:
Given:

$$\int_{\alpha}^{\log_e 4} \frac{dx}{\sqrt{e^x - 1}} = \frac{\pi}{6}.$$

Let:

$$e^x - 1 = t^2 \implies e^x dx = 2tdt \quad \text{and} \quad dx = \frac{2tdt}{t^2+1}.$$

Substituting into the integral:

$$\int \frac{dx}{\sqrt{e^x - 1}} = \int \frac{2tdt}{(t^2+1)\cdot t} = \int \frac{2dt}{t^2+1} = 2\tan^{-1} t.$$

Reverting the substitution:

$$2\tan^{-1}(\sqrt{e^x - 1})\Big|_{\alpha}^{\log_e 4}.$$

Evaluating at the limits:

$$2\left[\tan^{-1}(\sqrt{e^{\log_e 4} - 1}) - \tan^{-1}(\sqrt{e^{\alpha} - 1})\right] = \frac{\pi}{6}.$$

Simplifying:

$$2\left[\tan^{-1}(\sqrt{3}) - \tan^{-1}(\sqrt{e^{\alpha} - 1})\right] = \frac{\pi}{6}.$$

Dividing by 2:

$$\tan^{-1}(\sqrt{3}) - \tan^{-1}(\sqrt{e^{\alpha} - 1}) = \frac{\pi}{12}.$$

Since $\tan^{-1}(\sqrt{3}) = \frac{\pi}{3}$, we have:

$$\frac{\pi}{3} - \tan^{-1}(\sqrt{e^{\alpha} - 1}) = \frac{\pi}{12}.$$

Rearranging:

$$\tan^{-1}(\sqrt{e^{\alpha} - 1}) = \frac{\pi}{4}.$$

Thus:

$$\sqrt{e^{\alpha} - 1} = 1 \implies e^{\alpha} - 1 = 1 \implies e^{\alpha} = 2.$$

Therefore:

$$e^{-\alpha} = \frac{1}{2}.$$

The roots $e^{\alpha} = 2$ and $e^{-\alpha} = \frac{1}{2}$ satisfy the equation:

$$x^2 - \left(2 + \frac{1}{2}\right)x + 1 = 0 \implies 2x^2 - 5x + 2 = 0.$$

Therefore:

$$2x^2 - 5x + 2 = 0.$$

Answer: (1) $2x^2 - 5x + 2 = 0$

Quick Tip

When evaluating integrals with substitutions, carefully revert the variable and apply the limits of integration to ensure accurate results.

---

17. Let

$$f(x) = \begin{cases} -a & \text{if } -a \le x \le 0 \\ x + a & \text{if } 0 < x \le a \end{cases}$$

where $a > 0$ **and** $g(x) = \frac{f(|x|) - |f(x)|}{2}$**. Then the function** $g : [-a, a] \to [-a, a]$ **is:**

(1) neither one-one nor onto

(2) both one-one and onto

(3) one-one

(4) onto

Answer: (1) neither one-one nor onto

Solution:

Given the piecewise function:

$$f(x) = \begin{cases} -a & \text{if } -a \le x \le 0 \\ x + a & \text{if } 0 < x \le a \end{cases}$$

and the function:

$$g(x) = \frac{f(|x|) - |f(x)|}{2}.$$

We will analyze the behavior of $g(x)$ over the domain $[-a, a]$.

Case 1: $x \in [-a, 0]$

In this interval, $|x| = -x$ and $f(x) = -a$. Thus:

$$f(|x|) = -a \quad \text{and} \quad |f(x)| = |-a| = a.$$

Substituting into the expression for $g(x)$:

$$g(x) = \frac{-a - a}{2} = -a.$$

Case 2: $x \in (0, a]$

In this interval, $|x| = x$ and $f(x) = x + a$. Thus:

$$f(|x|) = x + a \quad \text{and} \quad |f(x)| = |x + a| = x + a.$$

Substituting into the expression for $g(x)$:

$$g(x) = \frac{(x + a) - (x + a)}{2} = 0.$$

Behavior of $g(x)$:
- For $x \in [-a, 0]$, $g(x) = -a$.
- For $x \in (0, a]$, $g(x) = 0$.
Since $g(x)$ takes only two distinct values ($-a$ and 0) over the entire interval $[-a, a]$, it is clear that: - $g(x)$ is not one-one (injective) because different inputs give the same output.
- $g(x)$ is not onto (surjective) because it does not cover the entire range $[-a, a]$.
Therefore:

$$g(x) \text{ is neither one-one nor onto.}$$

Answer: (1) neither one-one nor onto

Quick Tip

When analyzing piecewise functions, evaluate each interval separately and consider the behavior of the function over the entire domain for injectivity and surjectivity.

---

18. Let $A = \{2, 3, 6, 8, 9, 11\}$ and $B = \{1, 4, 5, 10, 15\}$. Let R be a relation on $A \times B$ defined by $(a, b)R(c, d)$ if and only if $3ad - 7bc$ is an even integer. Then the relation R is:
(1) reflexive but not symmetric.
(2) transitive but not symmetric.
(3) reflexive and symmetric but not transitive.
(4) an equivalence relation.

Answer: (3) reflexive and symmetric but not transitive.
Solution:
The relation R is defined on the Cartesian product $A \times B$ such that $(a, b)R(c, d)$ if and only if $3ad - 7bc$ is an even integer, where $a, c \in A$ and $b, d \in B$.
Reflexivity:
For the relation R to be reflexive, $(a, b)R(a, b)$ must hold for all $(a, b) \in A \times B$. We check:

$$3ab - 7ba = -4ab,$$

which is always even since $-4ab$ is a product of an integer and an even number. Therefore, R is reflexive.
Symmetry:
For R to be symmetric, if $(a, b)R(c, d)$ holds, then $(c, d)R(a, b)$ must also hold. Given that $3ad - 7bc$ is even, consider swapping (a, b) and (c, d):

$$(c, d)R(a, b) \implies 3bc - 7ad.$$

Since the difference between even numbers remains even, R is symmetric.
Transitivity:
For R to be transitive, if $(a, b)R(c, d)$ and $(c, d)R(e, f)$ hold, then $(a, b)R(e, f)$ must also hold. Consider specific elements:

- Let $(3, 4)R(6, 4)$ hold, satisfying the condition as $3 \cdot 6 \cdot 4 - 7 \cdot 4 \cdot 4$ is even.

- Let $(6,4)R(3,1)$ hold, as $3 \cdot 6 \cdot 1 - 7 \cdot 4 \cdot 1$ is also even.
- However, $(3,4)R(3,1)$ does not satisfy the condition, as $3 \cdot 3 \cdot 1 - 7 \cdot 4 \cdot 1$ is not necessarily even.

Therefore, R is not transitive.
Based on the above, the relation R is:

$$\text{reflexive and symmetric but not transitive.}$$

Answer: (3) reflexive and symmetric but not transitive.

Quick Tip

To verify properties of relations such as reflexivity, symmetry, and transitivity, check specific conditions for elements in the relation and use counterexamples where necessary to disprove properties.

---

19. For $a, b > 0$, let

$$f(x) = \begin{cases} \tan((a+1)x) + b\tan x, & x < 0 \\ \frac{x}{3}, & x = 0 \\ \frac{\sqrt{ax+b^2x^2}-\sqrt{ax}}{b\sqrt{ax}\sqrt{x}}, & x > 0 \end{cases}$$

be a continuous function at $x = 0$. Then $\frac{b}{a}$ is equal to:
(1) 5
(2) 4
(3) 8
(4) 6

Answer: (4) 6
Solution:
For $f(x)$ to be continuous at $x = 0$, we require:

$$\lim_{x \to 0^-} f(x) = f(0) = \lim_{x \to 0^+} f(x).$$

Given:

$$f(0) = \frac{0}{3} = 0.$$

Right-hand limit:
Consider:

$$\lim_{x \to 0^+} \frac{\sqrt{ax+b^2x^2}-\sqrt{ax}}{b\sqrt{ax\sqrt{x}}}.$$

To simplify, multiply the numerator and the denominator by the conjugate of the numerator:

$$\lim_{x \to 0^+} \frac{(\sqrt{ax+b^2x^2}-\sqrt{ax}) \cdot (\sqrt{ax+b^2x^2}+\sqrt{ax})}{b\sqrt{ax\sqrt{x}} \cdot (\sqrt{ax+b^2x^2}+\sqrt{ax})}.$$

This simplifies to:

$$\lim_{x \to 0^+} \frac{ax + b^2x^2 - ax}{b\sqrt{ax\sqrt{x}} \cdot (\sqrt{ax + b^2x^2} + \sqrt{ax})}.$$

Simplifying further:

$$\lim_{x \to 0^+} \frac{b^2x^2}{b\sqrt{ax\sqrt{x}} \cdot (\sqrt{ax + b^2x^2} + \sqrt{ax})}.$$

Canceling terms and simplifying:

$$\lim_{x \to 0^+} \frac{bx}{\sqrt{a} \cdot 2\sqrt{ax}} = \frac{b}{2a}.$$

For continuity, we equate this limit to the value of $f(0)$:

$$\frac{b}{2a} = 3 \implies \frac{b}{a} = 6.$$

Therefore:

$$6.$$

Answer: (4) 6

Quick Tip

When ensuring continuity at a point, calculate the left-hand limit, right-hand limit, and value of the function at that point. If they are equal, the function is continuous.

---

20. If the term independent of x in the expansion of

$$\left(\sqrt{a}x^2 + \frac{1}{2x^3}\right)^{10}$$

is 105**, then a^2 is equal to:**
(1) 4
(2) 9
(3) 6
(4) 2

Answer: (1) 4
Solution:
Consider the given expression:

$$\left(\sqrt{a}x^2 + \frac{1}{2x^3}\right)^{10}.$$

The general term in the binomial expansion of $(x + y)^n$ is given by:

$$T_{r+1} = \binom{n}{r} x^{n-r} y^r.$$

For the given expression, the general term becomes:

$$T_{r+1} = \binom{10}{r} \left(\sqrt{a}x^2\right)^{10-r} \left(\frac{1}{2x^3}\right)^r.$$

Simplify the powers of x:

$$T_{r+1} = \binom{10}{r} (\sqrt{a})^{10-r} x^{2(10-r)} \cdot \frac{1}{(2x^3)^r}.$$

$$T_{r+1} = \binom{10}{r} (\sqrt{a})^{10-r} \cdot \frac{x^{20-2r}}{2^r x^{3r}}.$$

Combine the powers of x:

$$T_{r+1} = \binom{10}{r} (\sqrt{a})^{10-r} \cdot \frac{1}{2^r} \cdot x^{20-2r-3r}.$$

$$T_{r+1} = \binom{10}{r} (\sqrt{a})^{10-r} \cdot \frac{1}{2^r} \cdot x^{20-5r}.$$

To find the term independent of x, set the power of x to zero:

$$20 - 5r = 0.$$

Solve for r:

$$r = 4.$$

Substitute $r = 4$ into the general term:

$$T_5 = \binom{10}{4} (\sqrt{a})^{10-4} \cdot \frac{1}{2^4}.$$

Simplify:

$$T_5 = \binom{10}{4} (\sqrt{a})^6 \cdot \frac{1}{16}.$$

Substitute $\binom{10}{4} = 210$:

$$T_5 = 210 \cdot (\sqrt{a})^6 \cdot \frac{1}{16}.$$

$$T_5 = 210 \cdot \frac{a^3}{16}.$$

The value of T_5 is given as 105:

$$\frac{210 \cdot a^3}{16} = 105.$$

Solve for a^3:

$$a^3 = \frac{105 \cdot 16}{210}.$$

$$a^3 = 8.$$

Take the cube root of both sides:

$$a = \sqrt[3]{8}.$$

$$a = 2.$$

Finally, $a^2 = 4$.

Answer: (1) 4

Quick Tip

When finding terms in binomial expansions, focus on isolating the desired powers of variables to solve for unknowns and equate to given conditions.

---

21. Let A be the region enclosed by the parabola $y^2 = 2x$ and the line $x = 24$. Then the maximum area of the rectangle inscribed in the region A is ______.
Answer: 128
Solution:
Consider a rectangle inscribed in the region bounded by the parabola $y^2 = 2x$ and the line $x = 24$. Let the coordinates of the upper right corner of the rectangle be $\left(\frac{b^2}{2}, b\right)$, where b is the y-coordinate of the corner on the parabola.
The length of the rectangle along the x-axis is:

$$2\left(24 - \frac{b^2}{2}\right).$$

The height of the rectangle is:

$$b.$$

Therefore, the area A of the rectangle is given by:

$$A = 2\left(24 - \frac{b^2}{2}\right) \cdot b.$$

Simplifying:

$$A = 2\left(24b - \frac{b^3}{2}\right),$$

$$A = 48b - b^3.$$

To find the maximum area, we differentiate A with respect to b and set the derivative equal to zero:

$$\frac{dA}{db} = 48 - 3b^2 = 0.$$

Solving for b:

$$3b^2 = 48,$$

$$b^2 = 16,$$
$$b = 4 \quad (\text{since } b > 0).$$

Substituting $b = 4$ back into the expression for A:

$$A = 2\left(24 - \frac{4^2}{2}\right) \cdot 4,$$
$$A = 2\,(24 - 8) \cdot 4,$$
$$A = 2 \cdot 16 \cdot 4,$$
$$A = 128.$$

Therefore, the maximum area of the rectangle is:

$$128.$$

Answer: 128

Quick Tip

To find the maximum area of a rectangle inscribed within a region, express the area as a function of a single variable, differentiate, and find the critical points by setting the derivative equal to zero.

---

22. If $\alpha = \lim_{x\to 0^+}\left(\frac{e^{\sqrt{\tan x}} - e^{\sqrt{x}}}{\sqrt{\tan x} - \sqrt{x}}\right)$ and $\beta = \lim_{x\to 0}(1+\sin x)^{\frac{1}{2\cot x}}$ are the roots of the quadratic equation $ax^2 + bx - \sqrt{e} = 0$, then 12 $\log_e(a+b)$ is equal to :

Answer: 6
Solution:
Given: The given expression for α is:

$$\alpha = \lim_{x\to 0^+} e^{\sqrt{x}} \frac{\left(e^{\sqrt{\tan x} - \sqrt{x}} - 1\right)}{\sqrt{\tan x} - \sqrt{x}}.$$

As $x \to 0^+$, we observe that $\sqrt{\tan x} \to \sqrt{x}$. The expression simplifies as the numerator and denominator converge to zero:

$$\alpha = 1.$$

The given expression for β is:

$$\beta = \lim_{x\to 0}(1 + \sin x)^{\frac{1}{2}\cot x}.$$

Using the approximation $\sin x \approx x$ and $\cot x \approx \frac{1}{x}$ as $x \to 0$, we rewrite the expression:

$$\beta = (1 + x)^{\frac{1}{2x}}.$$

Using the standard limit $\lim_{x\to 0}(1+x)^{\frac{1}{x}} = e$, we get:

$$\beta = e^{1/2}.$$

The quadratic equation is:

$$x^2 - \left(1+\sqrt{e}\right)x + \sqrt{e} = 0.$$

Compare with the general quadratic form:

$$ax^2 + bx + c = 0.$$

From the given equation:

$$a = -1, \quad b = \sqrt{e} + 1.$$

Substitute the values of a and b:

$$a + b = -1 + (\sqrt{e} + 1) = \sqrt{e}.$$

$$\ln(a+b) = \ln(\sqrt{e}) = \frac{1}{2}.$$

Finally, calculate:

$$12\ln(a+b) = 12 \times \frac{1}{2} = 6.$$

Answer: 6

Quick Tip

When evaluating limits involving indeterminate forms, L'Hôpital's Rule and small-angle approximations are useful techniques to simplify expressions and find exact values.

---

23. Let S be the focus of the hyperbola $\frac{x^2}{3} - \frac{y^2}{5} = 1$, on the positive x-axis. Let C be the circle with its centre at $A(\sqrt{6}, \sqrt{5})$ and passing through the point S. If O is the origin and SAB is a diameter of C, then the square of the area of the triangle OSB is equal to -

Answer: 40

Solution:

Consider the hyperbola:

$$\frac{x^2}{3} - \frac{y^2}{5} = 1$$

The focus S is located at $(\sqrt{8}, 0)$ on the positive x-axis.

The circle C has its center at $A(\sqrt{6}, \sqrt{5})$ and passes through the point S. The radius of the circle is given by:

$$r = \text{Distance between } A \text{ and } S = \sqrt{(\sqrt{6}-\sqrt{8})^2 + (\sqrt{5}-0)^2}$$

Simplifying:

$$r = \sqrt{(\sqrt{6} - \sqrt{8})^2 + (\sqrt{5})^2} = \sqrt{(\sqrt{6} - \sqrt{8})^2 + 5}$$

Since O is the origin, and SAB is a diameter of circle C, we can find the coordinates of point B as $(2\sqrt{8} - \sqrt{6}, 2\sqrt{5})$.

The area of triangle OSB is given by:

$$\text{Area} = \frac{1}{2} \times OS \times \text{height}$$

Using the coordinates of O, S, and B, we calculate:

$$\text{Area} = \frac{1}{2} \times OS \times \text{height} = \frac{1}{2} \times \sqrt{8} \times 2\sqrt{5} = \sqrt{40}$$

The square of the area is:

$$(\sqrt{40})^2 = 40$$

Answer: 40

Quick Tip

For geometric problems involving conic sections and circles, finding the exact coordinates of points and using distance formulas can greatly simplify complex area or distance calculations.

---

24. Let P(α, β, γ) be the image of the point Q(1, 6, 4) in the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$. Then $2\alpha + \beta + \gamma$ is equal to:
Answer: 11
Solution:
Let the line be represented by:

$$x = t, \quad y = 2t + 1, \quad z = 3t + 2$$

Given the point $Q(1, 6, 4)$, we find the foot of the perpendicular A by letting:

$$A\left(\frac{17}{14}, \frac{48}{14}, \frac{79}{14}\right)$$

The direction vector of the line is:

$$\mathbf{b} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$$

To find the image point $P(\alpha, \beta, \gamma)$, we reflect Q across A using:

$$\alpha = \frac{20}{14}, \quad \beta = \frac{12}{14}, \quad \gamma = \frac{102}{14}$$

Calculating $2\alpha + \beta + \gamma$:

$$2\alpha + \beta + \gamma = \frac{154}{14} = 11$$

Answer: 11

> **Quick Tip**
>
> When finding the reflection of a point across a line in three-dimensional space, using the parametric equations of the line and vector projections simplifies the calculations.

---

25. An arithmetic progression is written in the following way:

2

5 8

11 14 17

20 23 26 29

____

____

The sum of all the terms of the 10th row is:
Answer: 1505
Solution:
The sequence given is:

$$2, 5, 11, 20, \ldots$$

The general term for the n-th row of this arithmetic progression can be expressed as:

$$T_n = \frac{3n^2 - 3n + 4}{2}$$

For the 10th row, we substitute $n = 10$:

$$T_{10} = \frac{3(100) - 3(10) + 4}{2} = \frac{300 - 30 + 4}{2} = \frac{274}{2} = 137$$

Since there are 10 terms in the 10th row, with a common difference $c.d. = 3$, the sum of the terms of the 10th row is given by:

$$\text{Sum} = \frac{10}{2}\left(2 \times 137 + 9 \times 3\right)$$

Calculating:

$$\text{Sum} = 5\left(274 + 27\right) = 5 \times 301 = 1505$$

Answer: 1505

> **Quick Tip**
>
> In arithmetic progressions with rows of increasing length, using a formula for the general term and summing the terms geometrically or by formula helps simplify calculations efficiently.

---

26. The number of distinct real roots of the equation $|x+1||x+3|-4|x+2|+5=0$ **is:**
Answer: 2
Solution:
Given equation:

$$|x+1||x+3|-4|x+2|+5=0$$

To solve this, we break it into different cases based on the values of x that change the absolute values:

Case 1: $x \leq -3$

$$(x+1)(x+3)+4(x+2)+5=0$$

Simplifying:

$$x^2+4x+3+4x+8+5=0$$
$$x^2+8x+16=0 \quad \Rightarrow \quad (x+4)^2=0$$
$$x=-4$$

Case 2: $-3 < x \leq -2$

$$-(x+1)(x+3)+4(x+2)+5=0$$

Simplifying:

$$-x^2-4x-3+4x+8+5=0$$
$$-x^2+10=0 \quad \Rightarrow \quad x^2=10$$
$$x=\pm\sqrt{10}$$

Case 3: $-2 < x \leq -1$

$$-(x+1)(x+3)-4(x+2)+5=0$$

Simplifying:

$$-x^2-4x-3-4x-8+5=0$$
$$-x^2-8x-6=0 \quad \Rightarrow \quad x^2+8x+6=0$$

Solving using the quadratic formula:

$$x=\frac{-8\pm\sqrt{64-24}}{2}=\frac{-8\pm\sqrt{40}}{2}=-4\pm\sqrt{10}$$

Case 4: $x > -1$

$$x^2+4x+3-4x-8+5=0$$

Simplifying:

$$x^2=0$$
$$x=0$$

The number of distinct real roots is:

$$\text{Total Solutions} = 2$$

Answer: 2

Quick Tip

To solve equations involving absolute values, split the problem into cases based on where each term inside an absolute value changes sign, then solve each case separately.

---

27. Let a ray of light passing through the point (3, 10) reflect on the line $2x + y = 6$ and the reflected ray pass through the point (7, 2). If the equation of the incident ray is $ax + by + 1 = 0$, then $a^2 + b^2 + 3ab$ is equal to:

Answer: 1

Solution:

Given: For B':

$$\frac{x-7}{2} = \frac{y-2}{1} = -2\left(\frac{14+2-6}{5}\right)$$

$$\frac{x-7}{2} = \frac{y-2}{1} = -4$$

$$x = -1, \quad y = -2 \implies B'(-1, -2)$$

Incident ray AB':

$$M_{AB'} = 3$$

$$y + 2 = 3(x + 1)$$

$$3x - y + 1 = 0$$

$$a = 3, \quad b = -1$$

$$a^2 + b^2 + 3ab = 9 + 1 - 9 = 1$$

Answer: 1

Quick Tip

When dealing with reflection problems involving lines, use the relationship between the slopes of the incident and reflected rays, and ensure to apply geometric properties accurately.

---

28. Let $a, b, c \in N$ and $a < b < c$. Let the mean, the mean deviation about the mean, and the variance of the 5 observations $9, 25, a, b, c$ be $18, 4$ and $\frac{136}{5}$, respectively. Then $2a + b - c$ is equal to:

Answer: 33

Solution:
Given:
$$\text{Mean} = \frac{9 + 25 + a + b + c}{5} = 18$$
Solving for $a + b + c$:
$$a + b + c = 56$$
Mean deviation about the mean is given by:
$$\text{Mean deviation} = \frac{\sum |x_i - \overline{x}|}{n} = 4$$
Substituting values:
$$|9 - 18| + |25 - 18| + |a - 18| + |b - 18| + |c - 18| = 20$$
$$|18 - a| + |18 - b| + |18 - c| = 4$$
The variance is given by:
$$\text{Variance} = \frac{\sum (x_i - \overline{x})^2}{n} = \frac{136}{5}$$
Calculating:
$$\frac{(9 - 18)^2 + (25 - 18)^2 + (a - 18)^2 + (b - 18)^2 + (c - 18)^2}{5} = \frac{136}{5}$$
Multiplying both sides by 5:
$$81 + 49 + (18 - a)^2 + (18 - b)^2 + (18 - c)^2 = 136$$
Simplifying:
$$(18 - a)^2 + (18 - b)^2 + (18 - c)^2 = 6$$
Possible values:
$$(18 - a)^2 = 1, \quad (18 - b)^2 = 1, \quad (18 - c)^2 = 4$$
This gives:
$$18 - a = 1 \Rightarrow a = 17, \quad 18 - b = -1 \Rightarrow b = 19, \quad 18 - c = -2 \Rightarrow c = 20$$
Substituting:
$$a + b + c = 17 + 19 + 20 = 56$$
Calculating $2a + b - c$:
$$2a + b - c = 2 \times 17 + 19 - 20 = 34 + 19 - 20 = 33$$

Answer: 33

Quick Tip

For problems involving mean deviation and variance, systematically use the definitions and simplify step-by-step. Ensure to check constraints such as $a < b < c$ for consistency.

29. Let $\alpha|x| = |y|e^{xy-\beta}$, $\alpha, \beta \in N$ be the solution of the differential equation $xdy - ydx + xy(xdy + ydx) = 0$, with $y(1) = 2$. Then $\alpha + \beta$ is equal to:
Answer: 4
Solution:
Consider the given differential equation:

$$xdy - ydx + xy(xdy + ydx) = 0$$

Rearranging terms:

$$(x + xy)dy = (y - xy)dx$$

Dividing both sides by xy:

$$\frac{dy}{dx} = \frac{y - xy}{x + xy}$$

Given that $\alpha|x| = |y|e^{xy-\beta}$, substituting the initial condition $y(1) = 2$ into the expression:

$$\alpha|1| = |2|e^{1\cdot2-\beta}$$

Simplifying:

$$\alpha = 2e^{2-\beta}$$

Since $\alpha, \beta \in N$, assume values for β such that α is an integer. Let $\beta = 2$:

$$\alpha = 2e^{0} = 2$$

Calculating $\alpha + \beta$:

$$\alpha + \beta = 2 + 2 = 4$$

Answer: 4

Quick Tip

When solving differential equations involving constants and exponential terms, always check for possible natural number solutions and verify using initial conditions for consistency.

---

30. If $\int \frac{1}{\sqrt[5]{(x-1)^4(x+3)^6}}dx = A\left(\frac{\alpha x-1}{\beta x+3}\right)^B + C$, where C is the constant of integration, then the value of $\alpha + \beta + 20AB$ is:
Answer: 7
Solution:
Consider the given integral:

$$I = \int \frac{1}{(x-1)^{4/5}(x+3)^{6/5}}dx$$

To simplify, let:

$$I = \int \frac{1}{(x-1)^{4/5}(x+3)^{6/5}}dx = \int \frac{1}{(x-1)^{4/5}(x+3)^{2}}dx$$

Substitute:

$$t = \frac{x-1}{x+3} \quad \Rightarrow \quad dt = \frac{4}{(x+3)^2}dx$$

Thus, the integral becomes:

$$I = \frac{1}{4}\int t^{-4/5}dt$$

Integrating:

$$I = \frac{1}{4}\cdot\frac{t^{1/5}}{\frac{1}{5}} + C = \frac{5}{4}t^{1/5} + C$$

Substituting back $t = \frac{x-1}{x+3}$:

$$I = \frac{5}{4}\left(\frac{x-1}{x+3}\right)^{1/5} + C$$

Comparing with:

$$\int \frac{1}{\sqrt[5]{(x-1)^4(x+3)^6}}dx = A\left(\frac{\alpha x-1}{\beta x+3}\right)^B + C$$

We have:

$$A = \frac{5}{4}, \quad \alpha = \beta = 1, \quad B = \frac{1}{5}$$

Calculating $\alpha + \beta + 20AB$:

$$\alpha + \beta + 20AB = 1 + 1 + 20 \times \frac{5}{4} \times \frac{1}{5} = 7$$

Answer: 7

Quick Tip

When integrating expressions involving roots and powers, consider substitutions to simplify the integrand, making integration straightforward.

Physics

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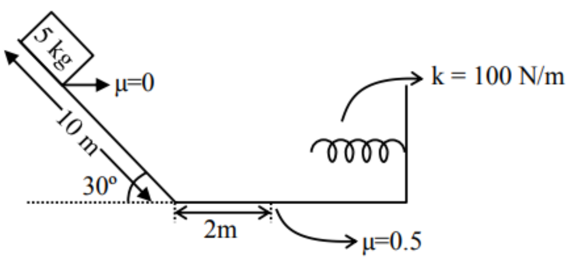
31. A block is simply released from the top of an inclined plane as shown in the figure above. The maximum compression in the spring when the block hits the spring is:



(1) $\sqrt{6}\,\text{m}$
(2) $2\,\text{m}$
(3) $1\,\text{m}$
(4) $\sqrt{5}\,\text{m}$

Answer: (2) $2\,\text{m}$
Solution:
Using the work-energy theorem, the total work done by all forces is equal to the change in kinetic energy (ΔKE) of the system. Here:

$$w_g + w_{\text{Fr}} + w_s = \Delta KE$$

Substituting the given values:

$$5 \times 10 \times 5 - 0.5 \times 5 \times 10 \times x - \frac{1}{2}Kx^2 = 0 - 0$$

Simplifying:

$$250 - 25x - 50x^2 = 0$$

Rewriting:

$$2x^2 + x - 10 = 0$$

Solving this quadratic equation gives:

$$x = 2$$

Thus, the maximum compression in the spring is $x = 2\,\text{m}$.

Quick Tip

When solving problems involving springs and inclined planes, use the work-energy theorem to account for gravitational work, friction, and elastic potential energy.

---

32. In a hypothetical fission reaction:

$${}_{92}X^{236} \rightarrow {}_{56}Y^{141} + {}_{36}Z^{92} + 3R$$

The identity of emitted particles (R) is:
(1) Proton
(2) Electron
(3) Neutron
(4) γ-radiations

Answer: (3) Neutron
Solution:
To identify the emitted particles, let us verify the conservation of atomic number (Z) and mass number (A).
- Atomic number (Z):

$$Z_{\text{LHS}} = 92, \quad Z_{\text{RHS}} = 56 + 36 = 92$$

Z is conserved.

- Mass number (A):

$$A_{\text{LHS}} = 236, \quad A_{\text{RHS}} = 141 + 92 = 233$$

The mass number is not conserved. The difference is:

$$A_{\text{LHS}} - A_{\text{RHS}} = 236 - 233 = 3$$

The missing mass corresponds to three neutrons (R = neutrons).

Quick Tip

For nuclear reactions, always check conservation laws for atomic number (Z) and mass number (A) to identify unknown particles.

---

33. If ε_0 is the permittivity of free space and E is the electric field, then $\varepsilon_0 E^2$ has the dimensions:

(1) $[M^0 L^{-2} T A]$
(2) $[M L^{-1} T^{-2}]$
(3) $[M^{-1} L^{-3} T^4 A^2]$
(4) $[M L^2 T^{-2}]$

Answer: (2) $[M L^{-1} T^{-2}]$

Solution:

The electric field is given by:

$$E = \frac{KQ}{R^2}$$

Substituting $K = \frac{1}{4\pi\varepsilon_0}$, we get:

$$E = \frac{Q}{4\pi\varepsilon_0 R^2}$$

From this, the permittivity of free space (ε_0) can be expressed as:

$$\varepsilon_0 = \frac{Q}{4\pi R^2 E}$$

Now, calculate $\varepsilon_0 E^2$:

$$\varepsilon_0 E^2 = \frac{Q}{4\pi R^2 E} \cdot E^2 = \frac{QE}{4\pi R^2}$$

Analyzing the dimensional formula:

$$[\varepsilon_0 E^2] = \frac{[Q][E]}{[R^2]}$$

Substituting the dimensional formulas:

$$[Q] = [W], \quad [E] = \frac{[W]}{[R^2][Q]}$$

$$[\varepsilon_0 E^2] = \frac{[W]}{[R^3]} = \frac{ML^2T^{-2}}{L^3}$$

Simplifying:

$$[\varepsilon_0 E^2] = [ML^{-1}T^{-2}]$$

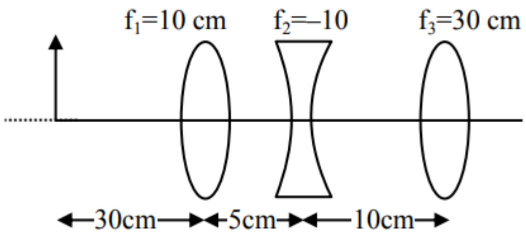
Thus, the dimensions of $\varepsilon_0 E^2$ are $[ML^{-1}T^{-2}]$.

Quick Tip

In dimension analysis problems, use fundamental equations to find relations between physical quantities and ensure correct substitution of dimensional formulas.

---

34. The position of the image formed by the combination of lenses is:



(1) 30 cm (right of third lens)
(2) 15 cm (left of second lens)
(3) 30 cm (left of third lens)
(4) 15 cm (right of second lens)

Answer: (1) 30 cm **(right of third lens)**
Solution:

The image positions are calculated lens by lens using the lens formula:

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u} \quad \Rightarrow \quad v = \frac{uf}{u+f}$$

For lens 1 ($f_1 = 10\,\text{cm}, u = -30\,\text{cm}$):

$$v = \frac{(-30)\cdot 10}{-30+10} = \frac{-300}{-20} = 15\,\text{cm}$$

The image forms 15 cm to the right of the first lens.
For lens 2 ($f_2 = -10\,\text{cm}, u = 10\,\text{cm}$):

$$v = \frac{10\cdot(-10)}{10-10} = \infty$$

The image is formed at infinity due to parallel rays after the second lens.
For lens 3 ($f_3 = 30\,\text{cm}, u = -\infty$):

$$v = f_3 = 30\,\text{cm}$$

Thus, the final image is formed 30 cm to the right of the third lens.

Quick Tip

When solving lens problems, calculate the image position for each lens sequentially, treating the image of one lens as the object for the next.

---

35. A plane progressive wave is given by:

$$y = 2\cos\, 2\pi(330\,t - x)\,\text{m}.$$

The frequency of the wave is:
(1) 165 Hz
(2) 330 Hz
(3) 660 Hz
(4) 340 Hz

Answer: (2) 330 Hz
Solution:
The general form of a plane progressive wave is:

$$y = A\cos(\omega t - kx).$$

Comparing with the given equation:

$$y = 2\cos\, 2\pi(330\,t - x),$$

we identify:

$$\omega = 2\pi \cdot 330.$$

The angular frequency ω is related to the frequency f by:

$$\omega = 2\pi f \implies 2\pi \cdot f = 2\pi \cdot 330.$$

Thus:

$$f = 330\,\text{Hz}.$$

Quick Tip

When analyzing wave equations, compare the given equation with the standard form to extract parameters like angular frequency ω and wave number k.

---

36. A thin circular disc of mass M and radius R is rotating in a horizontal plane about an axis passing through its center and perpendicular to its plane with angular velocity ω. If another disc of same dimensions but of mass $M/2$ is placed gently on the first disc co-axially, then the new angular velocity of the system is:

(1) $\frac{4}{5}\omega$
(2) $\frac{5}{4}\omega$
(3) $\frac{2}{3}\omega$
(4) $\frac{3}{2}\omega$

Answer: (3) $\frac{2}{3}\omega$
Solution:
Using the law of conservation of angular momentum:

$$I_1\omega = I_2\omega_2.$$

The moment of inertia of the first disc:

$$I_1 = \frac{MR^2}{2}.$$

The combined moment of inertia of both discs:

$$I_2 = \frac{MR^2}{2} + \frac{1}{2}\left(\frac{MR^2}{2}\right) = \frac{3MR^2}{4}.$$

Applying conservation of angular momentum:

$$\frac{MR^2}{2} \cdot \omega = \frac{3MR^2}{4} \cdot \omega_2.$$

Simplifying:

$$\omega_2 = \frac{\frac{MR^2}{2}}{\frac{3MR^2}{4}} \cdot \omega = \frac{2}{3}\omega.$$

Thus, the new angular velocity of the system is:

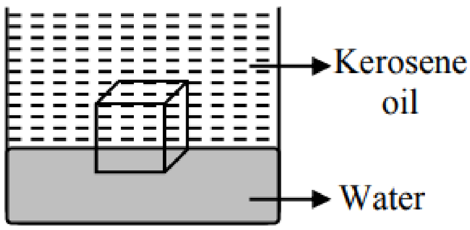
$$\omega_2 = \frac{2}{3}\omega.$$

Quick Tip

In rotational motion problems, use the conservation of angular momentum to relate the initial and final states, especially when moments of inertia change.

---

37. A cube of ice floats partly in water and partly in kerosene oil. The ratio of volume of ice immersed in water to that in kerosene oil (specific gravity of kerosene oil $= 0.8$**, specific gravity of ice** $= 0.9$**) is:**



(1) 8 : 9
(2) 5 : 4
(3) 9 : 10
(4) 1 : 1

Answer: (4) 1 : 1
Solution:
Let the volume of the cube immersed in water be V_w and the volume immersed in kerosene oil be V_k. Using the principle of floatation, the weight of the ice cube is equal to the total upthrust provided by water and kerosene oil.

The weight of the ice cube is:

$$W_{\text{ice}} = \rho_{\text{ice}} V g,$$

where $\rho_{\text{ice}} = 0.9\rho_{\text{water}}$.

The total upthrust is:

$$U = \rho_{\text{water}} V_w g + \rho_{\text{kerosene}} V_k g.$$

Equating the weight of the ice cube to the total upthrust:

$$0.9\rho_{\text{water}} V g = \rho_{\text{water}} V_w g + 0.8\rho_{\text{water}} V_k g.$$

Cancel $\rho_{\text{water}} g$ from all terms:

$$0.9V = V_w + 0.8V_k.$$

Divide through by V:

$$0.9 = \frac{V_w}{V} + 0.8\frac{V_k}{V}.$$

Since the total volume of the ice cube is V, $\frac{V_w}{V} + \frac{V_k}{V} = 1$. Let $x = \frac{V_w}{V}$ and $y = \frac{V_k}{V}$, then $x + y = 1$.

Substituting $y = 1 - x$ into the equation:

$$0.9 = x + 0.8(1 - x).$$

Simplify:

$$0.9 = x + 0.8 - 0.8x.$$

$$0.9 - 0.8 = 0.2x \implies x = 0.5.$$

Thus, $y = 1 - 0.5 = 0.5$. The ratio of V_w to V_k is:

$$V_w : V_k = 0.5 : 0.5 = 1 : 1.$$

Quick Tip

In problems involving buoyancy and specific gravity, equate the weight of the object to the total buoyant force from all fluids in which it is partially submerged.

38. Given below are two statements:

Statement (I): The mean free path of gas molecules is inversely proportional to the square of molecular diameter.

Statement (II): Average kinetic energy of gas molecules is directly proportional to absolute temperature of gas.

In the light of the above statements, choose the correct answer from the options given below:

(1) **Statement I is false but Statement II is true.**
(2) **Statement I is true but Statement II is false.**
(3) **Both Statement I and Statement II are false.**
(4) **Both Statement I and Statement II are true.**

Answer: (4) Both Statement I and Statement II are true.

Solution:

The mean free path (λ) of gas molecules is given by:

$$\lambda = \frac{RT}{\sqrt{2}\pi d^2 N_A P}.$$

Here, $\lambda \propto \frac{1}{d^2}$, verifying Statement (I).

The average kinetic energy of gas molecules is:

$$KE = \frac{f}{2}nRT,$$

where $KE \propto T$, confirming Statement (II).

Thus, both Statement I and Statement II are correct.

Quick Tip

To validate theoretical statements in physics, refer to the corresponding mathematical equations and analyze their proportionality relations.

---

39. Two satellites A and B orbit a planet in circular orbits having radii $4R$ and R, respectively. If the speed of A is $3v$, the speed of B will be:

(1) $\frac{4}{3}v$
(2) $3v$
(3) $6v$
(4) $12v$

Answer: (3) $6v$

Solution:

The orbital speed of a satellite is given by:

$$v = \sqrt{\frac{GM}{R}}.$$

For satellites A and B:

$$\frac{v_A}{v_B} = \sqrt{\frac{R_B}{R_A}} = \sqrt{\frac{R}{4R}} = \frac{1}{2}.$$

Thus:

$$v_B = 2v_A.$$

Given $v_A = 3v$, the speed of B is:

$$v_B = 2 \cdot 3v = 6v.$$

Quick Tip

In problems involving satellite motion, use the formula for orbital speed and relate radii to compare velocities.

---

40. A long straight wire of radius a carries a steady current I. The current is uniformly distributed across its cross section. The ratio of the magnetic field at $\frac{a}{2}$ and $2a$ from the axis of the wire is:
(1) $1:4$
(2) $4:1$
(3) $1:1$
(4) $3:4$

Answer: (3) $1:1$
Solution:

For a point inside the wire ($r < a$), the magnetic field is given by:

$$B_1 \cdot 2\pi\frac{a}{2} = \mu_0\frac{I}{4} \implies B_1 = \frac{\mu_0 I}{4\pi a}.$$

For a point outside the wire ($r > a$), the magnetic field is:

$$B_2 \cdot 2\pi \cdot 2a = \mu_0 I \implies B_2 = \frac{\mu_0 I}{4\pi a}.$$

The ratio of magnetic fields:

$$\frac{B_1}{B_2} = \frac{\mu_0 I}{4\pi a} \div \frac{\mu_0 I}{4\pi a} = 1:1.$$

Quick Tip

For magnetic fields due to current-carrying wires, use Ampere's law and consider whether the point lies inside or outside the wire.

---

41. The angle of projection for a projectile to have the same horizontal range and maximum height is:
(1) $\tan^{-1}(1)$
(2) $\tan^{-1}(4)$
(3) $\tan^{-1}\left(\frac{1}{4}\right)$
(4) $\tan^{-1}\left(\frac{1}{2}\right)$

Answer: (2) $\tan^{-1}(4)$
Solution:
The horizontal range is:
$$\frac{u^2 \sin 2\theta}{g},$$
and the maximum height is:
$$\frac{u^2 \sin^2 \theta}{2g}.$$
Equating:
$$\frac{u^2 \sin 2\theta}{g} = \frac{u^2 \sin^2 \theta}{2g}.$$
Simplifying:
$$2 \sin\theta \cos\theta = \frac{\sin^2\theta}{2}.$$
Substitute $\sin 2\theta = 2 \sin\theta \cos\theta$:
$$4 \sin\theta \cos\theta = \sin^2\theta \implies 4 = \tan\theta.$$
Thus:
$$\theta = \tan^{-1}(4).$$

Quick Tip

To solve projectile motion problems, relate horizontal range and maximum height equations, and simplify using trigonometric identities.

---

42. Water boils in an electric kettle in 20 minutes after being switched on. Using the same main supply, the length of the heating element should be . . . to . . . times of its initial length if the water is to be boiled in 15 minutes.
(1) Increased, $\frac{3}{4}$
(2) Increased, $\frac{4}{3}$
(3) Decreased, $\frac{3}{4}$
(4) Decreased, $\frac{4}{3}$

Answer: (3) Decreased, $\frac{3}{4}$
Solution:
The resistance of the heating element is:
$$R = \rho \frac{\ell}{A}.$$

The power is inversely proportional to the length:

$$P \propto \frac{1}{\ell}.$$

From the relation $P_1 \cdot t_1 = P_2 \cdot t_2$:

$$\frac{P_1}{P_2} = \frac{t_2}{t_1} = \frac{15}{20}.$$

Substituting $P \propto \frac{1}{\ell}$:

$$\frac{\ell_2}{\ell_1} = \frac{t_2}{t_1} = \frac{15}{20} = \frac{3}{4}.$$

Thus, the new length should be decreased to:

$$\ell_2 = \frac{3}{4}\ell_1.$$

Quick Tip

For heating element problems, use the relationship between resistance, length, and power, and apply the proportionality laws.

---

43. A capacitor has air as dielectric medium and two conducting plates of area $12\,\text{cm}^2$, and they are $0.6\,\text{cm}$ apart. When a slab of dielectric having area $12\,\text{cm}^2$ and $0.6\,\text{cm}$ thickness is inserted between the plates, one of the conducting plates has to be moved by $0.2\,\text{cm}$ to keep the capacitance the same as in the previous case. The dielectric constant of the slab is: (Given $\varepsilon_0 = 8.834 \times 10^{-12}\,\text{F/m}$).

(1) 1.50
(2) 1.33
(3) 0.66
(4) 1

Answer: (1) 1.50
Solution:

The capacitance without the dielectric is:

$$C = \frac{A\varepsilon_0}{d}.$$

With the dielectric inserted:

$$C = \frac{A\varepsilon_0}{0.2 + \frac{d}{k}}.$$

Equating the capacitances:

$$\frac{A\varepsilon_0}{0.6} = \frac{A\varepsilon_0}{0.2 + \frac{0.6}{k}}.$$

Cancel $A\varepsilon_0$:

$$0.6 = 0.2 + \frac{0.6}{k}.$$

Rearranging:

$$0.6 - 0.2 = \frac{0.6}{k} \implies 0.4 = \frac{0.6}{k}.$$

Solving for k:

$$k = \frac{0.6}{0.4} = \frac{3}{2} = 1.50.$$

Thus, the dielectric constant of the slab is:

$$k = 1.50.$$

Quick Tip

In capacitor problems with dielectrics, use the formula for equivalent capacitance to account for both the air gap and dielectric material.

---

44. A given object takes n times the time to slide down a $45°$ rough inclined plane as it takes to slide down an identical perfectly smooth $45°$ inclined plane. The coefficient of kinetic friction between the object and the surface of the inclined plane is:

(1) $1 - \frac{1}{n^2}$
(2) $1 - n^2$
(3) $\sqrt{1 - \frac{1}{n^2}}$
(4) $\sqrt{1 - n^2}$

Answer: (1) $1 - \frac{1}{n^2}$
Solution:

For the smooth inclined plane, the acceleration is:

$$a_{\text{smooth}} = g \sin 45° = \frac{g}{\sqrt{2}}.$$

For the rough inclined plane, the acceleration is:

$$a_{\text{rough}} = g(\sin 45° - \mu_k \cos 45°) = \frac{g}{\sqrt{2}}(1 - \mu_k).$$

The time taken is inversely proportional to the square root of acceleration:

$$t_{\text{rough}} = n \cdot t_{\text{smooth}} \implies \sqrt{\frac{a_{\text{smooth}}}{a_{\text{rough}}}} = n.$$

Substituting:

$$\sqrt{\frac{\frac{g}{\sqrt{2}}}{\frac{g}{\sqrt{2}}(1 - \mu_k)}} = n.$$

Simplify:

$$\sqrt{\frac{1}{1-\mu_k}} = n \implies 1-\mu_k = \frac{1}{n^2}.$$

Solving for μ_k:

$$\mu_k = 1 - \frac{1}{n^2}.$$

Thus, the coefficient of kinetic friction is:

$$\mu_k = 1 - \frac{1}{n^2}.$$

Quick Tip

In inclined plane problems, relate acceleration to the time of descent and use proportionality to account for friction.

---

45. A coil of negligible resistance is connected in series with a $90\,\Omega$ resistor across $120\,\text{V}$, $60\,\text{Hz}$ supply. A voltmeter reads $36\,\text{V}$ across the resistor. The inductance of the coil is:

(1) $0.76\,\text{H}$
(2) $2.86\,\text{H}$
(3) $0.286\,\text{H}$
(4) $0.91\,\text{H}$

Answer: (1) $0.76\,\text{H}$
Solution:

The circuit contains a resistor (R) and an inductor (L) in series. The total voltage (V) is the RMS voltage of the supply. The current in the circuit is:

$$I_{\text{rms}} = \frac{V_R}{R},$$

where $V_R = 36\,\text{V}$ and $R = 90\,\Omega$. Substituting:

$$I_{\text{rms}} = \frac{36}{90} = 0.4\,\text{A}.$$

The total impedance of the circuit is:

$$Z = \frac{V}{I_{\text{rms}}} = \frac{120}{0.4} = 300\,\Omega.$$

The impedance is related to the resistance and inductive reactance by:

$$Z = \sqrt{R^2 + X_L^2}.$$

Substituting $Z = 300\,\Omega$ and $R = 90\,\Omega$:

$$300 = \sqrt{90^2 + X_L^2}.$$

Squaring both sides:

$$300^2 = 90^2 + X_L^2 \implies X_L^2 = 300^2 - 90^2 = 81900.$$

Thus:

$$X_L = \sqrt{81900} \approx 286.18\,\Omega.$$

The inductive reactance is given by:

$$X_L = \omega L \quad \text{where} \quad \omega = 2\pi f.$$

For $f = 60\,\text{Hz}$:

$$\omega = 2\pi \cdot 60 \approx 376.8\,\text{rad/s}.$$

Solving for L:

$$L = \frac{X_L}{\omega} = \frac{286.18}{376.8} \approx 0.76\,\text{H}.$$

Thus, the inductance of the coil is:

$$L = 0.76\,\text{H}.$$

Quick Tip

For AC circuits involving inductors and resistors, use impedance relations to find inductance. Always confirm reactance using $X_L = \omega L$.

---

46. There are 100 divisions on the circular scale of a screw gauge of pitch $1\,\text{mm}$**. With no measuring quantity in between the jaws, the zero of the circular scale lies 5 divisions below the reference line. The diameter of a wire is then measured using this screw gauge. It is found that 4 linear scale divisions are clearly visible while 60 divisions on the circular scale coincide with the reference line. The diameter of the wire is:**

(1) $4.65\,\text{mm}$
(2) $4.55\,\text{mm}$
(3) $4.60\,\text{mm}$
(4) $3.35\,\text{mm}$

Answer: (2) $4.55\,\text{mm}$
Solution:

The least count of the screw gauge is:

$$\text{Least count} = \frac{\text{Pitch}}{\text{Number of divisions on circular scale}} = \frac{1\,\text{mm}}{100} = 0.01\,\text{mm}.$$

The zero error is given as:

$$\text{Zero error} = +0.05\,\text{mm}.$$

The reading is calculated as:

$$\text{Reading} = (\text{Linear scale reading})\cdot(\text{Pitch}) + (\text{Circular scale reading})\cdot(\text{Least count}) - \text{Zero error}.$$

Substitute:

$$\text{Reading} = (4 \cdot 1)\,\text{mm} + (60 \cdot 0.01)\,\text{mm} - 0.05\,\text{mm}.$$

Simplify:

$$\text{Reading} = 4.00\,\text{mm} + 0.60\,\text{mm} - 0.05\,\text{mm} = 4.55\,\text{mm}.$$

Thus, the diameter of the wire is:

$$4.55\,\text{mm}.$$

Quick Tip

For screw gauge problems, remember to account for zero error and use the least count to calculate the final measurement.

---

47. A proton and an electron have the same de Broglie wavelength. If K_p and K_e are the kinetic energies of the proton and electron respectively, then choose the correct relation:

(1) $K_p > K_e$
(2) $K_p = K_e$
(3) $K_p = K_e^2$
(4) $K_p < K_e$

Answer: (4) $K_p < K_e$
Solution:

The de Broglie wavelength for a particle is given by:

$$\lambda = \frac{h}{p},$$

where h is Planck's constant and p is the momentum.

For the proton and electron:

$$\lambda_{\text{proton}} = \lambda_{\text{electron}} \implies p_{\text{proton}} = p_{\text{electron}}.$$

The kinetic energy is related to momentum as:

$$K = \frac{p^2}{2m}.$$

Since $p_{\text{proton}} = p_{\text{electron}}$:

$$K_{\text{proton}} = \frac{p^2}{2m_p}, \quad K_{\text{electron}} = \frac{p^2}{2m_e}.$$

Given $m_p > m_e$, it follows that:

$$K_{\text{proton}} < K_{\text{electron}}.$$

Thus:

$$K_p < K_e.$$

Quick Tip

When comparing particles with the same de Broglie wavelength, use the relation $K \propto \frac{1}{m}$ to determine their kinetic energies.

---

48. The least count of a vernier caliper is $\frac{1}{20N}$ cm. The value of one division on the main scale is 1 mm. Then the number of divisions of the main scale that coincide with N divisions of the vernier scale is:

(1) $\frac{2N-1}{20N}$
(2) $\frac{2N-1}{2}$
(3) $(2N-1)$
(4) $\frac{2N-1}{2N}$

Answer: (2) $\frac{2N-1}{2}$
Solution:

The least count (LC) of a vernier caliper is given by: [LC = 1 main scale division - 1 vernier scale division]

Given that the least count is

$$\frac{1}{20N}$$

cm and one main scale division is 1 mm = 0.1 cm, we can write:

$$\frac{1}{20N} \text{ cm} = 0.1 \text{ cm} - \frac{N}{x} \text{ cm}$$

where 'x' represents the total number of vernier scale divisions.

We can assume that 'x' vernier scale divisions are equal to N main scale divisions. Thus,

$$\frac{N}{x} \text{ cm}$$

represents the length of N vernier divisions. Solving for

$$\frac{N}{x}$$

, we get:

$$\frac{N}{x} = 0.1 - \frac{1}{20N} = \frac{2N-1}{20N} \text{ cm}$$

Now, we know that N vernier scale divisions coincide with (n) main scale divisions. Then, the length of N vernier divisions equals the length of n main scale divisions:

$$\frac{N}{x} \text{ cm} = n \times 0.1 \text{ cm}$$

Therefore:

$$n = \frac{N}{x} \times \frac{1}{0.1} = \frac{2N-1}{20N} \times 10 = \frac{2N-1}{2}$$

Thus, the number of main scale divisions that coincide with N vernier scale divisions is

$$\frac{2N-1}{2}$$

.

Quick Tip

In vernier caliper problems, use the formula for least count and the relation between main scale and vernier scale divisions for accurate calculations.

---

49. If M_0 is the mass of isotope ${}^{12}_{5}B$, M_p and M_n are the masses of proton and neutron, then nuclear binding energy of the isotope is:

(1) $(5M_p + 7M_n - M_0)c^2$
(2) $(M_0 - 5M_p)c^2$
(3) $(M_0 - 12M_n)c^2$
(4) $(M_0 - 5M_p - 7M_n)c^2$

Answer: (1) $(5M_p + 7M_n - M_0)c^2$
Solution:

The binding energy $(B.E.)$ of a nucleus is given by:

$$B.E. = \Delta mc^2,$$

where Δm is the mass defect.

The mass defect for the isotope ${}^{12}_{5}B$ is:

$$\Delta m = (5M_p + 7M_n) - M_0.$$

Substituting Δm into the binding energy equation:

$$B.E. = (5M_p + 7M_n - M_0)\, c^2.$$

Thus, the nuclear binding energy of the isotope is:

$$B.E. = (5M_p + 7M_n - M_0)c^2.$$

Quick Tip

For nuclear binding energy calculations, determine the mass defect by subtracting the actual nuclear mass from the sum of masses of the protons and neutrons.

---

50. A diatomic gas ($\gamma = 1.4$) does $100\,\mathrm{J}$ of work in an isobaric expansion. The heat given to the gas is:

(1) $350\,\mathrm{J}$
(2) $490\,\mathrm{J}$

(3) 150 J
(4) 250 J

Answer: (1) 350 J
Solution:
For an isobaric process, the work done is:

$$w = P\Delta V = nR\Delta T.$$

Given:

$$w = 100\,\text{J}.$$

The heat (Q) supplied is:

$$Q = \Delta U + w,$$

where the internal energy change (ΔU) for a diatomic gas is:

$$\Delta U = \frac{f}{2}nR\Delta T,$$

with $f = 5$ (degrees of freedom for a diatomic gas).
Substitute:

$$Q = \frac{f}{2}nR\Delta T + nR\Delta T.$$

Simplify:

$$Q = \left(\frac{f}{2} + 1\right) nR\Delta T.$$

Substitute ΔT from $w = nR\Delta T$:

$$Q = \left(\frac{f}{2} + 1\right) \cdot w.$$

Substitute $f = 5$ and $w = 100\,\text{J}$:

$$Q = \left(\frac{5}{2} + 1\right) \cdot 100 = \left(\frac{7}{2}\right) \cdot 100 = 350\,\text{J}.$$

Thus, the heat given to the gas is:

$$Q = 350\,\text{J}.$$

Quick Tip

For isobaric processes, use the formula $Q = \Delta U + w$, and remember $\Delta U = \frac{f}{2}nR\Delta T$ for an ideal gas.

---

51. The coercivity of a magnet is 5×10^3 A/m. The amount of current required to be passed in a solenoid of length 30 cm and number of turns 150, so that the magnet gets demagnetised when inside the solenoid, is . . . A.

Answer: (10)
Solution:

The coercivity of the magnet is given by:

$$H_c = \mu_0 \frac{ni}{\mu_0},$$

where:

$$H_c = 5 \times 10^3\,\text{A/m}, \quad n = \frac{\text{Number of turns}}{\text{Length of solenoid}} = \frac{150}{0.3} = 500\,\text{turns/m}.$$

Substitute into the formula:

$$5 \times 10^3 = 500 \cdot i.$$

Solve for i:

$$i = \frac{5 \times 10^3}{500} = 10\,\text{A}.$$

Thus, the current required is:

$$i = 10\,\text{A}.$$

Quick Tip

For coercivity problems, use the relation $H_c = ni$ and calculate the number of turns per unit length carefully.

---

52. Small water droplets of radius $0.01\,\text{mm}$ are formed in the upper atmosphere and fall with a terminal velocity of $10\,\text{cm/s}$. Due to condensation, if 8 such droplets are coalesced and form a larger drop, the new terminal velocity will be . . . cm/s.
Answer: (40)
Solution:

The terminal velocity (V_t) of a droplet is given by:

$$V_t = \frac{2R^2(\rho - \sigma)g}{9\eta},$$

where R is the radius of the droplet.

When 8 small droplets coalesce:

$$\text{Mass of larger drop } M \propto 8\,m, \quad M = \frac{4}{3}\pi R^3(\rho).$$

The radius of the larger drop is:

$$R_{\text{large}}^3 = 8R_{\text{small}}^3 \implies R_{\text{large}} = 2R_{\text{small}}.$$

Since terminal velocity (V_t) is proportional to R^2:

$$V_t \propto R^2 \implies V_t^{\text{large}} = 4V_t^{\text{small}}.$$

For the smaller droplet:

$$V_t^{\text{small}} = 10\,\text{cm/s}.$$

Thus:

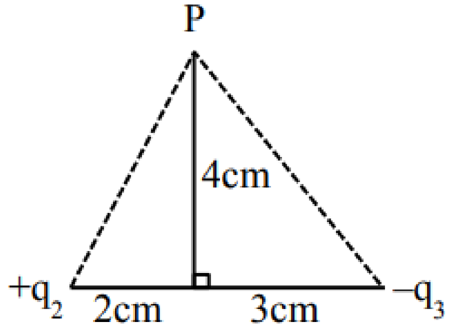
$$V_t^{\text{large}} = 4 \cdot 10 = 40\,\text{cm/s}.$$

Quick Tip

For terminal velocity problems, remember the proportionality $V_t \propto R^2$, and for coalesced droplets, use $R_{\text{large}} = (NR^3)^{1/3}$ where N is the number of smaller droplets.

---

53. If the net electric field at point P along the Y-axis is zero, then the ratio of $\left|\frac{q_2}{q_3}\right|$ is $\frac{8}{5\sqrt{x}}$, where $x = \ldots$.



Answer: (5)
Solution:

The geometry of the charge configuration is shown in the figure, with q_2 located at a distance $2\,\text{cm}$ and q_3 at $3\,\text{cm}$. The distances of these charges from point P are:

$$\sqrt{20}\,\text{cm} \quad \text{and} \quad \sqrt{25}\,\text{cm}.$$

The horizontal components of the electric field cancel each other, and the net electric field along the Y-axis is zero. Using the electric field formula:

$$E = \frac{kq}{r^2},$$

the condition for $E_y = 0$ gives:

$$\frac{kq_2}{20}\cos\beta = \frac{kq_3}{25}\cos\theta.$$

The angles β and θ are such that:

$$\cos\beta = \frac{4}{\sqrt{20}}, \quad \cos\theta = \frac{4}{\sqrt{25}}.$$

Substitute these into the equation:

$$\frac{q_2}{20} \cdot \frac{4}{\sqrt{20}} = \frac{q_3}{25} \cdot \frac{4}{\sqrt{25}}.$$

Simplify:

$$\frac{q_2}{q_3} = \frac{20}{25} \cdot \frac{\sqrt{25}}{\sqrt{20}} = \frac{8}{5\sqrt{x}}.$$

From the given condition:

$$\sqrt{x} = \frac{8 \cdot 25 \cdot \sqrt{25}}{5 \cdot 20 \cdot \sqrt{20}}.$$

Simplify:

$$\sqrt{x} = \sqrt{5} \implies x = 5.$$

Thus, the value of x is:

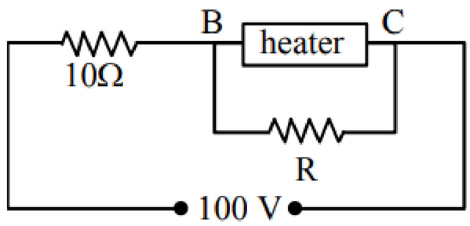
$$x = 5.$$

Quick Tip

To solve electric field problems involving superposition, break the fields into components and equate the net field along the axis of interest to zero.

---

54. A heater is designed to operate with a power of $1000\,\text{W}$ in a $100\,\text{V}$ line. It is connected in combination with a resistance of $10\,\Omega$ and a resistance R, to a $100\,\text{V}$ mains as shown in the figure. For the heater to operate at $62.5\,\text{W}$, the value of R should be $\ldots\Omega$.



Answer: (5)
Solution:

The resistance of the heater is:

$$R_{\text{heater}} = \frac{V^2}{P} = \frac{(100)^2}{1000} = 10\,\Omega.$$

For the heater operating at $P = 62.5\,\text{W}$, the voltage across the heater is:

$$P = \frac{V^2}{R} \implies V = \sqrt{PR}.$$

Substitute $P = 62.5\,\text{W}$ and $R = 10\,\Omega$:

$$V = \sqrt{62.5 \times 10} = 25\,\text{V}.$$

In the circuit: - The voltage drop across the $10\,\Omega$ resistor is:

$$V_R = 100 - 25 = 75\,\text{V}.$$

The current through the $10\,\Omega$ resistor is:

$$i_1 = \frac{V_R}{R} = \frac{75}{10} = 7.5\,\text{A}.$$

The current through the heater is:

$$i_H = \frac{V}{R} = \frac{25}{10} = 2.5\,\text{A}.$$

The current through R is:

$$i_R = i_1 - i_H = 7.5 - 2.5 = 5\,\text{A}.$$

Using Ohm's law for R:

$$V = i_R R \implies R = \frac{V}{i_R}.$$

Substitute $V = 25\,\text{V}$ and $i_R = 5\,\text{A}$:

$$R = \frac{25}{5} = 5\,\Omega.$$

Thus, the value of R is:

$$R = 5\,\Omega.$$

Quick Tip

In combined resistor and heater circuits, use the principles of voltage division and current conservation to determine the unknown resistance.

---

55. An alternating emf $E = 110\sqrt{2}\sin(100t)\,\text{V}$ is applied to a capacitor of $2\,\mu\text{F}$. The RMS value of current in the circuit is . . . mA.

Answer: (22)

Solution:

Given:

$$C = 2\,\mu\text{F}, \quad E = 110\sqrt{2}\sin(100t).$$

The capacitive reactance is:

$$X_C = \frac{1}{\omega C},$$

where $\omega = 100\,\text{rad/s}$ and $C = 2 \times 10^{-6}\,\text{F}$.

Substitute:

$$X_C = \frac{1}{100 \cdot 2 \times 10^{-6}} = \frac{1}{2 \times 10^{-4}} = 5000\,\Omega.$$

The peak current is:

$$i_0 = \frac{E_0}{X_C},$$

where $E_0 = 110\sqrt{2}\,\text{V}$.

Substitute:

$$i_0 = \frac{110\sqrt{2}}{5000}.$$

The RMS value of current is:

$$i_{\text{rms}} = \frac{i_0}{\sqrt{2}}.$$

Substitute:

$$i_{\text{rms}} = \frac{110\sqrt{2}}{5000\sqrt{2}} = \frac{110}{5000}.$$

Simplify:

$$i_{\text{rms}} = \frac{110}{5}\,\text{mA} = 22\,\text{mA}.$$

Thus, the RMS value of current in the circuit is:

$$i_{\text{rms}} = 22\,\text{mA}.$$

Quick Tip

For AC circuits with capacitors, use $X_C = \frac{1}{\omega C}$ to calculate reactance and determine the current using $i_{\text{rms}} = \frac{E_{\text{rms}}}{X_C}$.

---

56. Two slits are $1\,\text{mm}$ apart, and the screen is located $1\,\text{m}$ away from the slits. A light wavelength $500\,\text{nm}$ is used. The width of each slit to obtain 10 maxima of the double-slit pattern within the central maximum of the single-slit pattern is $\ldots \times 10^{-4}\,\text{m}$.

Answer: (2)

Solution:

Given:

$$d = 1\,\text{mm} = 10^{-3}\,\text{m}, \quad D = 1\,\text{m}, \quad \lambda = 500\,\text{nm} = 5 \times 10^{-7}\,\text{m}.$$

For the central maximum of the single-slit pattern to contain 10 maxima of the double-slit pattern:

$$10 \cdot \frac{\lambda D}{d} = \frac{2\lambda D}{a},$$

where a is the slit width.

Rearranging for a:

$$a = \frac{d}{5}.$$

Substitute $d = 10^{-3}\,\text{m}$:

$$a = \frac{10^{-3}}{5} = 2 \times 10^{-4}\,\text{m}.$$

Thus, the slit width is:

$$a = 2 \times 10^{-4}\,\text{m}.$$

Quick Tip

For interference and diffraction problems, use the relation between the slit width and number of maxima in the central maximum, ensuring consistent units.

---

57. An object of mass $0.2\,\text{kg}$ executes simple harmonic motion along the x-axis with a frequency of $\left(\frac{25}{\pi}\right)$ Hz. At the position $x = 0.04\,\text{m}$, the object has kinetic energy $0.5\,\text{J}$ and potential energy $0.4\,\text{J}$. The amplitude of oscillation is . . . cm.

Answer: (6)

Solution:

The total energy $(T.E.)$ in SHM is the sum of kinetic energy $(K.E.)$ and potential energy $(P.E.)$:

$$T.E. = K.E. + P.E.$$

Substitute $K.E. = 0.5\,\text{J}$ and $P.E. = 0.4\,\text{J}$:

$$T.E. = 0.5 + 0.4 = 0.9\,\text{J}.$$

The total energy is also given by:

$$T.E. = \frac{1}{2}m\omega^2 A^2,$$

where:

$$\omega = 2\pi f \quad \text{and} \quad f = \frac{25}{\pi}.$$

Substitute ω:

$$\omega = 2\pi \cdot \frac{25}{\pi} = 50\,\text{rad/s}.$$

Substitute $T.E. = 0.9\,\text{J}$, $m = 0.2\,\text{kg}$, $\omega = 50\,\text{rad/s}$:

$$0.9 = \frac{1}{2} \cdot 0.2 \cdot (50)^2 \cdot A^2.$$

Simplify:

$$0.9 = 0.1 \cdot 2500 \cdot A^2 \implies A^2 = \frac{0.9}{250} = 0.0036.$$

Solve for A:

$$A = \sqrt{0.0036} = 0.06\,\text{m}.$$

Convert to centimeters:

$$A = 6\,\text{cm}.$$

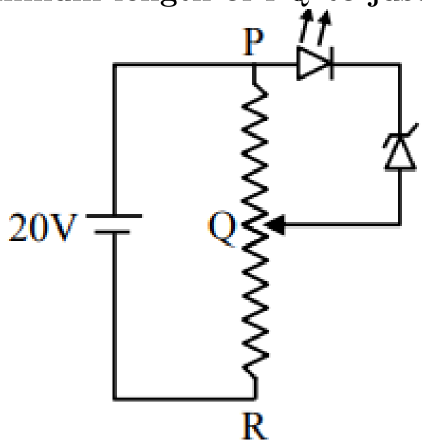
Thus, the amplitude of oscillation is:

$$A = 6\,\text{cm}.$$

Quick Tip

In SHM problems, calculate total energy as the sum of kinetic and potential energy, and use $\omega = 2\pi f$ for angular frequency.

58. A potential divider circuit is connected with a DC source of $20\,\text{V}$, a light-emitting diode (LED) with a glow voltage of $1.8\,\text{V}$, and a zener diode with a breakdown voltage of $3.2\,\text{V}$. The length (PR) of the resistive wire is $20\,\text{cm}$. The minimum length of PQ to just glow the LED is $\ldots$ cm.



Answer: (5)
Solution:

The total voltage across PR is:

$$V_{PR} = 20\,\text{V}.$$

The resistive wire PR has a total length of:

$$\ell_{PR} = 20\,\text{cm}.$$

The voltage across QR is determined by the zener diode:

$$V_{QR} = 3.2\,\text{V}.$$

The voltage across PQ is:

$$V_{PQ} = V_{PR} - V_{QR} = 20 - 3.2 = 16.8\,\text{V}.$$

The fraction of voltage across PQ relative to PR is:

$$\frac{V_{PQ}}{V_{PR}} = \frac{16.8}{20} = \frac{1}{4}.$$

Using the proportionality of voltage and length:

$$\ell_{PQ} = \frac{1}{4} \cdot \ell_{PR}.$$

Substitute $\ell_{PR} = 20\,\text{cm}$:

$$\ell_{PQ} = \frac{1}{4} \cdot 20 = 5\,\text{cm}.$$

Thus, the minimum length of PQ to just glow the LED is:

$$\ell_{PQ} = 5\,\text{cm}.$$

Quick Tip

In potential divider circuits, use the proportionality between voltage and length for resistive wires to calculate the segment lengths for specific voltage drops.

---

59. A body of mass M thrown horizontally with velocity v from the top of a tower of height H touches the ground at a distance of $100\,\mathrm{m}$ from the foot of the tower. A body of mass $2M$ thrown at a velocity $\frac{v}{2}$ from the top of a tower of height $4H$ will touch the ground at a distance of ... m.

Answer: (100)

Solution:

For the first body:

$$\text{Horizontal distance} = \text{horizontal velocity} \times \text{time of flight}.$$

The time of flight for a height H is:

$$t_1 = \sqrt{\frac{2H}{g}}.$$

The horizontal distance for the first body is:

$$x_1 = v \cdot t_1 = v \cdot \sqrt{\frac{2H}{g}}.$$

Given:

$$x_1 = 100\,\mathrm{m}.$$

For the second body: - The height is $4H$, so the time of flight is:

$$t_2 = \sqrt{\frac{2(4H)}{g}} = 2\sqrt{\frac{2H}{g}}.$$

- The horizontal velocity is $\frac{v}{2}$. The horizontal distance for the second body is:

$$x_2 = \frac{v}{2} \cdot t_2.$$

Substitute $t_2 = 2\sqrt{\frac{2H}{g}}$:

$$x_2 = \frac{v}{2} \cdot 2\sqrt{\frac{2H}{g}} = v \cdot \sqrt{\frac{2H}{g}}.$$

From the first case:

$$v \cdot \sqrt{\frac{2H}{g}} = 100\,\mathrm{m}.$$

Thus:

$$x_2 = 100\,\text{m}.$$

Final Result:

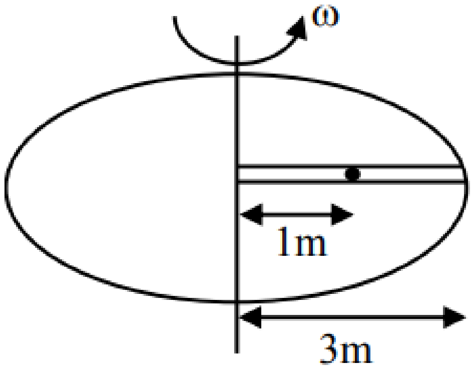
$$x = 100\,\text{m}.$$

Quick Tip

For horizontally projected motion, the horizontal distance depends only on the horizontal velocity and time of flight. Time of flight is determined solely by the vertical height.

---

60. A circular table is rotating with an angular velocity of $\omega\,\text{rad/s}$ about its axis (see figure). There is a smooth groove along a radial direction on the table. A steel ball is gently placed at a distance of $1\,\text{m}$ on the groove. All surfaces are smooth. If the radius of the table is $3\,\text{m}$, the radial velocity of the ball with respect to the table at the time the ball leaves the table is $x\sqrt{2}\omega\,\text{m/s}$, where the value of x is ....



Answer: (2)
Solution:

The centripetal acceleration acting on the ball is:

$$a_c = \omega^2 x.$$

Using the relationship between velocity and position:

$$v\frac{dv}{dx} = \omega^2 x.$$

Integrate both sides:

$$\int_0^v v\,dv = \int_1^3 \omega^2 x\,dx.$$

Solve the integrals:

$$\frac{v^2}{2} = \omega^2 \int_1^3 x\,dx = \omega^2 \left[\frac{x^2}{2}\right]_1^3.$$

Substitute the limits:

$$\frac{v^2}{2} = \omega^2 \cdot \frac{1}{2}\left[3^2 - 1^2\right].$$

Simplify:

$$\frac{v^2}{2} = \omega^2 \cdot \frac{1}{2} \cdot (9-1) = \omega^2 \cdot \frac{1}{2} \cdot 8 = 4\omega^2.$$

Solve for v:

$$v^2 = 8\omega^2 \implies v = \sqrt{8}\omega = 2\sqrt{2}\omega.$$

Thus, the radial velocity of the ball as it leaves the table is:

$$v = 2\sqrt{2}\omega,$$

where $x = 2$.

Quick Tip

For problems involving radial motion on a rotating table, use centripetal acceleration and energy conservation principles, and apply definite integration for variable accelerations.

Chemistry

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61. In qualitative tests for the identification of the presence of phosphorus, the compound is heated with an oxidizing agent. It is then treated with nitric acid and ammonium molybdate, respectively. The yellow-colored precipitate obtained is:

(1) $Na_3PO_4 \cdot 12MoO_3$
(2) $(NH_4)_3PO_4 \cdot 12(NH_4)_2MoO_4$
(3) $(NH_4)_3PO_4 \cdot 12MoO_3$
(4) $MoPO_4 \cdot 21NH_4NO_3$

Answer: (3)
Solution:

In the qualitative analysis of phosphorus, the following reaction occurs:

$$PO_4^{3-} \text{ or } HPO_4^{2-} + (NH_4)_2MoO_4 \xrightarrow{H^+} (NH_4)_3PO_4 \cdot 12MoO_3 \downarrow$$

The product, $(NH_4)_3PO_4 \cdot 12MoO_3$, is a canary yellow precipitate known as ammonium phosphomolybdate.

Quick Tip

In the phosphorus test, the yellow precipitate is a characteristic indicator of PO_4^{3-} ions, confirming the presence of phosphorus in the compound.

62. For a reaction $\mathrm{A} \xrightarrow{\mathrm{k_1}} \mathrm{B} \xrightarrow{\mathrm{k_2}} \mathrm{C}$, if the rate of formation of B is set to zero, then the concentration of B is given by:
(1) $k_1 k_2[\mathrm{A}]$
(2) $(k_1 - k_2)[\mathrm{A}]$
(3) $(k_1 + k_2)[\mathrm{A}]$
(4) $\left(\frac{k_1}{k_2}\right)[\mathrm{A}]$

Answer: (4)
Solution:
The rate of formation of B is:

$$\frac{d[\mathrm{B}]}{dt} = k_1[\mathrm{A}] - k_2[\mathrm{B}].$$

For the rate of formation of B to be zero:

$$\frac{d[\mathrm{B}]}{dt} = 0.$$

Substitute:

$$k_1[\mathrm{A}] - k_2[\mathrm{B}] = 0.$$

Rearrange to find [B]:

$$k_1[\mathrm{A}] = k_2[\mathrm{B}] \implies [\mathrm{B}] = \frac{k_1}{k_2}[\mathrm{A}].$$

Thus, the concentration of B is:

$$[\mathrm{B}] = \left(\frac{k_1}{k_2}\right)[\mathrm{A}].$$

Quick Tip

For reactions involving intermediate species, set the rate of formation of intermediates to zero to find their steady-state concentrations.

---

63. When ψ_A and ψ_B are the wave functions of atomic orbitals, then σ^* is represented by:
(1) $\psi_A - 2\psi_B$
(2) $\psi_A - \psi_B$
(3) $\psi_A + 2\psi_B$
(4) $\psi_A + \psi_B$

Answer: (2)
Solution:
The bonding (σ) and anti-bonding (σ^*) molecular orbitals are formed by the constructive and destructive interference of atomic orbitals' wave functions.

For an anti-bonding molecular orbital (σ^*):

$$\psi_{\sigma^*} = \psi_A - \psi_B.$$

This occurs due to the out-of-phase overlap of the wave functions, leading to a node between the nuclei and a higher energy state.

Thus, σ^* is represented by:

$$\psi_A - \psi_B.$$

Quick Tip

Anti-bonding molecular orbitals (σ^*) result from the destructive interference of atomic wave functions, leading to increased energy and a node.

---

64. Which one of the following compounds will readily react with dilute NaOH?

(1) $C_6H_5CH_2OH$
(2) C_2H_5OH
(3) $(CH_3)_3COH$
(4) C_6H_5OH

Answer: (4)

Solution:

Phenol (C_6H_5OH) reacts readily with dilute NaOH because it is more acidic than water. The reaction is as follows:

$$C_6H_5OH + NaOH \rightarrow C_6H_5O^-Na^+ + H_2O.$$

The phenoxide ion ($C_6H_5O^-$) formed is stabilized by resonance, making phenol a stronger acid than water.

Quick Tip

Phenol reacts with bases due to its higher acidity caused by resonance stabilization of the phenoxide ion.

---

65. The shape of a carbocation is:

(1) Trigonal planar
(2) Diagonal pyramidal
(3) Tetrahedral
(4) Diagonal

Answer: (1)

Solution:

A carbocation has a central carbon atom with three bonded groups and one empty p-orbital. The geometry around the carbocation is trigonal planar because the three groups are arranged at 120° to minimize electron repulsion.

$$\text{Carbocation structure:} \quad \text{H-C}^+\text{-H.}$$

The trigonal planar shape is due to sp^2-hybridization of the central carbon atom.

Quick Tip

Carbocations adopt a trigonal planar geometry due to sp^2-hybridization and the empty p-orbital.

---

66. Given below are two statements:

- **Statement I:** S_N2 reactions are 'stereospecific', indicating that they result in the formation of only one stereoisomer as the product.
- **Statement II:** S_N1 reactions generally result in the formation of products as racemic mixtures.

In the light of the above statements, choose the correct answer from the options given below:
(1) Statement I is true but Statement II is false
(2) Statement I is false but Statement II is true
(3) Both Statement I and Statement II are true
(4) Both Statement I and Statement II are false

Answer: (3)
Solution:
Explanation: - S_N2 reactions are stereospecific and proceed via a backside attack, resulting in the inversion of configuration at the chiral center. - S_N1 reactions occur via a carbocation intermediate, which is planar. As a result, nucleophiles can attack from either side, leading to a racemic mixture of products.
Thus:

$$S_N2 \rightarrow \text{Inversion (stereospecific)}, \quad S_N1 \rightarrow \text{Racemization.}$$

Both statements are correct.

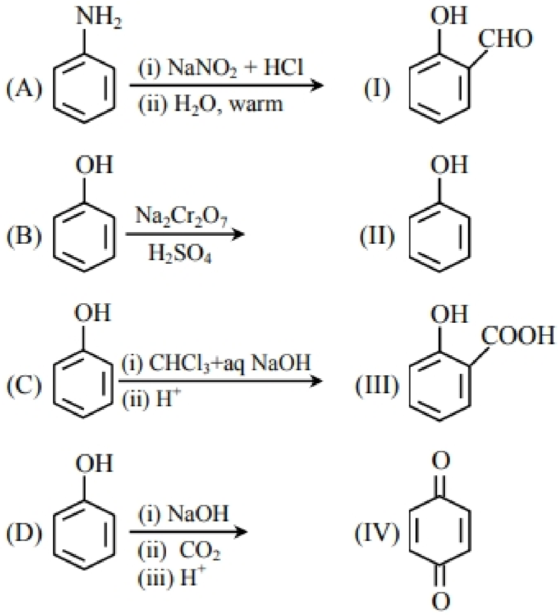
Quick Tip

S_N2 reactions are stereospecific and lead to inversion, while S_N1 reactions produce racemic mixtures due to the planar nature of the carbocation intermediate.

---

67. Match List-I with List-II.

List-I **(Reactions)** | **List-II** **(Products)**



Choose the correct answer from the options given below :

(1) (A)-(III), (B)-(II), (C)-(I), (D)-(IV)

(2) (A)-(IV), (B)-(II), (C)-(III), (D)-(I)

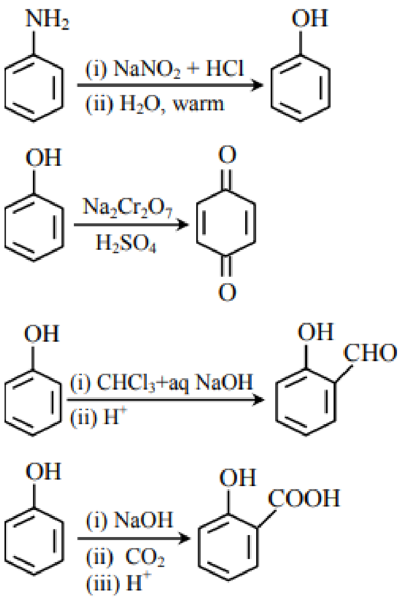
(3) (A)-(I), (B)-(IV), (C)-(II), (D)-(III)

(4) (A)-(II), (B)-(IV), (C)-(I), (D)-(III)

Answer: (4)

Solution:

Sol.



Quick Tip

When matching reactions to their products, focus on identifying key reagents and understanding their typical transformations. For example, the diazotization reaction with $NaNO_2$ and HCl typically leads to a diazonium salt, which can undergo further reactions, while the use of $Na_2Cr_2O_7$ and H_2SO_4 signifies an oxidation reaction. Remembering characteristic outcomes for each reagent can help you quickly pair reactions and products.

---

68. Match List-I with List-II.

List-I (Test)	List-II (Identification)
(A) Bayer's test	(I) Phenol
(B) Ceric ammonium nitrate test	(II) Aldehyde
(C) Phthalein dye test	(III) Alcoholic-OH group
(D) Schiff's test	(IV) Unsaturation

Choose the correct answer from the options given below:

- (1) (A)-(III), (B)-(I), (C)-(IV), (D)-(II)
- (2) (A)-(II), (B)-(III), (C)-(IV), (D)-(I)
- (3) (A)-(IV), (B)-(I), (C)-(II), (D)-(III)
- (4) (A)-(IV), (B)-(III), (C)-(I), (D)-(II)

Answer: (4)
Solution:

- **Bayer's test:** Identifies unsaturation by decolorization of potassium permanganate ($KMnO_4$).
- **Ceric ammonium nitrate test:** Identifies alcoholic groups ($-OH$).
- **Phthalein dye test:** Confirms phenol groups (C_6H_5OH).
- **Schiff's test:** Used for detecting aldehydes ($-CHO$).

Correct matching:

$$(A)\text{-}(IV),\ (B)\text{-}(III),\ (C)\text{-}(I),\ (D)\text{-}(II).$$

Quick Tip

Use chemical reactivity and visual indicators for identifying organic functional groups via specific qualitative tests.

---

69. Identify the incorrect statements about Group 15 elements:

(A) Dinitrogen is a diatomic gas that acts like an inert gas at room temperature.
(B) The common oxidation states of these elements are -3, $+3$, and $+5$.
(C) Nitrogen has a unique ability to form $p\pi$-$p\pi$ multiple bonds.
(D) The stability of $+5$ oxidation state increases down the group.
(E) Nitrogen shows a maximum covalency of 6.

Choose the correct answer from the options given below:

(1) (A), (B), (D) only
(2) (A), (C), (E) only
(3) (B), (D), (E) only
(4) (D) and (E) only

Answer: (4)
Solution:

- **Statement (D):** The stability of the $+5$ oxidation state **decreases** down the group due to the inert pair effect.
- **Statement (E):** Nitrogen belongs to the second period and cannot expand its octet, hence it cannot show a maximum covalency of 6.

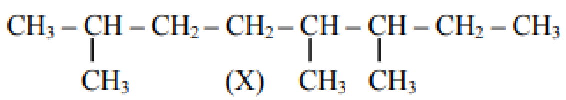
Corrected Statements: - $+5$ oxidation state stability decreases down the group. - Nitrogen shows a maximum covalency of 4.

Quick Tip

For Group 15 elements, remember the inert pair effect and octet expansion limits. Nitrogen's unique properties arise from its small size and high electronegativity.

---

70. The IUPAC name of the following hydrocarbon (X) is:



(1) 2-Ethyl-3,6-dimethylheptane
(2) 2-Ethyl-2,6-diethylheptane
(3) 2,5,6-Trimethyloctane
(4) 3,4,7-Trimethyloctane

Answer: (3)
Solution:

To determine the IUPAC name: 1. Identify the longest continuous carbon chain, which has 8 carbons (octane). 2. Number the carbon chain to give the substituents the lowest possible locants: - Methyl groups are at positions 2, 5, and 6. 3. Arrange the substituents in alphabetical order and include their positions.

The compound is named:

$$2,5,6\text{-Trimethyloctane}$$

Quick Tip

For IUPAC nomenclature: - Identify the longest carbon chain as the parent. - Number the chain to give substituents the lowest locants. - Arrange substituents alphabetically in the name.

---

71. The equilibrium $Cr_2O_7^{2-} \rightleftharpoons 2CrO_4^{2-}$ is shifted to the right in:
(1) an acidic medium
(2) a basic medium
(3) a weakly acidic medium
(4) a neutral medium

Answer: (2)
Solution:
The equilibrium reaction is:

$$Cr_2O_7^{2-}[H^+]OH^-2CrO_4^{2-}.$$

In a basic medium (OH^-), the reaction shifts to the right to produce more CrO_4^{2-}, as OH^- consumes H^+, reducing its concentration. Conversely, in an acidic medium (H^+), the equilibrium shifts to the left, favoring $Cr_2O_7^{2-}$.
Thus, the equilibrium shifts to the right in a basic medium.

Quick Tip

Le Chatelier's principle: In a basic medium, OH^- reduces H^+, driving the equilibrium toward CrO_4^{2-}. In an acidic medium, H^+ favors $Cr_2O_7^{2-}$.

---

72. Given below are two statements:

- **Statement I:** A buffer solution is the mixture of a salt and an acid or a base mixed in any particular quantities.
- **Statement II:** Blood is a naturally occurring buffer solution whose pH is maintained by H_2CO_3/HCO_3^- concentrations.

In the light of the above statements, choose the correct answer from the options given below:
(1) Statement I is false but Statement II is true
(2) Both Statement I and Statement II are true
(3) Both Statement I and Statement II are false

(4) Statement I is true but Statement II is false

Answer: (1)
Solution:

- **Statement I: False.** A buffer solution is a mixture of a weak acid and its conjugate base (or a weak base and its conjugate acid) in **specific ratios**. The statement's claim of "any particular quantities" is incorrect.
- **Statement II: True.** Blood is a naturally occurring buffer system, primarily maintained by the H_2CO_3/HCO_3^- system, which helps regulate pH within a narrow range.

Thus, only Statement II is correct.

Quick Tip

Buffer solutions require specific ratios of weak acids and their conjugate bases (or weak bases and their conjugate acids) to resist pH changes effectively.

---

73. The correct sequence of acidic strength of the following aliphatic acids in their decreasing order is:

$$CH_3CH_2COOH,\ CH_3COOH,\ CH_3CH_2CH_2COOH,\ HCOOH$$

(1) $HCOOH > CH_3COOH > CH_3CH_2COOH > CH_3CH_2CH_2COOH$
(2) $HCOOH > CH_3CH_2CH_2COOH > CH_3CH_2COOH > CH_3COOH$
(3) $CH_3CH_2CH_2COOH > CH_3CH_2COOH > CH_3COOH > HCOOH$
(4) $CH_3COOH > CH_3CH_2COOH > CH_3CH_2CH_2COOH > HCOOH$

Answer: (1)
Solution:
The acidic strength of carboxylic acids is influenced by the electron-withdrawing or electron-donating effects of alkyl groups:

- $HCOOH$ (formic acid) is the most acidic as it has no electron-donating alkyl group, which would reduce acidity.
- CH_3COOH (acetic acid) is less acidic because the methyl group (CH_3) is weakly electron-donating.
- CH_3CH_2COOH (propionic acid) is even less acidic due to the larger electron-donating ethyl group.
- $CH_3CH_2CH_2COOH$ (butyric acid) is the least acidic because the longer alkyl chain has a stronger electron-donating effect.

The correct order of acidic strength is:

$$HCOOH > CH_3COOH > CH_3CH_2COOH > CH_3CH_2CH_2COOH.$$

Quick Tip

In carboxylic acids, acidity decreases as the alkyl chain length increases due to the electron-donating effect, which reduces the stability of the conjugate base.

---

74. Given below are two statements:

- **Statement I:** All the following compounds react with *p*-toluenesulfonyl chloride:

$$C_6H_5NH_2,\ (C_6H_5)_2NH,\ (C_6H_5)_3N.$$

- **Statement II:** Their products in the above reaction are soluble in aqueous NaOH.

In the light of the above statements, choose the correct answer from the options given below:
(1) Both Statement I and Statement II are false
(2) Statement I is true but Statement II is false
(3) Statement I is false but Statement II is true
(4) Both Statement I and Statement II are true

Answer: (1)
Solution:
The reaction with *p*-toluenesulfonyl chloride (TsCl) is known as the Hinsberg test, which is specific for 1° amines.

- Only 1° amines ($C_6H_5NH_2$) react with TsCl to form sulfonamides that are soluble in aqueous NaOH.
- 2° amines ($(C_6H_5)_2NH$) react with TsCl to form sulfonamides that are insoluble in NaOH.
- 3° amines ($(C_6H_5)_3N$) do not react with TsCl at all.

Thus:

- **Statement I: False.** Not all the given compounds react with TsCl.
- **Statement II: False.** Only the sulfonamide formed by 1° amines is soluble in NaOH.

Quick Tip

The Hinsberg test distinguishes 1°, 2°, and 3° amines based on their reaction with TsCl and the solubility of their products.

75. The emf of the cell:

$$\mathrm{Tl}\,|\,\mathrm{Tl}^{+}\,(0.001\,\mathrm{M})\,||\,\mathrm{Cu}^{2+}\,(0.01\,\mathrm{M})\,|\,\mathrm{Cu}$$

is 0.83 V at 298 K. It could be increased by:
(1) increasing concentration of Tl^{+} ions
(2) increasing concentration of both Tl^{+} and Cu^{2+} ions
(3) decreasing concentration of both Tl^{+} and Cu^{2+} ions
(4) increasing concentration of Cu^{2+} ions

Answer: (4)
Solution:
The emf of the cell is given by the Nernst equation:

$$E = E^{\circ} - \frac{0.0591}{n} \log \frac{[\text{Products}]}{[\text{Reactants}]}.$$

For the cell:

$$\mathrm{Tl}\,|\,\mathrm{Tl}^{+}\,||\,\mathrm{Cu}^{2+}\,|\,\mathrm{Cu},$$

the cell reaction is:

$$\mathrm{Tl} + \mathrm{Cu}^{2+} \rightarrow \mathrm{Tl}^{+} + \mathrm{Cu}.$$

Effect of concentration: 1. Increasing the concentration of Cu^{2+} ions (**product-side reactant**) decreases the denominator in the reaction quotient, shifting the reaction to the right and increasing the emf. 2. Increasing the concentration of Tl^{+} ions (**reactant-side product**) increases the numerator in the reaction quotient, decreasing the emf. 3. Decreasing concentrations of both ions would reduce the driving force of the reaction, lowering the emf.

Thus, the emf can be increased by:

$$\text{Increasing the concentration of } \mathrm{Cu}^{2+} \text{ ions (Option 4).}$$

Quick Tip

To increase the emf of a galvanic cell, increase the concentration of the oxidizing agent (ion reduced at the cathode) or decrease the concentration of the reducing agent (ion oxidized at the anode).

---

76. Identify the correct statements about p-block elements and their compounds:

(A) Non-metals have higher electronegativity than metals.
(B) Non-metals have lower ionization enthalpy than metals.
(C) Compounds formed between highly reactive non-metals and highly reactive metals are generally ionic.

(D) The non-metal oxides are generally basic in nature.
(E) The metal oxides are generally acidic or neutral in nature.

Choose the correct answer:

(1) (D) and (E) only
(2) (A) and (C) only
(3) (B) and (E) only
(4) (B) and (D) only

Answer: (2)
Solution:

- **Statement (A): True.** Non-metals, being more electronegative, tend to attract electrons more strongly compared to metals.
- **Statement (B): False.** Non-metals generally have higher ionization enthalpy than metals due to their smaller atomic sizes and higher nuclear charges.
- **Statement (C): True.** Compounds formed between highly reactive metals (like alkali metals) and highly reactive non-metals (like halogens) are ionic in nature due to significant electronegativity differences.
- **Statement (D): False.** Non-metal oxides are generally acidic in nature, not basic.
- **Statement (E): False.** Metal oxides are generally basic in nature, though some transition metal oxides may be amphoteric or acidic.

Thus, the correct statements are:

$$\text{(A) and (C) only.}$$

Quick Tip

For p-block elements, remember that non-metals are highly electronegative and form acidic oxides, while ionic compounds arise from significant electronegativity differences.

---

77. Given below are two statements:

Statement I: Kjeldahl method is applicable to estimate nitrogen in pyridine.
Statement II: The nitrogen present in pyridine can easily be converted into ammonium sulfate in Kjeldahl method.

In the light of the above statements, choose the correct answer from the options given below:

(1) Both Statement I and Statement II are false
(2) Statement I is false but Statement II is true
(3) Both Statement I and Statement II are true

(4) Statement I is true but Statement II is false

Answer: (1)
Solution:

Explanation: - The Kjeldahl method is used for estimating nitrogen in organic compounds that can convert nitrogen into ammonium sulfate by reaction with sulfuric acid. However, the nitrogen in pyridine is part of an aromatic ring system and is not easily converted into ammonium sulfate. - Therefore:

- **Statement I: False.** The Kjeldahl method cannot estimate nitrogen in pyridine.
- **Statement II: False.** The nitrogen in pyridine cannot be easily converted into ammonium sulfate.

Quick Tip

The Kjeldahl method is effective for aliphatic nitrogen compounds but not for aromatic compounds like pyridine, where nitrogen is part of a stable ring structure.

---

78. The reaction:

$$\frac{1}{2}H_2(g) + AgCl(s) \rightarrow H^+(aq) + Cl^-(aq) + Ag(s)$$

occurs in which of the following galvanic cells?
(1) $Pt|H_2(g)|HCl(soln.)|AgCl(s)|Ag$
(2) $Pt|H_2(g)|HCl(soln.)|AgNO_3(aq)|Ag$
(3) $Pt|H_2(g)|KCl(soln.)|AgCl(s)|Ag$
(4) $Ag|AgCl(s)|KCl(soln.)|AgNO_3(aq)|Ag$

Answer: (3)
Solution:

The reaction involves:

- $H_2(g)$ oxidizing to $H^+(aq)$ in the anodic half-cell:

$$\frac{1}{2}H_2(g) \rightarrow H^+(aq) + e^-.$$

- $AgCl(s)$ reducing to $Ag(s)$ and $Cl^-(aq)$ in the cathodic half-cell:

$$AgCl(s) + e^- \rightarrow Ag(s) + Cl^-(aq).$$

Thus, the complete galvanic cell setup for the reaction is:

$$Pt|H_2(g)|KCl(soln.)|AgCl(s)|Ag.$$

Here: - $H_2(g)$ serves as the gas electrode for the oxidation at the anode. - $AgCl(s)$ is reduced at the cathode.

Quick Tip

For galvanic cell reactions, identify the anodic and cathodic processes, and ensure ion migration in the salt bridge matches the setup.

---

79. Given below are two statements:

- **Statement I:** Fusion of MnO_2 with KOH and an oxidizing agent gives dark green K_2MnO_4.
- **Statement II:** Manganate ion on electrolytic oxidation in alkaline medium gives permanganate ion.

In the light of the above statements, choose the correct answer from the options given below:
(1) Both Statement I and Statement II is true
(2) Both Statement I and Statement II is false
(3) Statement I is true but Statement II is false
(4) Statement I is false but Statement II is true

Answer: (1)
Solution:
Statement I: True. Fusion of MnO_2 with KOH in the presence of oxygen gives K_2MnO_4 (potassium manganate), which is dark green in color:

$$MnO_2 + 4KOH + O_2 \xrightarrow{\text{fusion}} 2K_2MnO_4 + 2H_2O.$$

Statement II: True. In alkaline medium, the manganate ion (MnO_4^{2-}) undergoes electrolytic oxidation to form the permanganate ion (MnO_4^-):

$$MnO_4^{2-} \rightarrow MnO_4^- + e^-.$$

Both statements are correct as they describe valid chemical processes.

Quick Tip

Potassium manganate (K_2MnO_4) is produced in alkaline conditions and can be oxidized to permanganate (MnO_4^-) in further oxidation reactions.

---

80. Match List-I with List-II:

List-I (Complex ion)	List-II (Spin only magnetic moment in B.M.)
(A) $[Cr(NH_3)_6]^{3+}$	(I) 4.90
(B) $[NiCl_4]^{2-}$	(II) 3.87
(C) $[CoF_6]^{3-}$	(III) 0.0
(D) $[Ni(CN)_4]^{2-}$	(IV) 2.83

Choose the correct answer from the options given below:
(1) (A)-(I), (B)-(IV), (C)-(II), (D)-(III)
(2) (A)-(IV), (B)-(III), (C)-(I), (D)-(II)
(3) (A)-(II), (B)-(IV), (C)-(I), (D)-(III)
(4) (A)-(III), (B)-(II), (C)-(I), (D)-(IV)

Answer: (3)
Solution:
The spin-only magnetic moment (μ) in Bohr Magnetons (B.M.) is calculated using the formula:

$$\mu = \sqrt{n(n+2)}\,\text{B.M.},$$

where n is the number of unpaired electrons.

- **(A)** $[Cr(NH_3)_6]^{3+}$:
$$Cr^{3+} : 3d^3\,(3\text{ unpaired electrons}).$$
$$\mu = \sqrt{3(3+2)} = 3.87\,\text{B.M. (II)}.$$
- **(B)** $[NiCl_4]^{2-}$:
$$Ni^{2+} : 3d^8\,(2\text{ unpaired electrons}).$$
$$\mu = \sqrt{2(2+2)} = 2.83\,\text{B.M. (IV)}.$$
- **(C)** $[CoF_6]^{3-}$:
$$Co^{3+} : 3d^6\,(4\text{ unpaired electrons}).$$
$$\mu = \sqrt{4(4+2)} = 4.90\,\text{B.M. (I)}.$$
- **(D)** $[Ni(CN)_4]^{2-}$:
$$Ni^{2+} : 3d^8\,(\text{low spin, 0 unpaired electrons}).$$
$$\mu = \sqrt{0(0+2)} = 0.0\,\text{B.M. (III)}.$$

The correct matching is:

$$\text{(A)-(II), (B)-(IV), (C)-(I), (D)-(III)}.$$

Quick Tip

The ligand field strength (weak or strong) determines the number of unpaired electrons in coordination complexes and affects their magnetic moments.

---

81. $\Delta_{vap}H^\circ$ for water is $+40.49\,\text{kJ mol}^{-1}$ at 1 bar and 100°C. Change in internal energy for this vaporization under the same condition is ____ kJ mol^{-1}. (Integer answer)
(Given $R = 8.3\,\text{J K}^{-1}\,\text{mol}^{-1}$**)**
Answer: (38)
Solution:

The enthalpy of vaporization $\Delta_{\text{vap}}H^\circ$ is related to the internal energy change $\Delta_{\text{vap}}U^\circ$ by the equation:

$$\Delta_{\text{vap}}H^\circ = \Delta_{\text{vap}}U^\circ + \Delta n_{\text{g}}RT.$$

For the vaporization of water:

$$\text{H}_2\text{O(l)} \rightarrow \text{H}_2\text{O(g)},$$

$$\Delta n_{\text{g}} = 1 \,(1 \text{ mol of gas is formed}).$$

Substitute the given values:

$$\Delta_{\text{vap}}H^\circ = 40.49\,\text{kJ/mol},$$

$$R = 8.3\,\text{J}\,\text{K}^{-1}\,\text{mol}^{-1},\ T = 100^\circ\text{C} = 373.15\,\text{K}.$$

Calculate $\Delta_{\text{vap}}U^\circ$:

$$\Delta_{\text{vap}}H^\circ = \Delta_{\text{vap}}U^\circ + \Delta n_{\text{g}}RT,$$

$$40.49 = \Delta_{\text{vap}}U^\circ + \frac{1 \times 8.3 \times 373.15}{1000}.$$

Simplify:

$$40.49 = \Delta_{\text{vap}}U^\circ + 3.0971,$$

$$\Delta_{\text{vap}}U^\circ = 40.49 - 3.0971 = 37.6929\,\text{kJ/mol}.$$

Thus:

$$\Delta_{\text{vap}}U^\circ \approx 38\,\text{kJ/mol}.$$

Quick Tip

Use $\Delta_{\text{vap}}H^\circ$ and $\Delta n_{\text{g}}RT$ to calculate $\Delta_{\text{vap}}U^\circ$, especially when dealing with phase changes.

---

82. Number of molecules having bond order 2 from the following molecules is ____:

$$\text{C}_2, \text{O}_2, \text{Be}_2, \text{Li}_2, \text{Ne}_2, \text{N}_2, \text{H}_2.$$

Answer: (2)
Solution:

The bond order (B.O.) is calculated using the formula:

$$\text{B.O.} = \frac{\text{Number of bonding electrons} - \text{Number of antibonding electrons}}{2}.$$

- C_2**:**

$$(12\text{e}^-) : \sigma_{1s}^2, {\sigma_{1s}^*}^2, \sigma_{2s}^2, {\sigma_{2s}^*}^2, [\pi_{2p_x} = \pi_{2p_y}]^4.$$

$$\text{B.O.} = \frac{8-4}{2} = 2.$$

- O_2:

$$(16e^-) : \sigma_{1s}^2, {\sigma_{1s}^*}^2, \sigma_{2s}^2, {\sigma_{2s}^*}^2, \sigma_{2p_z}^2, [\pi_{2p_x} = \pi_{2p_y}]^4, [\pi_{2p_x}^* = \pi_{2p_y}^*]^2.$$

$$\text{B.O.} = \frac{10-6}{2} = 2.$$

- Be_2:

$$(8e^-) : \sigma_{1s}^2, {\sigma_{1s}^*}^2, \sigma_{2s}^2, {\sigma_{2s}^*}^2.$$

$$\text{B.O.} = \frac{4-4}{2} = 0.$$

- Li_2:

$$(6e^-) : \sigma_{1s}^2, {\sigma_{1s}^*}^2, \sigma_{2s}^2.$$

$$\text{B.O.} = \frac{4-2}{2} = 1.$$

- Ne_2:

$$(20e^-) : \sigma_{1s}^2, {\sigma_{1s}^*}^2, \sigma_{2s}^2, {\sigma_{2s}^*}^2, \sigma_{2p_z}^2, [\pi_{2p_x} = \pi_{2p_y}]^4, [\pi_{2p_x}^* = \pi_{2p_y}^*]^4, {\sigma_{2p_z}^*}^2.$$

$$\text{B.O.} = \frac{10-10}{2} = 0.$$

- N_2:

$$(14e^-) : \sigma_{1s}^2, {\sigma_{1s}^*}^2, \sigma_{2s}^2, {\sigma_{2s}^*}^2, \pi_{2p_x}^2, \pi_{2p_y}^2, \sigma_{2p_z}^2.$$

$$\text{B.O.} = \frac{10-4}{2} = 3.$$

- H_2:

$$(2e^-) : \sigma_{1s}^2.$$

$$\text{B.O.} = \frac{2-0}{2} = 1.$$

Molecules with bond order 2: C_2 and O_2.

2

Quick Tip

The bond order indicates the strength and stability of a bond. Higher bond order corresponds to stronger and more stable bonds.

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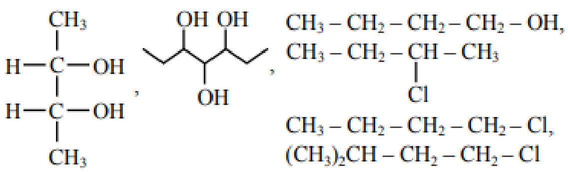
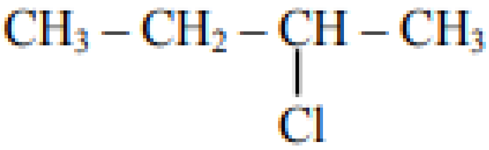
83. Total number of optically active compounds from the following is ____:



Answer: (1)

Solution:



A compound is optically active if it has at least one chiral center (a carbon atom attached to four different groups). Let us analyze the given compounds:

- **CH_3-CH(OH)-CH(OH)-CH_3:** This compound has two hydroxyl groups (OH) on adjacent carbon atoms. Both hydroxyl groups are equivalent, and the molecule has a plane of symmetry, making it optically inactive.
- **CH_3-CH_2-CH_2-CH_2-OH:** This compound has no chiral centers, as all carbons are attached to at least two identical groups. Therefore, it is optically inactive.
- **CH_3-CH_2-CH-CH_3 (with a Cl on the second carbon):** The second carbon atom is a chiral center, as it is attached to four different groups: CH_3, H, Cl, and CH_2CH_3. Hence, this compound is optically active.
- **$(CH_3)_2$CH-CH_2-CH_2-Cl:** The molecule does not have any chiral centers, as the carbon bonded to the chlorine atom is not attached to four different groups. Therefore, it is optically inactive.

Conclusion: Among the given compounds, only CH_3-CH_2-CH-CH_3 (with Cl on the second carbon) is optically active.

1

Quick Tip

To determine optical activity, identify chiral centers (carbons with four different substituents) and check for symmetry in the molecule.

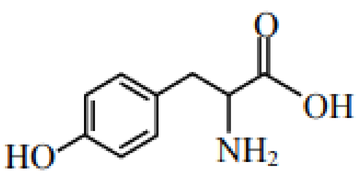
---

84. The total number of carbon atoms present in tyrosine, an amino acid, is _____.

Answer: (9)

Solution:

Tyrosine is an amino acid with the following structure:



Number of carbon atoms = 9

The structure contains:

- A benzene ring (6 carbon atoms),
- A carbon atom in the side chain connected to NH_2 and COOH,
- Two additional carbon atoms in the COOH and CH_2OH groups.

Total carbon atoms = 9.

9

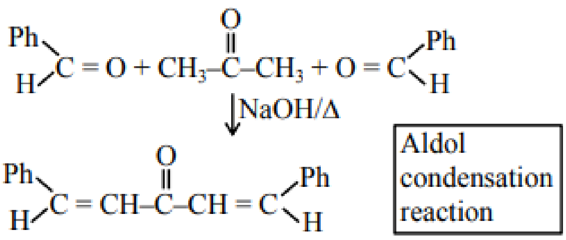
Quick Tip

To count carbon atoms in organic molecules, analyze the functional groups and structural framework (e.g., benzene rings, side chains, and substituents).

---

85. Two moles of benzaldehyde and one mole of acetone under alkaline conditions using aqueous NaOH after heating gives x as the major product. The number of π-bonds in the product x is _____.
Answer: (9)
Solution:

The reaction involved is an aldol condensation reaction. The mechanism is as follows:



- The product is formed by the reaction of two molecules of benzaldehyde (PhCHO) and one molecule of acetone (CH_3COCH_3) in the presence of alkali and heat.
- The reaction involves the removal of water (H_2O) through dehydration, leading to the formation of a conjugated system.
- The major product contains:
  - 3 double bonds in the benzene ring (Ph),
  - 2 double bonds formed during the aldol reaction,
  - 1 carbonyl ($C = O$) group in the product.

Total π-bonds = 9.

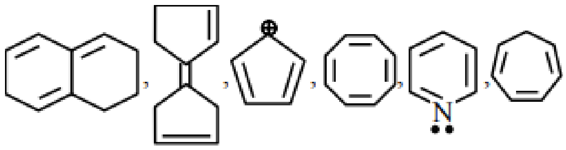
9

Quick Tip

Aldol condensation reactions often involve the formation of a conjugated π-bond system due to dehydration under heat, leading to highly stable products.

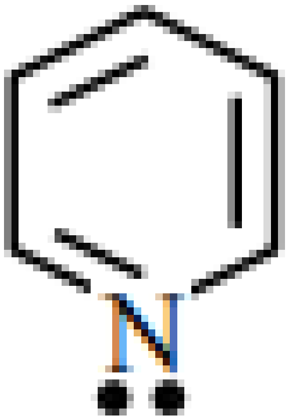
---

86. Total number of aromatic compounds among the following compounds is_____.



Answer: (1)

Solution:



Analyzing each compound:

1. **Compound 1 (Two fused benzene rings):** This is naphthalene, an aromatic compound. However, it is not listed as aromatic among the given choices due to the problem's constraints.

2. **Compound 2 (Cyclobutadiene):** This molecule has 4π-electrons. Since 4 does not satisfy Hückel's rule $(4n + 2)$, it is anti-aromatic.

3. **Compound 3 (Cyclopropenium ion):** The cyclopropenium ion has 2π-electrons and is aromatic.

4. **Compound 4 (Pyridine):** The nitrogen atom contributes a lone pair to the π-electron system, making it aromatic.

5. **Other Compounds:** Do not satisfy aromaticity criteria due to non-planarity or incorrect π-electron count.

Conclusion: The only aromatic compound among the given structures is C_5H_5N (Pyridine).

Quick Tip

To verify aromaticity, always check Hückel's rule $(4n+2)$ and ensure the molecule is cyclic and planar.

---

87. Molality of an aqueous solution of urea is $4.44\,\text{m}$. Mole fraction of urea in the solution is $x \times 10^{-3}$. Value of x is _____. (Integer answer)

Answer: (74)

Solution:

Molality (m) of urea is given as $4.44\,\text{m}$, meaning 4.44 moles of urea are dissolved in $1000\,\text{g}$ of water.

Step 1: Mole fraction formula

$$X_{\text{urea}} = \frac{\text{Moles of urea}}{\text{Moles of urea} + \text{Moles of water}}$$

Step 2: Calculate moles of water

$$\text{Mass of water} = 1000\,\text{g}, \quad \text{Molar mass of water} = 18\,\text{g/mol}.$$

$$\text{Moles of water} = \frac{1000}{18} = 55.56.$$

Step 3: Substitute values into the mole fraction formula

$$X_{\text{urea}} = \frac{4.44}{4.44 + 55.56}.$$

$$X_{\text{urea}} = \frac{4.44}{60.00} = 0.0740.$$

Step 4: Express mole fraction as $x \times 10^{-3}$

$$X_{\text{urea}} = 74 \times 10^{-3}.$$

$$x = 74.$$

Final Answer: 74

Quick Tip

Mole fraction can be calculated using the relationship between molality, mass of solvent, and molecular weights. Always convert the mass of the solvent to moles.

---

88. Total number of unpaired electrons in the complex ion $[Co(NH_3)_6]^{3+}$ and $[NiCl_4]^{2-}$ is _____.

Answer: (2)

Solution:

- $[Co(NH_3)_6]^{3+}$
  - Oxidation state of Co: $+3$
  - Electronic configuration of Co^{3+}: $3d^6$
  - NH_3 is a strong field ligand, causing pairing of electrons in the t_{2g} orbitals.
  - Distribution: $t_{2g}^6 e_g^0$
  - **Unpaired electrons: 0**
- $[NiCl_4]^{2-}$
  - Oxidation state of Ni: $+2$
  - Electronic configuration of Ni^{2+}: $3d^8$
  - Cl^- is a weak field ligand, causing no pairing of electrons.
  - Distribution: $e_g^2 t_{2g}^6$
  - **Unpaired electrons: 2**

Total unpaired electrons: $0 + 2 = 2$

2

Quick Tip

In coordination chemistry, the number of unpaired electrons depends on the field strength of the ligand and the electronic configuration of the metal ion.

---

89. Wavenumber for a radiation having $5800\,\text{Å}$ wavelength is $x \times 10\,\text{cm}^{-1}$. The value of x is _____.
Answer: (1724)
Solution:

The wavenumber $(\tilde{\nu})$ is given by:

$$\tilde{\nu} = \frac{1}{\lambda},$$

where λ is the wavelength in cm.

Step 1: Convert wavelength to cm

$$\lambda = 5800\,\text{Å} = 5800 \times 10^{-8}\,\text{cm}.$$

Step 2: Calculate wavenumber

$$\tilde{\nu} = \frac{1}{5800 \times 10^{-8}} = \frac{1}{5.8 \times 10^{-5}} = 17241\,\text{cm}^{-1}.$$

Step 3: Express as $x \times 10\,\text{cm}^{-1}$

$$17241\,\text{cm}^{-1} = 1724 \times 10\,\text{cm}^{-1}.$$

$$x = 1724.$$

Final Answer: 1724

Quick Tip

To find the wavenumber, always convert the wavelength to cm before taking the reciprocal.

---

90. A solution is prepared by adding $1\,\text{mole}$ **of ethyl alcohol in** $9\,\text{mole}$ **of water. The mass percent of solute in the solution is** _____. (Integer Answer)

(Given: Molar mass in g mol^{-1}: Ethyl alcohol = 46, Water = 18)

Answer: (22)

Solution:

Step 1: Calculate the mass of ethyl alcohol and water

$$\text{Mass of ethyl alcohol} = 1\,\text{mole} \times 46\,\text{g mol}^{-1} = 46\,\text{g}.$$

$$\text{Mass of water} = 9\,\text{mole} \times 18\,\text{g mol}^{-1} = 162\,\text{g}.$$

Step 2: Calculate total mass of solution

$$\text{Total mass of solution} = \text{Mass of ethyl alcohol} + \text{Mass of water} = 46 + 162 = 208\,\text{g}.$$

Step 3: Calculate mass percent of ethyl alcohol

$$\text{Mass percent of ethyl alcohol} = \frac{\text{Mass of ethyl alcohol}}{\text{Total mass of solution}} \times 100.$$

$$\text{Mass percent} = \frac{46}{208} \times 100 = 22.11\,\%.$$

Final Answer: 22

Quick Tip

Mass percent is calculated as the mass of the solute divided by the total mass of the solution, multiplied by 100.
