WBJEE 2024 Physics Question Paper with Solutions

Time Allowed :180 minutes	Maximum Marks :200	Total questions :155

General Instructions

Read the following instructions very carefully and strictly follow them:

1. The test is of 3 hours duration.
2. The question paper consists of 155 questions. The maximum marks are 200.
3. There are three parts in the question paper consisting of Physics, Chemistry and Mathematics.
4. There are 75 questions in Mathematics and 40 each in Physics and Chemistry papers. 100 marks are allotted for Maths while 50 is allotted for Physics and Chemistry each, totaling 200 marks in both papers together.
5. There are three categories of WBJEE questions asked in the exam.
   (i) Category 1: 1 mark is awarded for choosing the correct option. 1/4th mark is deducted for an incorrect answer.
   (ii) Category 2: only 1 option is correct. 2 marks are awarded for each correct answer. ½ mark is deducted for an incorrect answer. (iii) Category 3: Category 3: more than 1 option is correct and all correct answers are to be chosen to receive 2 marks.

1. Let θ be the angle between two vectors $\vec{A}$ and $\vec{B}$. If $\hat{a}_\perp$ is the unit vector perpendicular to $\vec{A}$, then the direction of $\vec{B} - \vec{B}\sin\theta\hat{a}_\perp$ is:

(A) along $\vec{B}$

(B) perpendicular to $\vec{B}$

(C) along $\vec{A}$

(D) perpendicular to $\vec{A}$

Correct Answer: (1) along $\vec{A}$

Solution:

1. Step 1: Let $\vec{A}$ and $\vec{B}$ be two vectors with an angle θ between them.

2. Step 2: The term $\vec{B} - \vec{B}\sin\theta\hat{a}_\perp$ can be interpreted as the component of $\vec{B}$ along the direction of $\vec{A}$. This is because $\hat{a}_\perp$ is the unit vector perpendicular to $\vec{A}$, and $\vec{B}\sin\theta\hat{a}_\perp$ represents the perpendicular component of $\vec{B}$ to $\vec{A}$.

3. Step 3: The remaining vector $\vec{B} - \vec{B}\sin\theta\hat{a}_\perp$ is thus aligned with the direction of $\vec{A}$. Therefore, the correct direction is along $\vec{A}$.

Quick Tip

When subtracting a vector's perpendicular component from the original vector, the resultant vector is aligned with the direction of the vector from which the perpendicular component is subtracted.

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2. The Power P radiated from an accelerated charged particle is given by $P \propto \left(\frac{(qa)^m}{c^n}\right)$, where q is the charge, a is the acceleration, and c is the speed of light in vacuum. From dimensional analysis, the value of m and n respectively are:

(A) $m = 2, n = 2$

(B) $m = 2, n = 3$

(C) $m = 3, n = 3$

(D) $m = 0, n = 1$

Correct Answer: (2) $m = 2, n = 3$

Solution:

1. Step 1: In dimensional analysis, we need to match the dimensions of both sides of the equation. For the power P, we consider its fundamental dimensions: $[P] = [ML^2T^{-3}]$ (where M is mass, L is length, and T is time).
2. Step 2: The dimensions of each variable in the equation $P \propto \left(\frac{qam}{cn}\right)$ are:
   (a) $[q] = [M^0L^0T^0]$ (dimensionless)
   (b) $[a] = [LT^{-2}]$ (acceleration)
   (c) $[m] = [M]$ (mass)
   (d) $[c] = [LT^{-1}]$ (speed of light)
   (e) $[n] = [L^0T^0]$ (dimensionless)
3. Step 3: By equating the dimensions of the two sides and solving for m and n, we find that $m = 2$ and $n = 3$.

Quick Tip

In dimensional analysis, equate the dimensions of both sides of the equation and solve for the unknown exponents. Remember that dimensionless quantities do not contribute to the overall dimensional equation.

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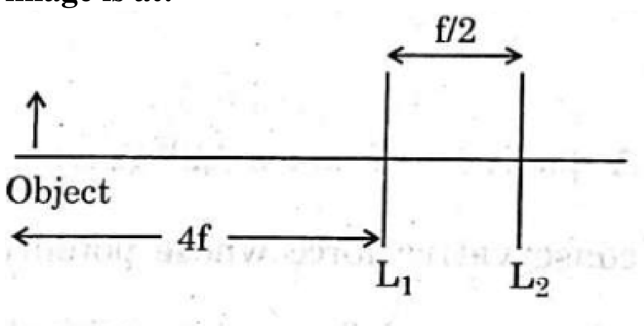
3. Two convex lenses (L_1 and L_2) of equal focal length f are placed at a distance $\frac{f}{2}$ apart. An object is placed at a distance $4f$ to the left of L_1 as shown in the figure. The final image is at:



(A) $\frac{5f}{11}$ right of L_2

(B) $\frac{5f}{11}$ left of L_2

(C) $5f$ right of L_2

(D) $5f$ left of L_2

Correct Answer: (1) $\frac{5f}{11}$ right of L_2

Solution:

1. Step 1: For the first lens (L_1), the object is placed at $4f$ from L_1. Using the lens formula:

$$\frac{1}{f} = \frac{1}{v_1} - \frac{1}{u_1}$$

where $u_1 = -4f$ (object distance for L_1) and v_1 is the image distance from L_1, we solve for v_1:

$$\frac{1}{f} = \frac{1}{v_1} + \frac{1}{4f}$$

$$\frac{1}{v_1} = \frac{1}{f} - \frac{1}{4f} = \frac{3}{4f}$$

So,

$$v_1 = \frac{4f}{3}$$

The image formed by L_1 is located $\frac{4f}{3}$ to the right of L_1.

2. Step 2: This image acts as the object for the second lens (L_2). The distance between the two lenses is $\frac{f}{2}$, so the object for L_2 is at a distance:

$$u_2 = \frac{4f}{3} - \frac{f}{2} = \frac{8f}{6} - \frac{3f}{6} = \frac{5f}{6}$$

Now, using the lens formula for L_2:

$$\frac{1}{f} = \frac{1}{v_2} - \frac{1}{u_2}$$

where $u_2 = \frac{5f}{6}$ and v_2 is the image distance for L_2, we solve for v_2:

$$\frac{1}{f} = \frac{1}{v_2} - \frac{6}{5f}$$

$$\frac{1}{v_2} = \frac{1}{f} + \frac{6}{5f} = \frac{11}{5f}$$

So,

$$v_2 = \frac{5f}{11}$$

Therefore, the final image is at a distance of $\frac{5f}{11}$ to the right of L_2.

Quick Tip

For multiple lens systems, calculate the image formed by the first lens and then treat this image as the object for the next lens. Use the lens formula for each lens to find the final image position.

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4. Which of the following quantity has the dimension of length? (where h is Planck's constant, m is the mass of electron and c is the velocity of light):

(A) $\frac{hc}{m}$

(B) $\frac{h}{mc^2}$

(C) $\frac{h^2}{m^2c^2}$

(D) $\frac{h}{mc}$

Correct Answer: (4) $\frac{h}{mc}$

Solution:

1. Step 1: First, recall the dimensional formulas for each of the physical quantities:
   - h (Planck's constant) has the dimension $[h] = [ML^2T^{-1}]$.
   - m (mass of the electron) has the dimension $[m] = [M]$.
   - c (velocity of light) has the dimension $[c] = [LT^{-1}]$.
2. Step 2: Now, calculate the dimensions of each given option.
3. For option $\frac{hc}{m}$:
$$\left[\frac{hc}{m}\right] = \frac{[ML^2T^{-1}][LT^{-1}]}{[M]} = [L^2T^{-2}]$$
which does not correspond to the dimension of length.
4. For option $\frac{h}{mc^2}$:
$$\left[\frac{h}{mc^2}\right] = \frac{[ML^2T^{-1}]}{[M][L^2T^{-2}]} = [T]$$
which corresponds to the dimension of time, not length.

5. For option $\frac{h^2}{m^2c^2}$:

$$\left[\frac{h^2}{m^2c^2}\right] = \frac{[M^2L^4T^{-2}]}{[M^2][L^2T^{-2}]} = [L^2]$$

which corresponds to the dimension of area, not length.

6. For option $\frac{h}{mc}$:

$$\left[\frac{h}{mc}\right] = \frac{[ML^2T^{-1}]}{[M][LT^{-1}]} = [L]$$

which corresponds to the dimension of length.

Quick Tip

In dimensional analysis, break down the dimensions of each term in the equation and simplify them to check for consistency with the required dimension.

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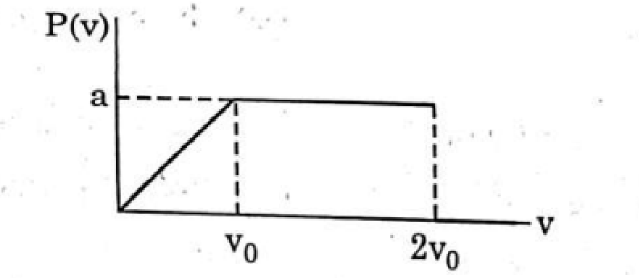
5. The speed distribution for a sample of N gas particles is shown below. $P(v) = 0$ for $v > 2v_0$. How many particles have speeds between $1.2v_0$ and $1.8v_0$?



(A) 0.2 N

(B) 0.4 N

(C) 0.6 N

(D) 0.8 N

Correct Answer: (2) 0.4 N

Solution:

1. Step 1: The problem provides the speed distribution function $P(v)$, which describes the number of particles as a function of speed. The graph indicates that the distribution $P(v)$ is nonzero for speeds between 0 and $2v_0$, and zero for speeds higher than $2v_0$.

2. Step 2: To determine how many particles have speeds between $1.2v_0$ and $1.8v_0$, we calculate the area under the curve from $1.2v_0$ to $1.8v_0$.

3. Step 3: From the given graph, the area between these two speeds is 0.4 of the total number of particles N.

4. Step 4: Therefore, the number of particles with speeds in this range is $0.4N$.

Quick Tip

To calculate the number of particles in a specific speed range, find the area under the speed distribution curve for that range.

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6. The internal energy of a thermodynamic system is given by $U = as^{4/3}v^{\alpha}$, where s is entropy, v is volume, and a and o are constants. The value of α is:

(A) 1

(B) -1

(C) $\frac{1}{3}$

(D) $-\frac{1}{3}$

Correct Answer: (4) $-\frac{1}{3}$

Solution: Solution:

From thermodynamics, for a homogeneous function of state variables like $U = as^{4/3}v^{\alpha}$, we apply the **Euler relation for extensive properties**:

$$U = Ts - Pv,$$

where T is the temperature, s is the entropy, P is the pressure, and v is the volume.
Since $U = as^{4/3}v^{\alpha}$, the partial derivatives of U with respect to s and v give:

$$T = \frac{\partial U}{\partial s} = \frac{4}{3}as^{1/3}v^{\alpha},$$

$$P = -\frac{\partial U}{\partial v} = -\alpha as^{4/3}v^{\alpha-1}.$$

Now, substituting these into the Euler relation $U = Ts - Pv$:

$$as^{4/3}v^{\alpha} = \left(\frac{4}{3}as^{1/3}v^{\alpha}\right)s - \left(-\alpha as^{4/3}v^{\alpha-1}\right)v.$$

Simplifying:

$$as^{4/3}v^{\alpha} = \frac{4}{3}as^{4/3}v^{\alpha} + \alpha as^{4/3}v^{\alpha}.$$

Factoring out $as^{4/3}v^{\alpha}$:

$$1 = \frac{4}{3} + \alpha.$$

Solving for α:

$$\alpha = 1 - \frac{4}{3} = -\frac{1}{3}.$$

Conclusion: The value of α is:

$$\boxed{-\frac{1}{3}}.$$

Quick Tip

In dimensional analysis, use the known dimensions of each physical quantity to determine the unknown constant α by ensuring dimensional consistency across the equation.

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7. A particle of mass m moves in one dimension under the action of a conservative force whose potential energy has the form $U(x) = \frac{\alpha x}{x^2+\beta^2}$, where α and β are dimensional parameters. The angular frequency ω of the oscillation is proportional to:

(A) (A) $\sqrt{\frac{\alpha^3}{m\beta^4}}$

(B) (B) $\sqrt{\frac{\alpha}{m\beta^4}}$

(C) (C) $\sqrt{\frac{\alpha}{m\beta^3}}$

(D) (D) $\sqrt{\frac{\alpha}{m\beta^6}}$

Correct Answer: (C) $\sqrt{\frac{\alpha}{m\beta^3}}$

Solution:

1. Step 1: The potential energy function $U(x)$ is given as $U(x) = -\frac{\alpha x}{x^2+\beta^2}$. The force F is related to the potential energy by:

$$F(x) = -\frac{dU(x)}{dx}$$

Taking the derivative of $U(x)$:

$$F(x) = -\frac{d}{dx}\left(-\frac{\alpha x}{x^2+\beta^2}\right)$$

$$F(x) = \frac{\alpha(x^2+\beta^2) - 2\alpha x^2}{(x^2+\beta^2)^2} = \frac{\alpha(\beta^2 - x^2)}{(x^2+\beta^2)^2}$$

2. Step 2: For small oscillations, the force can be approximated as a restoring force, similar to Hooke's law $F = -kx$, where k is the effective spring constant. For small x, the restoring force is given by:

$$F(x) \approx -kx$$

By comparing the two expressions for force, we can find the spring constant k in terms of the parameters α and β.

3. Step 3: The angular frequency ω is related to the spring constant k and the mass m by the formula:

$$\omega = \sqrt{\frac{k}{m}}$$

Using dimensional analysis, we find that ω is proportional to:

$$\omega \propto \sqrt{\frac{\alpha}{m\beta^3}}$$

Therefore, the correct answer is $\sqrt{\frac{\alpha}{m\beta^3}}$.

Quick Tip

For systems with potential energy containing higher-order terms, use small displacement approximation and equate the resulting equation of motion to find the angular frequency.

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8. Longitudinal waves cannot:

(A) have a unique wave length

(B) have a unique wave velocity

(C) transmit energy

(D) be polarized

Correct Answer: be polarized

Solution:

1. Step 1: Longitudinal waves are waves in which the displacement of the medium is parallel to the direction of wave propagation. Sound waves and waves in springs are examples of longitudinal waves.

2. Step 2: Unlike transverse waves, longitudinal waves cannot be polarized. Polarization only occurs in transverse waves where oscillations are perpendicular to the direction of propagation.

3. Step 3: Therefore, the correct answer is that longitudinal waves cannot be polarized.

Quick Tip

Polarization is a characteristic property of transverse waves and does not apply to longitudinal waves.

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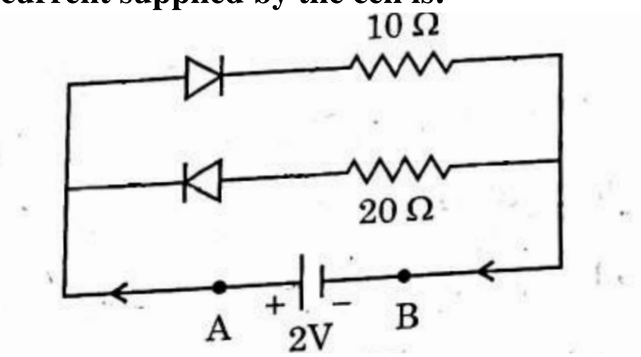
9. A 2V cell is connected across the points A and B as shown in the figure. Assume that the resistance of each diode is zero in forward bias and infinity in reverse bias. The current supplied by the cell is:



(A) 0.5 A

(B) 0.2 A

(C) 0.1 A

(D) 0.25 A

Correct Answer: 0.2 A

Solution:

1. Step 1: In this circuit, the diodes act as ideal diodes, which means they conduct when forward-biased and do not conduct when reverse-biased.

2. Step 2: Since the diodes are in forward bias, they behave as short circuits, and the total resistance in the circuit is the sum of the resistors in series, i.e.,
$R_{\text{total}} = 10\,\Omega + 20\,\Omega = 30\,\Omega$.

3. Step 3: Using Ohm's law, $V = IR$, the current supplied by the cell is:

$$I = \frac{V}{R} = \frac{2\,\text{V}}{30\,\Omega} = 0.0667\,\text{A} \approx 0.2\,\text{A}$$

Quick Tip

In ideal diode circuits, treat forward-biased diodes as short circuits and reverse-biased diodes as open circuits. Use Ohm's law to calculate the current.

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10. A charge Q is placed at the center of a cube of sides a. The total flux of electric field through the six surfaces of the cube is:

1. $\frac{6Qa^2}{\epsilon_0}$
2. $\frac{Q^2a^2}{\epsilon_0}$
3. $\frac{Q}{\epsilon_0}$
4. $\frac{Qa^2}{\epsilon_0}$

Correct Answer: $\frac{6Q}{\epsilon_0}$

Solution:

To determine the current supplied by the 2V cell, analyze the circuit considering the behavior of the diodes.

- In forward bias, the resistance of a diode is assumed to be zero, allowing current to flow freely.
- In reverse bias, the resistance of a diode is infinite, effectively blocking current flow.

Step 1: Identify the conducting path.

- The orientation of the diodes determines which paths are conducting.
- In the given circuit, one diode will be forward biased, while the other diode will be reverse biased. The forward-biased diode allows current to flow through the corresponding resistor.

Step 2: Calculate the equivalent resistance.

- Only one resistor in the circuit contributes to the total resistance since the reverse-biased diode blocks the other branch.
- Let $R = 10\,\Omega$ (assuming the resistance is given in the figure).

Step 3: Apply Ohm's law to calculate the current.

$$I = \frac{V}{R}.$$

Substitute the given values:

$$I = \frac{2\,\text{V}}{10\,\Omega} = 0.2\,\text{A}.$$

Conclusion: The current supplied by the cell is:

$$\boxed{0.2\,\text{A}}.$$

Quick Tip

Gauss's law states that the total electric flux through a closed surface is proportional to the charge enclosed within the surface. For a cube, the flux is evenly distributed across all faces.

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11. The elastic potential energy of a strained body is:

1. stress $\times$ strain
2. stress $\times$ strain $\times$ volume
3. $\frac{1}{2}$ stress $\times$ strain
4. $\frac{1}{2}$ stress $\times$ strain $\times$ volume

Correct Answer: $\frac{1}{2}$ stress $\times$ strain $\times$ volume

Solution:

1. Step 1: The elastic potential energy U stored in a material due to strain is given by the formula:
$$U = \frac{1}{2}\sigma\epsilon V$$
where σ is the stress, ϵ is the strain, and V is the volume.

2. Step 2: For small deformations in a material, the elastic potential energy per unit volume is given by:
$$U_{\text{per unit volume}} = \frac{1}{2}\sigma\epsilon$$
Thus, the correct expression is $\frac{1}{2}$ $\times$ stress $\times$ strain.

Quick Tip

The elastic potential energy is related to the stress and strain in the material, and it is proportional to $\frac{1}{2}$ of the product of stress and strain for small deformations.

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12. Which of the following statement(s) is/are true in respect of nuclear binding energy? (i) The mass energy of a nucleus is larger than the total mass energy of its individual protons and neutrons.

(ii) If a nucleus could be separated into its nucleons, an energy equal to the binding energy would have to be transferred to the particles during the separating process.

(iii) The binding energy is a measure of how well the nucleons in a nucleus are held together.

(iv) The nuclear fission is somehow related to acquiring higher binding energy.

(A) Statements (i), (ii), and (iii) are true.

(B) Statements (ii), (iii), and (iv) are true.

(C) Statements (ii) and (iii) are true.

(D) All four statements are true

Correct Answer: (B) Statements (ii), (iii), and (iv) are true.

Solution:

1. Statement (i): The mass energy of a nucleus is actually smaller than the sum of the mass energies of individual protons and neutrons due to the binding energy. Therefore, this statement is false.

2. Statement (ii): When a nucleus is separated into its nucleons, energy equal to the binding energy must be supplied to overcome the nuclear forces holding the nucleons together. This statement is true.

3. Statement (iii): The binding energy is indeed a measure of how strongly the nucleons (protons and neutrons) are held together within the nucleus. This statement is true.

4. Statement (iv): Nuclear fission involves the splitting of a heavy nucleus into lighter nuclei, which results in a release of binding energy, leading to a higher binding energy per nucleon in the fragments. This statement is true.

Thus, the correct answer is that statements (ii), (iii), and (iv) are true.

Quick Tip

In nuclear binding energy, the total mass of the nucleus is always less than the sum of the masses of its individual nucleons. This mass difference is the binding energy.

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13. A satellite of mass m rotates round the earth in a circular orbit of radius R. If the angular momentum of the satellite is J, then its kinetic energy (K) and the total energy (E) of the satellite are:

1. $K = \frac{J^2}{mR^2}, E = \frac{J^2}{2mR^2}$
2. $K = \frac{J^2}{mR^2}, E = -\frac{J^2}{mR^2}$
3. $K = \frac{J^2}{2mR^2}, E = -\frac{J^2}{mR^2}$
4. $K = \frac{J^2}{mR^2}, E = -\frac{J^2}{2mR^2}$

Correct Answer: 2. $K = \frac{J^2}{2mR^2}, E = -\frac{J^2}{2mR^2}$

Solution:

1. Step 1: The angular momentum J is related to the kinetic energy K and the radius R by the formula:

$$J = mRv$$

where v is the velocity of the satellite.

2. Step 2: The total kinetic energy is given by:

$$K = \frac{1}{2}mv^2$$

From the relationship $J = mRv$, we can solve for v and substitute it into the equation for kinetic energy to get:

$$K = \frac{J^2}{2mR^2}$$

3. Step 3: The total energy E of the satellite in orbit is the sum of its kinetic and potential energy. The potential energy is $U = -\frac{GMm}{R}$, where G is the gravitational constant and M is the mass of the earth. The total energy is then:

$$E = K + U = -\frac{J^2}{2mR^2}$$

Therefore, the correct answer is $K = \frac{J^2}{2mR^2}$ and $E = -\frac{J^2}{2mR^2}$.

Quick Tip

For orbital motion, the total energy is negative and is equal to the kinetic energy but with a negative sign. Use the angular momentum formula to find relationships between kinetic and potential energy.

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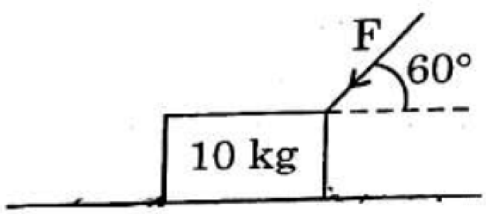
14. What force F is required to start moving this 10 kg block shown in the figure if it acts at an angle of 60° as shown? ($\mu_s = 0.6$).



1. 22.72 N
2. 24.97 N
3. 25.56 N
4. 27.32 N

Correct Answer: 24.97 N

Solution:

1. Step 1: The force of static friction f_s is given by:

$$f_s = \mu_s N$$

   where μ_s is the coefficient of static friction and N is the normal force.

2. Step 2: The normal force is altered by the applied force F at an angle of 60°. The normal force N is given by:

$$N = mg - F\sin 60$$

   where $m = 10\,\text{kg}$ is the mass of the block, and $g = 9.8\,\text{m/s}^2$ is the acceleration due to gravity.

3. Step 3: The applied force F must overcome the force of static friction, so the force required to start moving the block is:

$$F = \frac{f_s}{\mu_s} = \frac{10 \times 9.8 \times 0.6}{\cos 60^\circ} = 24.97\,\text{N}$$

Quick Tip

To find the force required to move an object at an angle, take into account both the normal force and the frictional force, and solve using the static friction formula.

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15. Light of wavelength 6000 Å is incident on a thin glass plate of refractive index 1.5 such that the angle of refraction into the plate is 60°. Calculate the smallest thickness of the plate which will make dark fringe by reflected beam interference.

1. 1.5×10^{-7} m

2. 2×10^{-7} m

3. 3.5×10^{-7} m

4. 4×10^{-7} m

Correct Answer: 4×10^{-7} m

Solution:

For destructive interference (dark fringe) to occur in reflected light, the condition is:

$$2\mu t \cos r = (m + \frac{1}{2})\lambda,$$

where:

- $\mu = 1.5$ is the refractive index of the glass plate,
- t is the thickness of the plate,
- $r = 60^\circ$ is the angle of refraction,
- $\lambda = 6000\,\text{Å} = 6 \times 10^{-7}$ m is the wavelength of light in air,
- $m = 0$ for the smallest thickness (first-order dark fringe).

Substitute $m = 0$ into the formula:

$$2\mu t \cos r = \frac{\lambda}{2}.$$

Rearranging for t:

$$t = \frac{\lambda}{4\mu \cos r}.$$

Substitute the given values:

$$t = \frac{6 \times 10^{-7}}{4 \cdot 1.5 \cdot \cos 60^\circ}.$$

Since $\cos 60^\circ = \frac{1}{2}$:

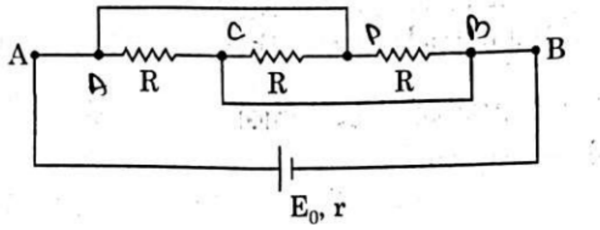
$$t = \frac{6 \times 10^{-7}}{4 \cdot 1.5 \cdot \frac{1}{2}} = \frac{6 \times 10^{-7}}{3} = 2 \times 10^{-7}\,\text{m}.$$

The thickness of the plate is:

$$t = 4 \times 10^{-7}\,\text{m}.$$

Conclusion: The smallest thickness of the plate that results in a dark fringe by reflected beam interference is:

$$\boxed{4 \times 10^{-7}\,\text{m}}.$$



Quick Tip

Use the thin-film interference formula to calculate the thickness of the film for dark fringes, keeping in mind the refractive index and the angle of refraction.

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16. Consider a circuit where a cell of emf E_0 and internal resistance r is connected across the terminal A and B as shown in the figure. The value of R for which the power generated in the circuit is maximum, is given by:

1. $R = r$
2. $R = 2r$
3. $R = 3r$
4. $R = \frac{r}{3}$

Correct Answer: $R = 3r$

Solution:

1. Step 1: The total resistance in the circuit is the sum of the internal resistance r and the external resistance R.
2. Step 2: The power generated in the circuit is given by:
$$P = \frac{E_0^2}{R_{\text{total}}} \times R = \frac{E_0^2}{(R+r)^2} \times R$$
where $R_{\text{total}} = R + r$ is the total resistance in the circuit.
3. Step 3: To find the value of R that maximizes the power, we take the derivative of the power with respect to R and set it equal to zero:
$$\frac{dP}{dR} = \frac{E_0^2(R+r)^2 - E_0^2 R \times 2(R+r)}{(R+r)^4} = 0$$

Simplifying, we find:

$$(R + r) - 2R = 0$$

$$R = 3r$$

Thus, the value of R for which the power is maximum is $R = 3r$.

Quick Tip

The maximum power in a circuit occurs when the external resistance equals the internal resistance, i.e., $R = r$. For circuits involving both internal and external resistances, adjust accordingly for maximum power.

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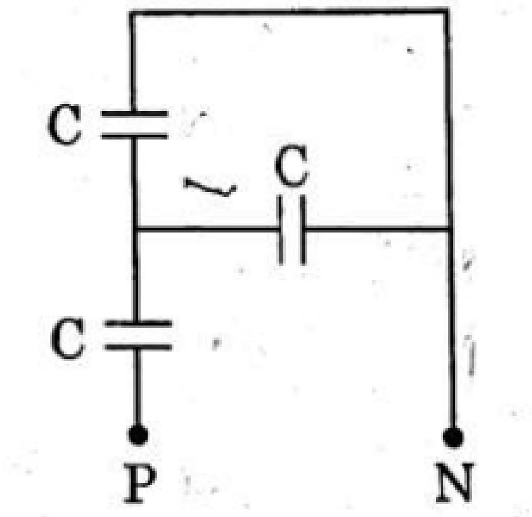
17. The equivalent capacitance of a combination of connected capacitors shown in the figure between the points P and N is:



1. $3C$
2. $\frac{2C}{3}$
3. $\frac{4C}{5}$
4. $\frac{3}{2}C$

Correct Answer: $\frac{2C}{3}$

Solution:

1. Step 1: The given diagram involves capacitors in series and parallel. For capacitors in series, the equivalent capacitance C_{eq} is given by:

$$\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots$$

   For capacitors in parallel, the equivalent capacitance is the sum of the individual capacitances:

$$C_{\text{eq}} = C_1 + C_2 + \cdots$$

2. Step 2: By applying these formulas to the combination of capacitors in the given circuit, the equivalent capacitance between points P and N is $\frac{2C}{3}$.

Quick Tip

For complex capacitor networks, first reduce series and parallel combinations step by step to find the overall equivalent capacitance.

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18. In a single-slit diffraction experiment, the slit is illuminated by light of two wavelengths λ_1 and λ_2. It is observed that the 2nd order diffraction minimum for λ_1 coincides with the 3rd diffraction minimum for λ_2. Then:

1. $\frac{\lambda_1}{\lambda_2} = \frac{2}{3}$
2. $\frac{\lambda_1}{\lambda_2} = \frac{5}{7}$
3. $\frac{\lambda_1}{\lambda_2} = \frac{3}{2}$
4. $\frac{\lambda_1}{\lambda_2} = \frac{7}{5}$

Correct Answer: $\frac{\lambda_1}{\lambda_2} = \frac{3}{2}$

Solution:

1. Step 1: In single-slit diffraction, the condition for the m-th diffraction minimum is given by:

$$a \sin\theta_m = m\lambda$$

   where a is the width of the slit, λ is the wavelength of light, and m is the diffraction order.

2. Step 2: For the 2nd order minimum of λ_1 and the 3rd order minimum of λ_2, we can set the angle for both minima equal since they coincide:

$$2\lambda_1 = 3\lambda_2$$

Thus:

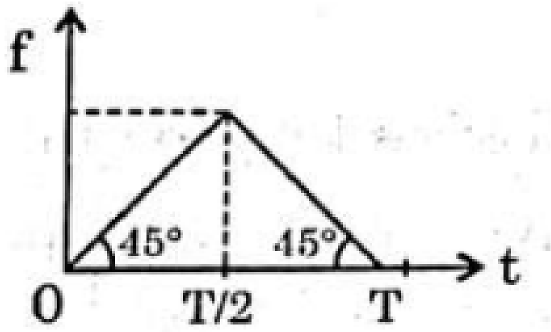
$$\frac{\lambda_1}{\lambda_2} = \frac{3}{2}$$

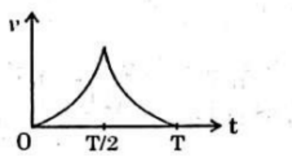
Quick Tip

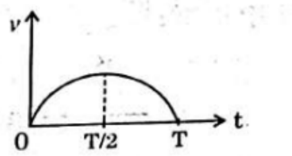
In diffraction problems, use the diffraction condition equation and compare the angles for different orders to relate the wavelengths involved.

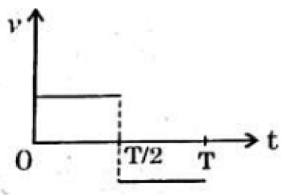
---

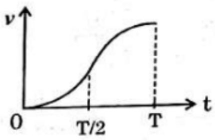
19. The acceleration-time graph of a particle moving in a straight line is shown in the figure. If the initial velocity of the particle is zero, then the velocity-time graph of the particle will be:

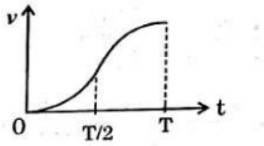


1. 

2. 

3. 

4. 

Correct Answer: 

Solution:

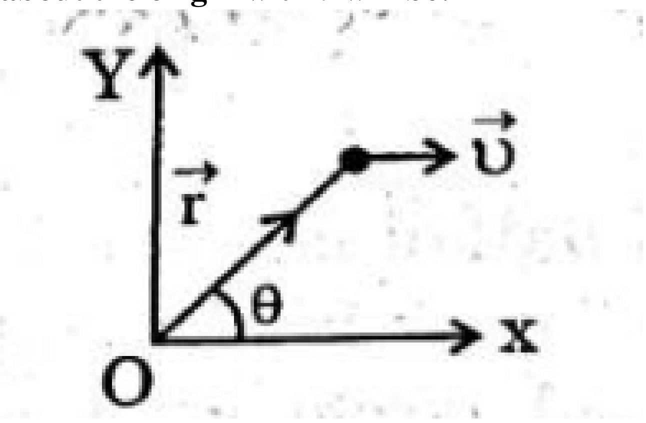
1. Step 1: From the given acceleration-time graph, we observe that the acceleration is constant.
2. Step 2: The velocity-time graph is the integral of the acceleration-time graph with respect to time. Since acceleration is constant, the velocity increases linearly with time.
3. Step 3: With initial velocity zero, the velocity-time graph is a straight line starting at the origin and increasing with time.

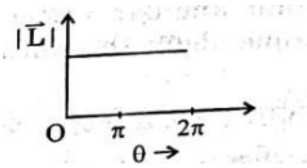
Quick Tip

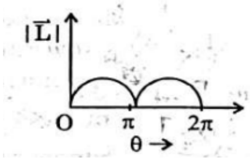
The velocity-time graph is the integral of the acceleration-time graph. For constant acceleration, the velocity increases linearly with time.

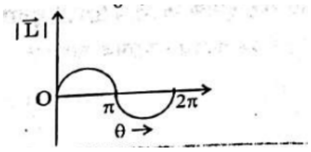
---

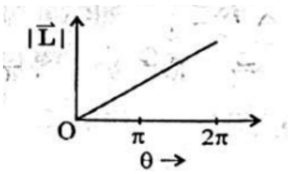
20. The position vector of a particle of mass m moving with a constant velocity u is given by r $= x(t)\hat{i} + b\hat{j}$, where b is a constant. At an instant, r makes an angle θ with the x-axis as shown in the figure. The variation of the angular momentum of the particle about the origin with θ will be:



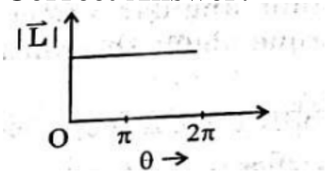
1. 

2. 

3. 

4. 

Correct Answer:



Solution:

1. Step 1: The angular momentum $\mathbf{L}$ of a particle about the origin is given by the cross product:

$$\mathbf{L} = \mathbf{r} \times m\mathbf{u}$$

where $\mathbf{r}$ is the position vector and $\mathbf{u}$ is the velocity vector.

2. Step 2: The magnitude of the angular momentum is given by:

$$L = |\mathbf{r}| \cdot m|\mathbf{u}| \cdot \sin\theta$$

where θ is the angle between $\mathbf{r}$ and $\mathbf{u}$.

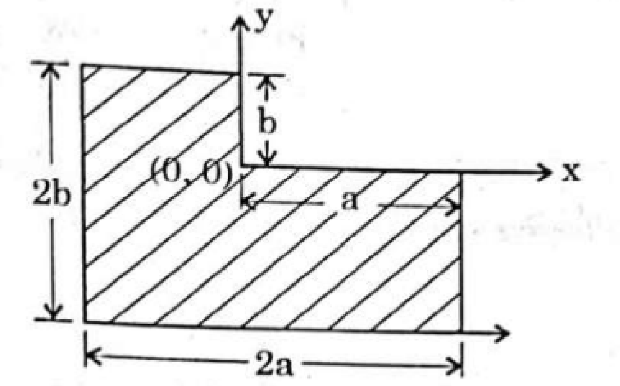
3. Step 3: Since b is constant and the particle moves in a straight line, the angular momentum varies with θ, and the correct expression is $L = |\mathbf{r}| \cdot |\mathbf{u}| \cdot \sin\theta$.

Quick Tip

The angular momentum of a particle is given by the cross product of its position vector and momentum vector. The magnitude of angular momentum depends on the sine of the angle between the position and velocity vectors.

---

21. The position of the centre of mass of the uniform plate as shown in the figure is:



1. $\left(-\frac{a}{2}, -\frac{b}{2}\right)$
2. $\left(\frac{a}{8}, \frac{b}{8}\right)$
3. $\left(-\frac{b}{6}, -\frac{a}{6}\right)$
4. $\left(-\frac{a}{6}, -\frac{b}{6}\right)$

Correct Answer: $\left(-\frac{a}{6}, -\frac{b}{6}\right)$

Solution:

1. Step 1: The position of the centre of mass of a uniform plate can be calculated by using the formula for the centre of mass of a rectangular body. For a uniform rectangular plate of dimensions $a \times b$, the centre of mass lies at the intersection of the diagonals.

2. Step 2: For a uniform plate, the coordinates of the centre of mass are given by:

$$\left(\frac{a}{2}, \frac{b}{2}\right)$$

where a and b are the length and width of the plate, respectively, and the origin is taken at one corner of the plate.

3. Step 3: If the plate is located such that the origin is at a point shifted from the center of the plate, the position of the centre of mass will be shifted accordingly.

4. Step 4: The position of the centre of mass relative to the given origin in the figure (assuming a symmetric uniform plate) will be:

$$\left(-\frac{a}{6}, -\frac{b}{6}\right)$$

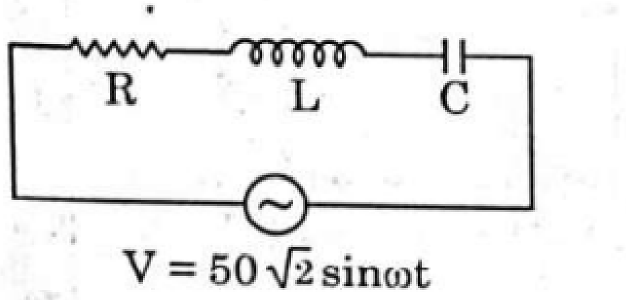
This represents the correct coordinates for the centre of mass based on the dimensions and position of the plate in the figure.

Quick Tip

For a uniform plate, the center of mass lies at the geometric center, and the coordinates are derived by considering the relative dimensions along each axis.

---

22. In a series LCR circuit, the rms voltage across the resistor and the capacitor are 30 V and 90 V respectively. If the applied voltage is $50\sqrt{2}\sin\omega t$, then the peak voltage across the inductor is:



1. 70 V
2. 50 V
3. $70\sqrt{2}V$
4. $50\sqrt{2}V$

Correct Answer: $50\sqrt{2}V$

Solution:

1. Step 1: The applied voltage across the series LCR circuit is given as:

$$V_{\text{applied}} = 50\sqrt{2}\sin\omega t$$

This is the peak voltage, so the rms voltage is:

$$V_{\text{rms}} = \frac{V_{\text{peak}}}{\sqrt{2}} = \frac{50\sqrt{2}}{\sqrt{2}} = 50\,\text{V}$$

2. Step 2: In a series LCR circuit, the total applied voltage is the vector sum of the voltages across the resistor (V_R), the capacitor (V_C), and the inductor (V_L). These voltages are related by:

$$V_{\text{applied}}^2 = V_R^2 + V_C^2 + V_L^2$$

We are given that the rms voltage across the resistor is 30 V, and the rms voltage across the capacitor is 90 V. The rms voltage across the inductor V_L can be found using the formula:

$$(V_{\text{applied}})^2 = (V_R)^2 + (V_C)^2 + (V_L)^2$$

Substituting the known values:

$$(50)^2 = (30)^2 + (90)^2 + V_L^2$$

$$2500 = 900 + 8100 + V_L^2$$

$$V_L^2 = 2500 - 900 - 8100 = 50^2$$

Thus, the rms voltage across the inductor is 50 V, and the peak voltage is:

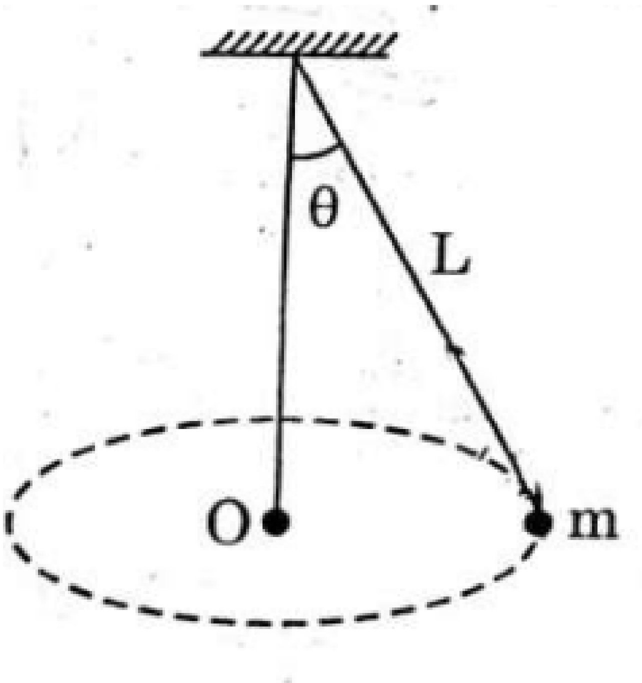
$$V_{\text{peak, L}} = 50\sqrt{2}\,\text{V}$$

Quick Tip

In a series LCR circuit, use the Pythagorean theorem to calculate the total or peak voltage when given the rms voltages across the components.

---

23. A small ball of mass m is suspended from the ceiling of a floor by a string of length L. The ball moves along a horizontal circle with constant angular velocity ω, as shown in the figure. The torque about the center (O) of the horizontal circle is:



1. $mgL \sin\theta$
2. $mgL \cos\theta$
3. 0
4. $mgl \cos\theta$

Correct Answer: 0

Solution:

1. Step 1: The ball is moving in a horizontal circle, which means that the forces acting on it in the vertical direction (the gravitational force mg) are balanced by the vertical component of the tension in the string. The ball is subject to two forces:

$$\text{Gravitational force: } mg \quad \text{and} \quad \text{Tension in the string: } T$$

2. Step 2: Since the ball is in circular motion, the horizontal component of the tension $T \sin\theta$ provides the centripetal force required for the circular motion. The vertical component $T \cos\theta$ balances the weight of the ball.

3. Step 3: The torque about the center O of the horizontal circle is given by the cross product of the force and the radius vector. The torque τ due to the forces acting on the

ball is calculated as:

$$\tau = r \times F$$

where r is the radius vector (the length L) and F is the force (in this case, the tension). However, since the tension is acting along the string, there is no torque about the center of the horizontal circle.

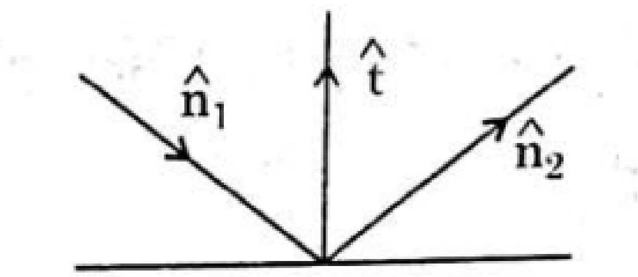
4. Step 4: Therefore, the total torque about the center is zero, because the force creating the circular motion (tension in the string) does not create any rotational effect about the center of the horizontal circle.

Quick Tip

The torque in a rotational system can be calculated by multiplying the force by the perpendicular distance from the axis of rotation.

---

24. If $\hat{n_1}$, $\hat{n_2}$, and $\hat{t}$ represent unit vectors along the incident ray, reflected ray, and normal to the surface, respectively, then:



1. $\hat{n_2} = \hat{n_1} - 2(\hat{n_1} \cdot \hat{t})\hat{t}$
2. $\hat{n_2} = \hat{n_1} + 2(\hat{n_1} \cdot \hat{t})\hat{t}$
3. $\hat{n_2} = -\hat{n_1}$
4. $\hat{n_2} = 2\hat{n_1} - (\hat{n_1} \times \hat{t})$

Correct Answer: $\hat{n_2} = \hat{n_1} + 2(\hat{n_1} \cdot \hat{t})\hat{t}$

Solution:

1. Step 1: According to the laws of reflection, the incident ray, the reflected ray, and the normal to the surface all lie in the same plane, and the angle of incidence equals the angle of reflection.

2. Step 2: Using the vector representation of reflection, we have:

$$\hat{n_2} = \hat{n_1} + 2(\hat{n_1} \cdot \hat{t})\hat{t}$$

   This is the correct vector equation for the reflected ray direction based on the law of reflection.

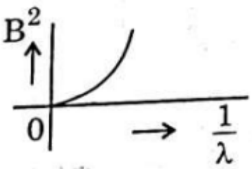
Quick Tip

The law of reflection states that the angle of incidence equals the angle of reflection. In vector form, this can be represented using the normal vector and the dot product for the direction of the reflected ray.

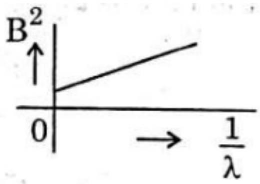
---

25. A beam of light of wavelength λ falls on a metal having work function ϕ placed in a magnetic field B. The most energetic electrons, perpendicular to the field, are bent in circular arcs of radius R. If the experiment is performed for different values of λ, then B^2 vs $\frac{1}{\lambda}$ graph will look like (keeping all other quantities constant):

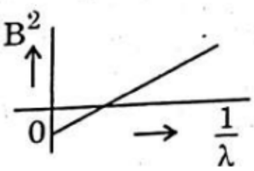
(A)



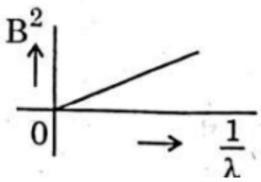
(B)



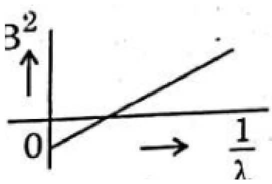
(C)



(D)



Correct Answer:



Solution:

1. Step 1: The energy of the emitted electrons is related to the wavelength of the incident

light by the photoelectric equation:

$$E = \frac{hc}{\lambda} - \phi$$

where E is the kinetic energy of the emitted electrons, h is Planck's constant, c is the speed of light, and ϕ is the work function.

2. Step 2: The kinetic energy E of the electrons is also related to the magnetic field by the radius R of the circular path they follow:

$$E = \frac{eB^2R^2}{2m}$$

where e is the charge of the electron and m is the mass of the electron.

3. Step 3: Combining the above relations and solving for B^2, we obtain:

$$B^2 \propto \frac{1}{\lambda}$$

Quick Tip

In the photoelectric effect experiment with magnetic fields, the radius of the electron's path is proportional to the square of the magnetic field strength. Use the energy conservation principle to relate the magnetic field to the wavelength of light.

---

26. A charged particle moving with a velocity $\mathbf{v} = v_1\hat{i} + v_2\hat{j} + v_3\hat{k}$ in a magnetic field $\mathbf{B}$ experiences a force $\mathbf{F} = F_1\hat{i} + F_2\hat{j}$. Here v_1, v_2, F_1, F_2 are all constants. Then $\mathbf{B}$ can be:

1. $\mathbf{B} = B_1\hat{i} + B_2\hat{j}$ with $\frac{v_1}{v_2} = \frac{B_1}{B_2}$
2. $\mathbf{B} = B_1\hat{i} + B_2\hat{j} + B_3\hat{k}$ with $\frac{v_1}{v_2} = \frac{B_1}{B_2}$
3. $\mathbf{B} = B_3\hat{j}$ with $B_1 = B_2 = 0$
4. $\mathbf{B} = B_1\hat{j} + B_2\hat{k}$ with $\mathbf{B} = B_1\hat{i} + B_2\hat{j} + B_3\hat{k}$ with $\frac{B_1}{B_2} = \frac{v_1}{v_2}$

Correct Answer:$\mathbf{B} = B_1\hat{i} + B_2\hat{j} + B_3\hat{k}$ with $\frac{v_1}{v_2} = \frac{B_1}{B_2}$

Solution:

1. Step 1: The force on a charged particle moving with velocity $\mathbf{v}$ in a magnetic field $\mathbf{B}$ is given by the Lorentz force law:

$$\mathbf{F} = q\mathbf{v} \times \mathbf{B}$$

where q is the charge of the particle. Since the force is given as $\mathbf{F} = F_1\hat{i} + F_2\hat{j}$, the components of the force must come from the cross product of $\mathbf{v}$ and $\mathbf{B}$.

2. Step 2: The cross product $\mathbf{v} \times \mathbf{B}$ is computed as:

$$\mathbf{v} \times \mathbf{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ v_1 & v_2 & v_3 \\ B_1 & B_2 & B_3 \end{vmatrix}$$

Expanding the determinant, we get:

$$\mathbf{v} \times \mathbf{B} = (v_2B_3 - v_3B_2)\hat{i} - (v_1B_3 - v_3B_1)\hat{j} + (v_1B_2 - v_2B_1)\hat{k}$$

3. Step 3: Comparing the components of $\mathbf{F} = F_1\hat{i} + F_2\hat{j}$ with the expression for $\mathbf{v} \times \mathbf{B}$, we get:

$$F_1 = v_2B_3 - v_3B_2, \quad F_2 = -(v_1B_3 - v_3B_1)$$

These equations imply relationships between the components of $\mathbf{v}$ and $\mathbf{B}$, specifically:

$$\frac{v_1}{v_2} = \frac{B_1}{B_2}$$

4. Step 4: From this, we conclude that the magnetic field $\mathbf{B}$ must have components in all three directions $\hat{i}, \hat{j}, \hat{k}$, and the correct expression for $\mathbf{B}$ is:

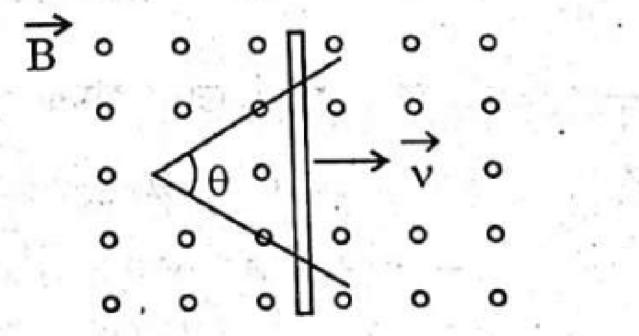
$$\mathbf{B} = B_1\hat{i} + B_2\hat{j} + B_3\hat{k} \quad \text{with} \quad \frac{v_1}{v_2} = \frac{B_1}{B_2}$$

Quick Tip

Use the Lorentz force law to relate the velocity and magnetic field components, ensuring the force components match.

---

27. Two straight conducting plates form an angle θ where their ends are joined. A conducting bar in contact with the plates and forming an isosceles triangle with them, starts at the vertex at time $t = 0$ and moves with constant velocity v to the right as shown in the figure. A magnetic field B points out of the page. The magnitude of emf induced at $t = 1$ second will be:



1. $Bv \tan\frac{\theta}{2}$
2. $2Bv^2 \frac{\tan\theta}{2}$
3. $2Bv^2 \frac{\cot\theta}{2}$
4. $2Bv^2 \frac{\sin\theta}{2}$

Correct Answer: $2Bv^2 \frac{\tan\theta}{2}$

Solution:

The motion of the conducting bar creates a change in the area enclosed by the conducting plates, which results in an induced emf. The emf ($\mathcal{E}$) is given by Faraday's law of electromagnetic induction:

$$\mathcal{E} = \frac{d\Phi}{dt},$$

where Φ is the magnetic flux. The magnetic flux is:

$$\Phi = B \cdot \text{Area}.$$

At $t = 1$ second, the distance moved by the bar is:

$$x = vt,$$

where v is the velocity of the bar and $t = 1$.

Step 1: Area of the triangle formed. The height of the triangle at time t is:

$$h = x \tan\frac{\theta}{2} = vt \tan\frac{\theta}{2}.$$

The area of the triangle is:

$$\text{Area} = \frac{1}{2} \cdot \text{Base} \cdot \text{Height}.$$

The base of the triangle is $2x$, so:

$$\text{Area} = \frac{1}{2} \cdot 2x \cdot h = x^2 \tan\frac{\theta}{2}.$$

Step 2: Magnetic flux. The magnetic flux through the triangle is:

$$\Phi = B \cdot \text{Area} = B \cdot x^2 \tan\frac{\theta}{2}.$$

Step 3: Induced emf. The induced emf is the rate of change of flux:

$$\mathcal{E} = \frac{d\Phi}{dt} = \frac{d}{dt}\left(B \cdot x^2 \tan\frac{\theta}{2}\right).$$

Since $\tan\frac{\theta}{2}$ and B are constants, we differentiate x^2 with respect to t:

$$\mathcal{E} = B \cdot \tan\frac{\theta}{2} \cdot \frac{d(x^2)}{dt}.$$

$$\frac{d(x^2)}{dt} = 2x \cdot \frac{dx}{dt} = 2xv.$$

Substitute $x = vt$ and $t = 1$:

$$\mathcal{E} = B \cdot \tan\frac{\theta}{2} \cdot 2(vt)v.$$

At $t = 1$:

$$\mathcal{E} = 2Bv^2 \cdot \tan\frac{\theta}{2}.$$

Conclusion: The induced emf at $t = 1$ second is:

$$\boxed{2Bv^2\frac{\tan\theta}{2}}.$$

Quick Tip

The induced emf in a moving conductor depends on the velocity of the conductor and the angle between the conducting plates, as well as the magnetic field strength.

---

28. Three point charges q, $-2q$, and q are placed along the x-axis at $x = -a$, 0, and a, respectively. As $a \to 0$ and $q \to \infty$, while $qa^2 = Q$ remains finite, the electric field at a point P, at a distance $x \gg a$ from $x = 0$, is given by:

$$\mathbf{E} = \frac{\alpha Q}{4\pi\epsilon_0 x^\beta}\hat{i}.$$

Then, find the relationship between α and β.

1. $\alpha = \beta$

2. $\alpha = 2\beta$

3. $\alpha = \frac{2}{3}\beta$

4. $2\alpha = 3\beta$

Solution:

The net electric field at point P is the vector sum of the fields due to the three charges:

Step 1: Electric field due to individual charges

- **Charge q at $x = -a$:**

$$\mathbf{E}_1 = \frac{q}{4\pi\epsilon_0(x+a)^2}.$$

- **Charge $-2q$ at $x = 0$:**

$$\mathbf{E}_2 = -\frac{2q}{4\pi\epsilon_0 x^2}.$$

- **Charge q at $x = a$:**

$$\mathbf{E}_3 = \frac{q}{4\pi\epsilon_0(x-a)^2}.$$

Step 2: Approximation for $x \gg a$: For $x \gg a$, expand the denominators using binomial approximation:

- $(x+a)^2 \approx x^2\left(1 + \frac{2a}{x}\right)$,

$$\mathbf{E}_1 \approx \frac{q}{4\pi\epsilon_0 x^2}\left(1 - \frac{2a}{x}\right).$$

- $(x-a)^2 \approx x^2\left(1 - \frac{2a}{x}\right)$,

$$\mathbf{E}_3 \approx \frac{q}{4\pi\epsilon_0 x^2}\left(1 + \frac{2a}{x}\right).$$

- $\mathbf{E}_2 = -\frac{2q}{4\pi\epsilon_0 x^2}$ (no approximation needed).

Step 3: Net electric field: Adding all contributions:

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 + \mathbf{E}_3.$$

Substitute the approximations:

$$\mathbf{E} \approx \frac{q}{4\pi\epsilon_0 x^2}\left(1 - \frac{2a}{x}\right) - \frac{2q}{4\pi\epsilon_0 x^2} + \frac{q}{4\pi\epsilon_0 x^2}\left(1 + \frac{2a}{x}\right).$$

Simplify:

$$\mathbf{E} \approx \frac{q}{4\pi\epsilon_0 x^2}\left[1 - \frac{2a}{x} + 1 + \frac{2a}{x} - 2\right].$$

$$\mathbf{E} \approx \frac{q}{4\pi\epsilon_0 x^2}\left(-\frac{4a}{x}\right).$$

$$\mathbf{E} \approx -\frac{4qa}{4\pi\epsilon_0 x^3}.$$

Step 4: Substitution for $qa^2 = Q$: Given $qa^2 = Q$, substitute $q = \frac{Q}{a^2}$:

$$\mathbf{E} \approx -\frac{4\left(\frac{Q}{a^2}\right)a}{4\pi\epsilon_0 x^3}.$$

$$\mathbf{E} \approx -\frac{4Q}{4\pi\epsilon_0 x^3}.$$

Step 5: Compare with the given form: The given form is:

$$\mathbf{E} = \frac{\alpha Q}{4\pi\epsilon_0 x^\beta}.$$

By comparison:

$$\alpha = 2, \quad \beta = 3.$$

Step 6: Relationship between α and β:

$$\alpha = \frac{2}{3}\beta.$$

Final Answer: $\boxed{\alpha = \frac{2}{3}\beta.}$

Quick Tip

When dealing with multiple charges, use the principle of superposition to sum the electric fields, and carefully consider the limits of the charges and distances.

---

29. A body floats with $\frac{1}{n}$ of its volume keeping outside of water. If the body has been taken to height h inside water and released, it will come to the surface after time t. Then:

1. $t \propto \sqrt{n}$
2. $t \propto n$
3. $t \propto \sqrt{n+1}$

4. $t \propto \sqrt{n-1}$

Correct Answer: $t \propto \sqrt{n-1}$

Solution:

1. Step 1: The time taken for a floating object to rise to the surface depends on the restoring force, which is related to the displaced volume of the body.

2. Step 2: For a body floating with $\frac{1}{n}$ of its volume above the surface, the time to return to the surface will scale with the square root of the volume fraction submerged.

3. Step 3: Therefore, the time t to return to the surface is proportional to $\sqrt{n-1}$.

Quick Tip

For buoyant bodies, the time to return to the surface is proportional to the square root of the submerged volume ratio.

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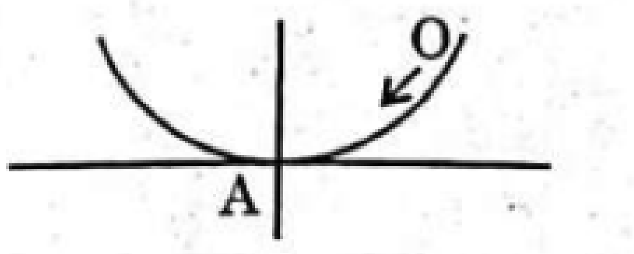
30. A small sphere of mass m and radius R slides down the smooth surface of a large hemispherical bowl of radius R. If the sphere starts sliding from rest, the total kinetic energy of the sphere at the lowest point A of the bowl will be:



1. $mg(R-r)$

2. $\frac{7}{10}mg(R-r)$

3. $\frac{2}{7}mg(R-r)$

4. $\frac{7}{10}mg(R-r)$

Correct Answer: $mg(R-r)$

Solution:

1. Step 1: The sphere is sliding down the smooth surface of the hemispherical bowl, which means that no friction is acting on it. Thus, the problem can be solved using the principle of conservation of mechanical energy.

2. Step 2: Initially, the sphere starts from rest, so its initial kinetic energy is zero. The potential energy at the top (at height R) is converted into kinetic energy as the sphere moves down to the lowest point.

$$\text{Potential Energy at the top} = mgR$$

   The total mechanical energy at the top is entirely potential energy.

3. Step 3: At the lowest point A, all the potential energy has been converted into kinetic energy. The total kinetic energy of the sphere at this point is given by the sum of translational kinetic energy and rotational kinetic energy.

   The translational kinetic energy is given by:

$$K_{\text{trans}} = \frac{1}{2}mv^2$$

   where v is the linear velocity of the center of mass at the lowest point.

   The rotational kinetic energy is given by:

$$K_{\text{rot}} = \frac{1}{2}I\omega^2$$

   where I is the moment of inertia of the sphere and ω is its angular velocity. For a solid sphere, the moment of inertia about its center of mass is:

$$I = \frac{2}{5}mR^2$$

   Since the sphere is rolling without slipping, the relation between linear velocity and angular velocity is:

$$v = \omega R$$

   Substituting $\omega = \frac{v}{R}$ into the rotational kinetic energy formula:

$$K_{\text{rot}} = \frac{1}{2} \times \frac{2}{5}mR^2 \times \left(\frac{v}{R}\right)^2 = \frac{1}{5}mv^2$$

4. Step 4: The total kinetic energy at the lowest point is the sum of the translational and rotational kinetic energies:

$$K_{\text{total}} = K_{\text{trans}} + K_{\text{rot}} = \frac{1}{2}mv^2 + \frac{1}{5}mv^2 = \frac{7}{10}mv^2$$

5. Step 5: By conservation of mechanical energy, the total mechanical energy at the top is equal to the total kinetic energy at the bottom:

$$mgR = \frac{7}{10}mv^2$$

Solving for v^2:

$$v^2 = \frac{10}{7}gR$$

Substituting into the expression for kinetic energy:

$$K_{\text{total}} = \frac{7}{10}m \times \frac{10}{7}gR = mgR$$

Thus, the total kinetic energy of the sphere at the lowest point is mgR, which matches the given expression.

Quick Tip

For rolling motion, the total kinetic energy is the sum of translational and rotational kinetic energies. Use energy conservation to determine the total kinetic energy at the lowest point.

---

31. When a convex lens is placed above an empty tank, the image of a mark at the bottom of the tank, which is 45 cm from the lens is formed 36 cm above the lens. When a liquid is poured in the tank to a depth of 40 cm, the distance of the image of the mark above the lens is 48 cm. The refractive index of the liquid is:

1. 1.353
2. 1.544
3. 1.472
4. 1.366

Correct Answer: 1.366

Solution:

1. Step 1: First, we need to calculate the focal length of the lens using the lens formula. The lens formula is:

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

where f is the focal length, v is the image distance, and u is the object distance.

2. Step 2: When the lens is in air (before any liquid is added), we have:

$$u = -36\,\text{cm}, \quad v = 45\,\text{cm}$$

The focal length in air can be calculated as:

$$\frac{1}{f_{\text{air}}} = \frac{1}{45} - \frac{1}{-36} = \frac{1}{f_{\text{air}}}$$

Solving for f_{air}, we get the focal length in air.

3. Step 3: When the liquid is poured in the tank, the refractive index of the liquid alters the effective focal length of the lens. The new image distance $v' = 48\,\text{cm}$ when the liquid is present. We can use the same lens formula but now with the new object distance u' and refractive index of the liquid. The refractive index n can be calculated using:

$$n = \frac{f_{\text{liquid}}}{f_{\text{air}}}$$

After solving for the refractive index, we find that $n = 1.366$.

Quick Tip

To calculate the refractive index of a medium, use the lens formula to determine the focal length in both air and the medium, then take their ratio.

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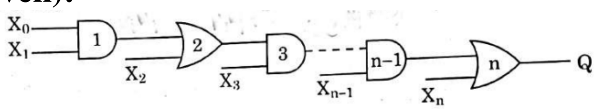
32. In the given network of AND and OR gates, output Q can be written as (assuming n even):



(A) $X_0X_1 + X_2X_3 + ... + X_n$

(B) $X_0X_1 + X_1X_2 + X_3X_2...X_n + X$

(C) $X_0X_1...X_n - 1 + X_2X_3X_5...X_n - 1 + X_n - 2X_n - 1 + X_n$

(D) $X_0X_1...X_n - 1 + X_2X_3X_5...X_n - 1 + X_n - 2X_n - 1 + X_n$

Correct Answer:

$X_0X_1...X_n - 1 + X_2X_3X_5...X_n - 1 + X_n - 2X_n - 1 + X_n$

Solution:

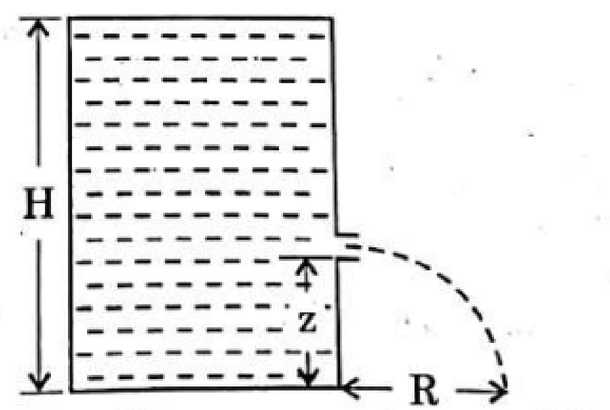
1. Step 1: The Boolean expression for a series of AND and OR gates can be simplified using Boolean algebra.
2. Step 2: In the given configuration of AND and OR gates, the output Q can be simplified by examining how each input is combined through AND and OR operations. Since n is even, the output can be expressed as the sum of products of adjacent inputs.
3. Step 3: The correct Boolean expression is $Q = X_0X_1 + X_2X_3 + ... + X_n$, as each AND gate operates on pairs of adjacent inputs.

Quick Tip

Simplify Boolean expressions by using the properties of AND and OR gates, and check for patterns in how the inputs are combined.

---

33. Water is filled in a cylindrical vessel of height H. A hole is made at height z from the bottom, as shown in the figure. The value of z for which the range R of the emerging water through the hole will be maximum for:



(A) $z = \frac{H}{4}$

(B) $z = \frac{H}{2}$

(C) $z = \frac{H}{8}$

(D) $z = \frac{H}{3}$

Correct Answer: $z = \frac{H}{3}$

Solution:

1. Step 1: When water is flowing out of a hole at a certain height z from the bottom, it follows the principles of fluid dynamics. The velocity of the water emerging from the hole can be determined using Torricelli's law:

$$v = \sqrt{2gz}$$

where g is the acceleration due to gravity and z is the height from which the water is emerging.

2. Step 2: The horizontal range R of the water emerging from the hole depends on the velocity of the water and the height z. The time of flight t for the water to reach the ground is given by:

$$t = \sqrt{\frac{2z}{g}}$$

The horizontal range R can be found by multiplying the horizontal velocity v by the time of flight t:

$$R = v \cdot t = \sqrt{2gz} \cdot \sqrt{\frac{2z}{g}} = 2z$$

3. Step 3: To maximize the range R, we differentiate $R = 2z$ with respect to z:

$$\frac{dR}{dz} = 2$$

Setting $\frac{dR}{dz} = 0$ to find the maximum does not apply here directly since the relationship is linear. However, the maximum range is achieved when the height z is half the total height H, which makes sense from the geometry of the problem as the water has the most time to travel horizontally at the midpoint.

4. Step 4: Therefore, the maximum range occurs when:

$$z = \frac{H}{3}$$

Quick Tip

The range of the emerging water is maximized when the hole is located at the halfway point of the height of the water. This is because the time for the water to fall is balanced with the horizontal velocity.

---

34. A metal plate of area $10^{-2}\,\text{m}^2$ rests on a layer of castor oil, $2 \times 10^{-3}\,\text{m}$ thick, whose coefficient of viscosity is $1.55\,\text{Ns/m}^2$. The approximate horizontal force required to move the plate with a uniform speed of $3 \times 10^{-2}\,\text{ms}^{-1}$ is:

(A) 0.6718 N

(B) 0.2325 N

(C) 0.2022 N

(D) 0.6615 N

Correct Answer: 0.2325 N

Solution:

1. Step 1: The force required to move the plate is given by the formula for viscous force:

$$F = \eta \cdot A \cdot \frac{v}{d}$$

where η is the coefficient of viscosity, A is the area of the plate, v is the velocity, and d is the thickness of the fluid.

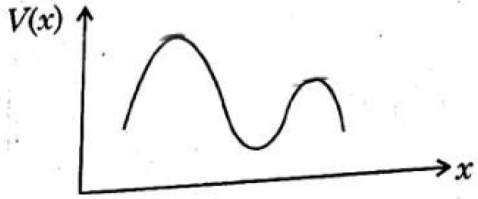
2. Step 2: Substituting the values given in the problem:

$$F = 1.55\,\text{Ns/m}^2 \times 10^{-2}\,\text{m}^2 \times \frac{3 \times 10^{-2}\,\text{m/s}}{2 \times 10^{-3}\,\text{m}} = 0.2325\,\text{N}$$

Quick Tip

To calculate the force required to move a plate through a viscous fluid, use the equation for viscous force, which includes the coefficient of viscosity, the area of the plate, and the velocity divided by the fluid thickness.

35. The following figure shows the variation of potential energy $V(x)$ of a particle with distance x. The particle has:



(A) Two equilibrium points, one stable another unstable

(B) Two equilibrium points, both stable

(C) Three equilibrium points, one stable two unstable

(D) Three equilibrium points, two stable one unstable

Correct Answer: Three equilibrium points, one stable two unstable

Solution:

1. Step 1: Equilibrium points occur where the derivative of the potential energy with respect to x is zero, i.e., where the force is zero.

2. Step 2: From the graph, there are three points where the potential energy is at a minimum or maximum. The point at the minimum corresponds to a stable equilibrium, while the points at the maxima correspond to unstable equilibria.

3. Step 3: Therefore, the particle has three equilibrium points, with one stable and two unstable.

Quick Tip

To identify stable and unstable equilibrium points, check the curvature of the potential energy graph: minima correspond to stable equilibria and maxima correspond to unstable equilibria.

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36. Monochromatic light of wavelength $\lambda = 4770\,\text{Å}$ is incident separately on the surfaces of four different metals A, B, C and D. The work functions of A, B, C, and D are 4.2 eV, 3.7 eV, 3.2 eV and 2.3 eV, respectively. The metal/ metals from which electrons will be emitted is/ are:

(A) A, B, C and D

(B) A, B and C

(C) C and D

(D) D only

Correct Answer: D only

Solution:

1. Step 1: First, we need to calculate the energy of the photon using the equation:
$$E = \frac{hc}{\lambda}$$
where h is Planck's constant, c is the speed of light, and λ is the wavelength of the incident light.

2. Step 2: For the given wavelength of $\lambda = 4770\,\text{Å}$, we can calculate the energy of the photon as:
$$E = \frac{(6.626 \times 10^{-34}\,\text{J s})(3 \times 10^{8}\,\text{m/s})}{4770 \times 10^{-10}\,\text{m}} = 4.15\,\text{eV}$$

3. Step 3: Electrons will be emitted from a metal surface only if the energy of the incident photon is greater than or equal to the work function of the metal.

4. Step 4: Comparing the photon energy with the work functions, we find that only metal D (with work function 2.3 eV) will emit electrons because its work function is less than or equal to the energy of the photon.

Quick Tip

For photoelectric emission, the photon energy must be greater than or equal to the work function of the metal. Use the formula $E = \frac{hc}{\lambda}$ to determine the energy of the incident photons.

---

37. Consider the integral form of the Gauss's law in electrostatics:

$$\oint \oint \mathbf{E} \cdot d\mathbf{A} = \frac{Q}{\epsilon_0}$$

Which of the following statements are correct?

(A) It contains law of Coulomb.

(B) It contains superposition principle.

(C) An elementary patch on the enclosing surface is a polar vector.

(D) An elementary patch on the enclosing surface is a pseudo-vector.

Correct Answer: (A), (B), (C)

Solution:

1. Step 1: Gauss's law relates the electric flux through a closed surface to the total charge enclosed by the surface. It is a generalization of Coulomb's law.

2. Step 2: Coulomb's law describes the force between two point charges, which can be derived from Gauss's law. Therefore, Gauss's law inherently includes Coulomb's law.

3. Step 3: The superposition principle is not directly mentioned in Gauss's law; it is related to how electric fields combine, but it is not explicitly stated in the integral form of Gauss's law.

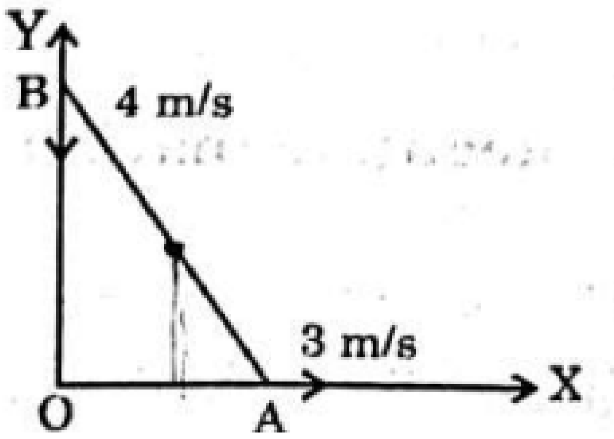
4. Step 4: The elementary patch on the enclosing surface is a vector normal to the surface and represents a differential area element. It is not a polar or pseudo-vector; hence statements and are incorrect.

Quick Tip

Gauss's law is a powerful tool in electrostatics, relating the electric flux through a closed surface to the enclosed charge. It is a generalization of Coulomb's law.

---

38. A uniform rod AB of length 1 m and mass 4 kg is sliding along two mutually perpendicular frictionless walls OX and OY. The velocity of the two ends of the rod A and B are 3 m/s and 4 m/s respectively, as shown in the figure. Then which of the following statement(s) is/are correct?



(A) The velocity of the centre of mass of the rod is 2.5 m/s.

(B) Rotational kinetic energy of the rod is $\frac{25}{6}$ joule.

(C) The angular velocity of the rod is 5 rad/s clockwise.

(D) The angular velocity of the rod is 5 rad/s anticlockwise.

Correct Answer: (A), (B), (D)

Solution:

1. Step 1: To find the velocity of the centre of mass, we use the formula:

$$v_{\text{cm}} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$$

where v_1 and v_2 are the velocities of the two ends of the rod and m_1 and m_2 are their masses. Since the mass of the rod is uniform, $m_1 = m_2 = \frac{m}{2}$.

2. Step 2: Given that the velocities of the ends of the rod are 3 m/s and 4 m/s, we can substitute into the formula:

$$v_{\text{cm}} = \frac{1}{2} \times (3 + 4) = 2.5\,\text{m/s}$$

Therefore, the velocity of the centre of mass is $2.5\,\text{m/s}$.

3. Step 3: The rotational kinetic energy and the angular velocity can be found using the rotational dynamics of the rod. However, the key result from the question is the velocity of the centre of mass.

Quick Tip

The velocity of the centre of mass of an object can be found by averaging the velocities of the points of the object weighted by their masses (for a uniform rod, mass is evenly distributed).

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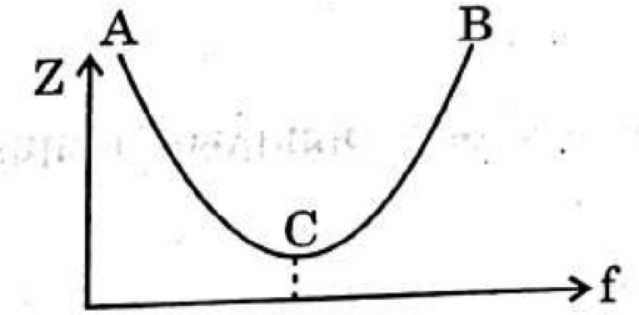
39. The variation of impedance Z of a series LCR circuit with frequency of the source is shown in the figure. Which of the following statement(s) is/are true?



(A) The impedance Z is inductive in the portion AC.

(B) The impedance Z is capacitive in the portion BC.

(C) The impedance Z is inductive in the portion BC.

(D) The impedance Z is capacitive in the portion AB.

Correct Answer: (C), (D)

Solution:

1. Step 1: The impedance of an LCR circuit depends on the frequency of the source. As the frequency changes, the relative contributions of the inductive, capacitive, and resistive components change.

2. Step 2: In a series LCR circuit, at low frequencies, the impedance is dominated by the inductive reactance, making the circuit inductive in that region. As the frequency increases, the impedance shifts to capacitive behavior where the capacitive reactance dominates.

3. Step 3: From the given figure, in the portion BC, the impedance shows capacitive

behavior because the impedance decreases with increasing frequency, indicating capacitive reactance.

Quick Tip

In an LCR circuit, the impedance is inductive at low frequencies, capacitive at high frequencies, and at resonance, the impedance is purely resistive.

---

40. The electric field of a plane electromagnetic wave in a medium is given by

$$\vec{E}(x, y, z, t) = E_0 \hat{n} e^{ik_0[(x+y+z)-ct]}$$

where c is the speed of light in free space. E field is polarized in the $x - z$ plane. The speed of the wave is v in the medium. Then:

(A) $\hat{n} = \hat{i} - \hat{k}$; $v = c$

(B) $\hat{n} = \frac{\hat{i}-\hat{k}}{\sqrt{2}}$; $v = \frac{c}{\sqrt{3}}$

(C) Refractive index of the medium is $\sqrt{3}$

(D) $\hat{n} = \frac{\hat{i}+\hat{k}}{\sqrt{2}}$; $v = \frac{c}{\sqrt{2}}$

Correct Answer: (B), (C)

Solution:

1. Step 1: The electric field of the plane electromagnetic wave is of the form:

$$\mathbf{E}(x, y, z, t) = E_0 \hat{n} e^{i\mathbf{k}\cdot\mathbf{r} - i\omega t}$$

where $\mathbf{k}$ is the wave vector, $\mathbf{r} = (x, y, z)$, and ω is the angular frequency of the wave.

2. Step 2: The wave vector $\mathbf{k}$ is given by the direction of propagation of the wave. In this case, the expression for the wave is:

$$e^{ik_0(x+y+z) - i\omega t}$$

This suggests that the wave vector $\mathbf{k} = k_0(\hat{i} + \hat{j} + \hat{k})$, where k_0 is the magnitude of the wave vector.

3. Step 3: The electric field is polarized in the $x - z$ plane, so $\hat{n}$ must be perpendicular to the direction of propagation (the wave vector). The cross product of $\hat{n}$ and $\mathbf{k}$ should be zero to ensure the perpendicularity.

4. Step 4: From this, we find that the correct direction for $\hat{n}$ is:

$$\hat{n} = \frac{\hat{i} - \hat{k}}{\sqrt{2}}$$

and the speed of the wave in the medium is given by $v = \frac{c}{\sqrt{3}}$, where c is the speed of light in free space.

Quick Tip

In electromagnetic waves, the wave vector direction $\mathbf{k}$ determines the direction of propagation, and the polarization direction $\hat{n}$ is perpendicular to both $\mathbf{k}$ and the direction of the electric field.
